Chapter 2-5

A COMPARATIVE STUDY OF THE MATHEMATICS TEXTBOOKS OF CHINA, ENGLAND, JAPAN, KOREA, AND THE UNITED STATES

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1. INTRODUCTION

The purpose of this study is to compare the eighth-grade mathematics textbooks of China, Japan and Korea on the one hand, and those of England and the United States on the other, and to explore the implications for mathematics education in East Asia and the West.

There have been a number of international studies in mathematics education such as TIMSS and IAEP where the mathematics curricula (and student achievements) in various countries were compared (Beaton *et al,* 1996; Fuson *et al*, 1988; Mullis *et al,* 1997, 2000, Lapointe *et al*, 1992). Such "macro" comparison usually focus on the curriculum or curricula that are followed by the majority of the student population, and hence by their very nature tend to be crude. On the other hand, classroom studies (e.g. Leung, 1995) typically focus on activities led by a single or a handful of teachers out of tens of thousands in the population (the TIMSS Video Survey (Stigler and Hiebert, 1999) may be an exception), and is by their nature a "micro" approach. But for textbook studies, since each country rarely has a very large number of mathematics textbooks, if we select a popular series of textbooks in each country to study, as is the practice of the present study, it will cover a fair amount of the student population already. So textbook studies are somewhere between a macro and a micro approach in terms of scale.

The present study is based on an analysis of a series of eighth-grade mathematics textbooks in each of China (People's Education Press, 1999), Japan (Fukumori, 1997), Korea (Kim and Kim, 1997), England (Briggs, 1996; McGuire, P. and Smith, K. 1996), and the United States (Billstein and Williamson, 1999). Each of the East Asian series has the largest market share in its respective country. The English and American series are also popular ones, but since there is such a large number of textbooks with different market shares, no single textbook can be said to be dominant.

2. MAJOR FINDINGS

2.1 Textbook development and publication policies: u*niformity vs. diversity*

Textbook development and publication policies differ from country to country. In each of China, Japan and Korea, there is a uniform curriculum determined at the national level. In China, before 1992, there was only one series of textbooks published by the People's Education Press, the official publisher appointed by the Government, for the entire country. From 1992, selected publishers in different provinces have been commissioned to publish textbooks that serve different needs.

The textbooks for elementary schools in Korea are developed and published by the Ministry of Education, and there is only one series of mathematics textbooks in the country. The textbooks for the secondary schools in Korea and all the textbooks in Japan are published by private companies upon obtaining prior approval from the Ministry of Education. Not only does the content reflect closely the national curriculum, but the terms and symbols used mostly adhere to the curriculum as well. The Ministry of Education even stipulates the size, the number of pages, and the number of colors to be used in those privately published textbooks. Thus, textbooks in Korea and Japan look more or less identical.

On the other hand, the approval processes by the national educational authorities of England and the United States for commercially published textbooks are less strict than those in Korea and Japan. Publishing companies exercise their own discretion and publish a broad spectrum of textbooks, ranging from "conservative" (e.g. Saxon Publishers of the United States publishes traditional textbooks under the principle of "methods of incremental development and continual practice and review (http://www2. Philosophy.html saxonpub.com/corp)) to "radical" (e.g. the *Contemporary*

Mathematics in Context, consequential on the Core-plus Mathematics Project, with content structured according to themes rather than mathematical domains) (cf. Stevenson and Stigler, 1992, pp.138-9).

2.2 Choice of content: essential vs. discretionary

An obvious difference between the textbooks in the two Western countries and the East Asian countries is that there are more elective elements in the Western textbooks. This is of course just a reflection of the differences in the curricula of the respective countries mentioned above. In the East Asian countries, students, irrespective of their abilities and inclinations, are expected to follow the same curriculum and to learn the same content. In Western countries, there are often a number of different curricula to cater for the different needs and interests of the students. Even within the same curriculum, there is more flexibility built in so that teachers and students are presented with more choices in the teaching and learning. These different emphases are reflected in the Western textbooks where there are either different contents catering for students of different abilities or ample elective elements for teachers and students to choose from. For example, the textbooks of England's Oxford University Press are published in three different strands (higher, intermediate and foundation) to accommodate various levels of individual ability. And in the American textbook, repetitive and exploratory activities are given to help the lower ability students. At the same time, there are higher-level problems to challenge and motivate the more able ones.

As a result of this difference, the American and English textbooks are larger and thicker than the East Asian ones (see Table 1). This is also a consequence of the fact that the contents in the East Asian textbooks are introduced in a compressed way. In Asian countries, textbooks are regarded as a body of the minimum and essential knowledge that everyone must learn and understand. However, in Western countries, textbooks are rather like "little encyclopaedia" that contain numerous and various contents, from which appropriate topics are selected as desired.

This difference is related to the typical class size and teaching practice in the two cultures. In Asian countries, teaching is conducted in classes with large class size, and teachers speak to the entire class most of the time instead of focusing on individual students. In contrast, the class size in Western countries is much smaller, and individualized teaching or small group activities are more commonly adopted. These different practices may be a reflection of the different values in the educational systems of the two societies in handling individual differences. This will be elaborated in the Discussion Section later.

2.3 The role of textbooks in teaching and learning: *absolute vs. relative*

In many East Asian countries, teachers and students regard the textbook as a "bible" which contains all the essential knowledge. This is related to the last point on the availability of elective elements in the textbooks, and also a consequence of the adherence of the textbooks to the national curriculum mentioned in the first Section above. Since the public examinations in these countries adhere closely to the national curriculum, students rely on the textbooks heavily in order to pass the public examinations to gain access to the next stage of schooling. The common mind-set in these counties is that the textbooks must be learned from cover to cover (Stevenson and Stigler, 1992, p.141). In fact, textbooks are virtually the sole teaching tool around which class activities are organized. In China, for example, textbooks are referred to as "jiaocai", which literally means "teaching materials". So the textbook is equated with *the* teaching materials used in class.

It is true that textbooks in the United States and England are an important source of knowledge for teaching as well, but they are not accorded the same importance as in the East Asian countries. Class activities may or may not follow those suggested in the textbooks, and even when they do, they may not be in the same order as appear in the textbooks. Teachers sometimes adapt and modify the content in the textbooks, taking into consideration students' level of understanding or their interest, and very often teachers use materials outside textbooks in their teaching.

2.4 Physical appearance of textbooks: *plain vs. colourful*

Different physical appearances of textbooks suggest that they are meant to be used in different ways, and thus the physical appearance of textbooks has implications for pedagogical strategies and approaches (Schmidt *et al*, 1997, pp.37-38).

There are marked differences in the physical appearances of textbooks in different countries. Chinese, Japanese and Korean textbooks are small, thin volumes with few pictures that are mostly in black and white. This contrasts with the visually attractive textbooks in England and the United States, which feature various page layouts and many pictures in full colour. The various physical features of typical textbooks in different countries are summarized in Table 1 below:

The visually unimpressive Asian textbooks may fail to attract student interest. On the other hand, while Western textbooks may be able to arouse student interest through their interesting pictures and stories, these features may also cause some distraction to students in their learning of mathematics

(Stevenson and Stigler, 1992, p.139). This point will be elaborated further in the Discussion Section later.

Table 2-5-1. Physical Features of Textbooks in Different Countries

	China	England	Japan	Korea	The United States
Size (mm)	130×185	210×275	148×210	*148×210*	*223×265*
Number of pages	268	*240*	*214*	*323*	*620*
Pictures	few	many	few	*few*	*many*
Colour	black and white	full colour	mainly black and white	mainly black and white	*full colour*

2.5 Characteristics of the content

2.5.1 Strand and linear vs. thematic and spiral

Firstly, for the East Asian textbooks, each chapter is titled according to a mathematical curricular strand (e.g. Equations first, followed by Functions, and then Probability), and each chapter is composed of homogeneous content within the strand. In contrast, the American textbook in this study consists of modules centred around themes such as "Making choices" or "Search and rescue" and each module has several sections in which heterogeneous mathematical concepts are introduced (e.g. Algebra, Function and Statistics all in one section) (There are other American textbooks which follow the "strand" approach too.). This is also the case for the English textbook. For example, chapter titles with themes such as "High flyers", "Beadcraft", 'Cycling" and "Sea life" are used to introduce various mathematical contents of that chapter.

Secondly, East Asian textbooks have a linear structure within each grade, and one strand is usually covered by one chapter only (across grades, the structure may still be considered as a spiral one, for example, Linear Equations in the 7^{th} grade, System of Linear equations in the 8^{th} grade, and Quadratic Equations in the 9^{th} grade). On the other hand, Western textbooks typically have several strands within one module, and any one specific strand is usually repeated over several modules. In contrast to the within grade linear structure seen in East Asian textbooks, there are many small spirals even within a grade.

Lastly, textbooks in East Asian countries consist of a limited number of components or features, and concentrate mostly on explanations, examples, and exercises. American and English textbooks, on the other hand, have various features (such as projects, practice and application exercises,

technology, student self-assessments, and career connections, etc) that are rarely found in East Asian countries.

2.5.2 deductive vs. inductive

East Asian textbooks in general take a deductive approach in which general concepts are introduced before specific examples are given. For example, the 8th grade textbook in China presents the following rigorous proof in Euclidean Geometry (People's Education Press, 1999, p. 66):

Theorem on the property of isosceles triangles: The two base angles of an isosceles triangle are equal.

Given: In ΔABC, AB=AC.

To prove: $\angle B = \angle C$

Proof: Construct the angle bisector AD of the apex angle (mark the two angles $\angle 1$ and $\angle 2$ in the diagram).

In ΔBAD and ΔCAD,

$AB = AC$ (given)

$\angle 1 = \angle 2$ (by construction)

$AD = AD$ (common side)

$\therefore \Delta BAD \cong \Delta CAD$ (SAS)

$\therefore \angle B = \angle C$ (corresponding angles of congruent triangles equal)

The approach above is in marked contrast to the inductive approach that the American and English textbooks take. Typically, Western textbooks first present various activities and explorations in a realistic context, which serve as the "scaffolding" based on which students grasp the meanings of abstract mathematical concepts step by step. For example, the American textbook introduces the concept of trapezoid gradually by asking students to identify the common features of the geometric figures in the Food Guide Pyramid (except the triangle at the top). This is followed by the definition of the trapezoid after students' explorations (Billstein and Williamson, 1999, pp. 512-513). Such an approach takes into consideration students' psychological state and different levels of understanding, and allows students to familiarize themselves with new concepts step by step in a rather easy and natural manner. On the other hand, the Eastern approach demands more effort on the part of the students because the content is presented in a compact and efficient way.

These deductive and inductive approaches might have their roots in the Eastern and Western ways of thinking. Eastern students are taught not to

doubt the teachings of their ancestors and great men of past generations, let alone argue against them. Thus, it is more likely for Eastern students to accept mathematical contents presented in a deductive way. However, this approach does not fit well with students of the West, who are used to accepting the facts only after ample evidence is given. In this regard, an inductive approach is more persuasive than a deductive approach for Western students.

2.5.3 content vs. context

East Asian textbooks usually contain minimal real world contexts, and give great weight to the mathematical content itself. On the other hand, mathematical contents in Western textbooks are usually presented in the context of various real world situations. For instance, the Korean textbook presents the concepts of Proposition, Hypothesis, and Conclusion immediately after only one simple question (Kim and Kim, 1997, pp. 202-203):

> Chapter 4 Properties of Geometric Figures
> 1. Proposition and Proof
> §1. Proposition
> Question: Which one is a true sentence?
> (1) The Earth is moving (2) 2-2>1
> (3) A triangle with two equal internal angles is a regular triangle
> The sentence in which we can decide whether it is true or false is a *proposition.*
> Question: Decide whether the following sentence is a proposition.
> When a, b, c are numbers, if a=b, then ac=bc.
> Usually, the part before "then" is a *hypothesis* and the part after "then" is a *conclusion.* For example, in the above proposition, a=b is a hypothesis and ac=bc is a conclusion.

Students are not led to think about why a proposition is needed, but are "forced" to internalize the new concepts of proposition, hypothesis, and conclusion. In contrast, Western textbooks typically present various introductory activities and explorations that are quite lengthy. Although they are supposed to set the mathematics content "in context", the "context" is not always effectively related to the mathematical concepts concerned, and the activities do not always guarantee that the mathematics is meaningful to the students. Sometimes they serve only as the "scaffolding" and students may fail to go beyond the scaffolding to grasp the mathematical ideas.

For instance, "Search and Rescue" is the theme of the second module of the American textbook. It begins with a paragraph quoted from *Hatchet* written by Gray Paulsen: A thirteen-year old boy was in a double-seater plane, when suddenly the pilot had a heart attack and the plane crashed. This section deals with several terms related to the concept of Angle such as angle, ray, vertex, degree, right angle, straight angle, acute angle, obtuse angle etc. The core mathematical concept in the section is only vaguely related to the example of the plane crash, i.e., through "reading a compass". Given the fact that a compass reads clockwise while angle measures in Geometry are usually done counter-clockwise, this example may even confuse students on how to read an angle and may lead to confusion that cannot be easily rectifiable.

2.5.4 Contrived vs. realistic

In China, Japan and Korea, calculators are rarely used in class. According to Mullis *et al* (2000), the percentages of Korean and Japanese students having access to calculators in class are 28% and 34% respectively, which were the lowest among the TIMSS-R countries. This situation affects the nature of the problems presented in the textbooks. Eastern textbooks include relatively few application problems, and even when application problems are presented in a real life context, the calculations involved are usually simple. For example, the following problem dealing with a system of linear equations is one of the application problems included in the Japanese textbook (Fukumori, 1997, p.36):

The prices of a rose and a lily are 200 yen and 300 yen respectively. How many flowers did you buy if you paid 2400 yen for 10 flowers combining roses and lilies?

Numerical values in the problem are artificially simplified because students have to do pencil-and-paper calculations. Thus the problems are contrived, even though they appear to be given in a real life context. This is why students in East Asian countries tend to regard mathematics as "petrified" knowledge only limited to the textbook.

In contrast, calculators are widely used in the Western classroom. Textbooks thus have more leeway to include various real life problems, unrestricted by the complexity of calculations. For instance, one of the tasks included in the English textbook presents the real exchange rates (up to the third decimal point) for the ten European countries (this was before the Euro replaced the respective European currencies), and asks about the amount of money exchanged from one currency to another (Briggs *et al*, 1996, p.126-127). An authentic data set can be used because the complexity of the calculations is not a concern. Through solving problems with realistic data,

Western students are relatively more apt to realize the usefulness of mathematics in their real lives.

3. DISCUSSION

3.1 Different views on the nature of mathematics: *"naked" mathematics* vs. *"dressed-up" mathematics*

East Asian textbooks tend to "force" students to learn the noble logical system of mathematics by presenting a combination of concepts, symbols, and algorithms in a decontextualised way. On the other hand, Western textbooks first lead students through various activities and explorative examples, and based on these, mathematical concepts are introduced within profound contexts. This difference may be a reflection of the different views on the nature of mathematics. If a Platonic view of mathematics is taken, and mathematics is considered as absolute truth in the realm of ideas, the teacher's role is naturally one of presenting mathematics concepts clearly and helping the "ignorant" students to acquire the mathematics. Real life examples may help arouse students' interest in the learning, but in the final analysis, the truth of mathematics does not depend on these real life examples. They may even distract students from the mathematical truth. But the contrasting fallibilist position sees mathematics as a human endeavour, with all the limitations and errors shared by all other human activities. Mathematics is born out of human activities, and so it is fitting that mathematics be learned "in context". The abstract mathematical concepts need to be "dressed up" in realistic situations again, not only to aid their learning, but also to return them to their true nature.

However, even if a fallibilist view is taken, such dressing-up of mathematics is a two-edged sword. The dressing-up is supposed to take into account student's different levels of understanding, which is of course desirable. On the other hand, such an approach may distract or mislead students when the scaffolding process, with various introductory activities, is only vaguely connected to the main content of mathematics. Sooner or later, students will have to face the hard mathematics that is concealed under the comfortable fancy outlook. The rich context contained in the Western textbooks may be an effective tool to draw students' interest momentarily. But they have their limitations since a continuing interest in mathematics is mostly acquired through earnest efforts to understand essentially difficult mathematical concepts.

In short, the difference between East Asian and Western textbooks derives from whether difficult, abstract mathematical concepts are presented in a "naked" or a "dressed-up" way. East Asian textbooks encourage students to encounter abstract ideas directly without side tracking while Western textbooks "embellish" the ideas in order to reach the students.

3.2 Different didactic phenomena in teaching and learning: *formal abidance vs. meta-cognitive shift*

As hinted at in the last Section, different views on the nature of mathematics have implications for different views on teaching and learning, which may in turn lead to different didactic phenomena. The didactic phenomenon which is likely to occur in the teaching and leaning with East Asian textbooks may be classified as "formal abidance", while the possible phenomenon which may occur in the teaching and learning with Western textbooks can be categorized as "meta-cognitive shift" (Brousseau, 1984, 1997; Kang, 1990).

East Asian textbooks tend to minimize the metaphorical use of knowledge and are full of logical presentation of formulated knowledge. Thus, the extreme didactic phenomena associated with East Asian textbooks is formal abidance, which is a consequence of de-emphasizing or ignoring students' personalization and contextualization while they are exposed to mathematical knowledge.

In contrast, meta-cognitive shift is a phenomenon which takes place when the process of personalization and contextualization of mathematical knowledge is overemphasized. In some cases, the didactical efforts of Westerns textbooks are shifted from the mathematical knowledge to a didactical device. For instance, in the example cited previously in Section 5C, there is a meta-cognitive shift from the content (angle) to the context (rescue situation). The problem with a meta-cognitive shift is that while it is desirable to draw students' interest with an appealing setting (context), the interest may not extend to the mathematical concept (content). Instead, students' interest may just linger on the situation itself and they may never get to learning the concept.

3.3 Different views of man: social vs. individual

The difference in the availability of elective elements mentioned in Section 2 is perhaps a reflection of a deeper philosophical difference between the two cultures. The East Asian culture believes in orthodoxy, and students are expected to adhere to the orthodoxy despite their individual

differences. In the Western culture however, the individual is of paramount importance. Hence the curriculum has to adjust to the needs of the individual rather than the individual adjusting to an orthodox curriculum.

This philosophical difference hinges on different views on the nature of man. In the typical East Asian "social orientation" philosophy, in contrast to the "individual orientation" in Western cultures (Yang 1981), integration and social harmony is of prime importance. Man is defined in relation to other human beings in the community, and it is the obligation of the individual to adjust himself or herself to the social hierarchy. On the other hand, the Western "individual orientation" philosophy treasures the value of the individual. Each individual has his or her own rights, and the social system itself is expected to cater for the needs of the individual.

This difference has resulted in different approaches; East Asian countries introduce content with less consideration for individual differences while Western countries take pains to cater for individual differences.

4. CONCLUSION

As the textbook is the medium that articulates what should be taught in the curriculum, a study of the mathematics textbooks will reveal critical characteristics of both the intended and the implemented curricula in different countries, which may in turn reflect important differences between the cultural values in these countries.

As shown above, differences in various aspects of the textbooks in East Asia and the West reflect dissimilar social and cultural values. As the textbook is a powerful means through which students acquire both knowledge and values, these textbook differences may in turn reinforce the underlying cultural differences. The discussions above also show that each of the Eastern and Western approaches has its own strengths and weaknesses. For instance, textbooks in the West may help students realize how useful mathematics can be in their lives, but if the link between a mathematical concept and the corresponding real life situation is not made clear, sometimes students may not be able to completely grasp the mathematical concept. By contrast, the East Asian textbooks may succeed in conveying ideas in an economical way, but they often fail to motivate students to learn. Therefore it is important to take a critical view of each approach. It is through a critical understanding of the differences between different cultures that we are able to learn from each other and to put the results of such comparative studies to better use in the future.

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