Chapter 2-1

SOME COMPARATIVE STUDIES BETWEEN FRENCH AND VIETNAMESE CURRICULA

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1. MAIN FEATURES OF THE GENESIS OF THE TEACHING OF MATHEMATICS IN FRENCH SCHOOLS

1.1 From the end of the 15th century to the end of the 18th century: the emergence of mathematics teaching[1]

During this period, mathematics was divided into two parts: pure mathematics, (made up of arithmetic, geometry and algebra) and mixed mathematics (Mechanics, Optics, Astronomy, Geography, Chronology, Military Architecture, Hydrostatics, Hydraulics, Hydrographics or Navigation etc., according to the list given by d'Alembert).

This distinction between pure mathematics and mixed mathematics extended the field of mathematics. Moreover, the role played by mathematics in society increased. Although at first limited to the area of economics, mathematics progressively became part of other institutions, as is demonstrated by two indicators. Firstly, certain contributions were made by mathematicians, such as Descartes' publication on the art of weighing,

[1] Ce paragraphe s'appuie sur Artaud (1998)

Huygens' work on clocks, or Pascal's invention of an arithmetic machine "for calculation without pain and without knowledge". Secondly, the social need for mathematics was evident through the number of works published and the quality of those involved in this publishing activity.

Hence, throughout this period, the social visibility of mathematics increased, founded on a solid cultural and social foundation, arising from the structure of the world of science. This world was made up of several concentric spheres. In the centre were the great scientists such as Galileo, Descartes, Pascal, Huygens, Newton, Leibniz – and later, Bernoulli or Euler: they defended their discoveries jealously and sometimes aggressively, and only communicated among themselves indirectly or in the privileged context of highly select meetings. A crowd of people revolved around them: the organizers of scientific life, who acted as their secretaries (like for instance Father Mersenne in the first half of the 17th century), and also individuals who busied themselves in the corridors of science: people who used Mathematics, who published books about mathematics written by other people, as well as mathematics tutors.

1.1.1 How did the teaching of mathematics emerge in this context?

The teaching system was, at the time, made up of two essential components, universities and religious colleges, as well as the *Collège Royal*, the forerunner of the *Collège de France.* Alongside these two components, there existed "small schools", which were mainly directly dependent on town mayors and clergy, colleges dependent on religious congregations, and specialised schools such as military schools. The university was made up of four faculties: the faculties of law, medecine and theology to which one had access through the faculty of liberal arts, the equivalent of today's secondary system. During the 16th century, Jesuit colleges emerged. They closely resembled the arts faculties, which declined as the Jesuit colleges gained in influence. These colleges provided eight or nine years of study : grammar, rhetoric and dialectic took up the first six years, the last two or three years being given over to the study of philosophy. It was in these colleges that the first chairs of mathematics were created in the early 17th century, and here that the first real teaching of mathematics was introduced. Most of the great mathematicians were educated there (Descartes, for example, attended the College de La Flèche). At the same time the need for the expansion of the State, through the army and the navy, increased this movement through the creation of royal chairs of mathematics and hydrography from the end of the 17th century: it was necessary to train military and naval officers by teaching them the theory of arithmetic, the sphere and rules of navigation. These chairs were held by Jesuits until their expulsion from France in 1762. In the last decades of the 17th century, this process of extension, which sprang from the energy of the Jesuits, encountered the development and consolidation of the State and its hold over land and sea.

However, of the forty Paris universities, only one dispensed a "real" mathematics course: Mazarin college (where Legendre studied and presented a thesis on infinite calculus in 1172).

Hence it can be said that the teaching of mathematics began to emerge at the end of the 18th century.

1.2 From the 1789 Revolution to the mid-20th century

The main ideas concerning children's education emerged shortly before the Revolution, and were formulated in different registers of grievances. These included:

- The organisation of an educational system designed to educate the citizens of the country,
- The extension of teaching to both sexes and to all social classes,
- The creation of a teaching body,
- A central agency to oversee the setting up of a plan throughout the kingdom.

Professional schools, training for knowledge specific to certain professions, were no longer enough to ensure the educational needs of a growing number of citizens.

The idea of education for all went along with the dislocation of religious congregations which had control of schools. The promotion of a scientific culture became a means to oust the elites, especially religious elites whose teaching was directed towards "humanities" based essentially on the learning of classical languages, mainly Latin.

Communal *lycées* and *collèges* were created in the early 19th century under the Consulate.

Under the 3rd Republic, Jules Ferry, the Minister for Public Instruction (1879-1883), set up a secular, free and compulsory primary school system and opened up secondary schools to girls[2]. At that time primary and secondary education were two parallel and opposed teaching orders: the former was free, taking in children of both sexes from all social classes, with utilitarian objectives, the latter was fee-paying, accepting only boys from the

[2] loi de 1882, Art.4

elites which made up the traditional ruling class, and with humanist objectives.

It was not until the beginning of the 20th century[3] that mathematics was accepted as an educational discipline, on the same level as classical languages, and "scientific humanities" were placed alongside classical humanities in secondary teaching.

1.3 From the 1950s to the end of the 20th century

Only a minority of pupils had access to scientific courses until the mid 20th century when higher primary schools (middle schools) were integrated into the secondary system and the secondary system became free. This period was characterised by the emergence of a new educational model that was based on the notion of cause and effect between scientific development and a country's economic development, making the training for the scientific and technical careers needed by France a priority.

Higher education was thoroughly reformed, the curricula of mathematics and science were modernised, and the curricula of human sciences (economy, sociology...) were modelled on those of the exact sciences. This general change improved the standing of mathematics and science in *lycées* and *collèges*. The country was subsequently confronted with the problem of democratic access to secondary education: how were the knowledge and skills which the secondary system were designed to teach to be defined?

1.3.1 The role of the development of research in education in the French educational system

After the failure of the modern mathematics reform in the 1970s, the IREM (*Instituts de Recherches sur l'Enseignement des Mathematiques*) were created, and a community of researchers emerged whose focus was didactics of mathematics – a science of the conditions of the spread and teaching of mathematical knowledge useful for human institutions. This line of work investigated the ways in which knowledge was structured in the thinking of scientists, of students, and within particular situations. Theories of didactics sought to discern and articulate patterns in the ways these structures overlapped and influenced one another. Some studies focused on "didactical transposition" (Chevallard, 1985), others on pupil's scientific knowledge within particular conceptual fields (Vergnaud, 1990), and others on "didactical situations" (Brousseau, 1997).

[3] loi de 1902

One of the effects of the development of this field of research was new views of mathematics teaching and of the teacher's role, and the emergence of crucial questions for the development of new curricula and teaching methods.

The results of research in mathematics education, and the creation of a body of knowledge about the learning of mathematics at primary and secondary school, influenced the sphere of people reflecting on and influencing mathematics, the orientations of the national programme of studies and the organisation of teaching in secondary school (Laborde, 2002):

- Practice of a scientific approach (mathematics as a place for experimenting, modelling and formulating)
- Interplay between points of view and ways of expression and representation
- Use of Information and Communication Technologies
- Interdisciplinary activities and projects
- Mastering mathematical techniques and importance of proof

2. MAIN FEATURES OF THE GENESIS OF THE TEACHING OF MATHEMATICS IN VIETNAMESE SCHOOLS[4]

2.1 The feudal era and the Mandarin competitive examinations

Vietnam gained its independence in the Xth century when it threw off Chinese feudal domination. The Ly dynasty was the first royal dynasty which achieved a centralised monarchy ruling over all the Vietnamese territories, and had to face three major problems: the building and maintenance of a widespread network of dams intended to control the floods of the Red River Delta, the safeguard of National Independence and the tackling of rural struggles.

Therefore, the Ly dynasty set up a centralised administration and, in the XIth century, created the first Mandarin competitive examinations. Their purpose was to recruit high-ranking civil servants; competitions were open

[4] Thanks very much to Professor Ngo Thuc Lanh for authorizing us to use his paper not again published intituled *Vai net ve lich su giao duc toan hoc o Vietnam* (Some features of the teaching of Mathematics in Vietnam).

to everyone except actors and women; the first examination took place in 1075 [5]. Regional juries awarded titles to the candidates who were subsequently allowed to sit for imperial examinations to obtain a "doctorate". The rate of success in regional examinations was very low: barely a hundred out of several thousand candidates. As regards the doctorate, the title was awarded to slightly more than 2000 men in ten centuries. Regional and imperial laureates could be appointed "civil servants" and could become "Mandarins", thus belonging to a body controlling the entire administration of the kingdom.

The monarchy dominated a cluster of very lively rural districts whose collective life was led at a cultural and ideological level by a Confucian *élite* composed of the unfortunate candidates who had failed the State examinations and became preceptors, masters of ceremonies, public writers or schoolmasters.

2.1.1 School and education

One school, One master; the principle applied everywhere; it was paid for by well-off families and left to private initiative; classes were devoted to reading, writing, reciting classical texts by heart, commenting on the traditional themes of the Confucian philosophy, and composing poems in view of the State examinations.

The tests intended to recruit the Mandarins included literary, moral, political essays, writing a poem and official texts, but generally ignored Mathematics, even if the book *Daiu Viet Su Ki Toan Thu*[6] mentions some questions on the subject (years 1077, 1261, 1373, 1404, 1437, 1475, 1477, 1483, 1507, 1722 and 1762). Those tests, probably devised to recruit scribes in charge of taxes, salaries, buildings, mostly dealt with dividing quantities into equal or proportional parts, as well as area or volume calculations. The papers[7] included relevant data but also unnecessary information connected with the real world. The candidates were supposed to answer the questions but equally to comment on the superfluous data unlinked to Mathematics. Basically they had to achieve a literary work.

Furthermore, in this largely rural country whose inhabitants were mostly farmers, the *élite* remained unfamiliar with production problems. In a word,

[5] The last one took place in 1919.

[6] Compilation of historic annals, from Le Van Huu (1912) completed by Ngo Si Lien (1479) and, in the XVIIIth century by Pham Cong Tru and Le Hi.

[7] As we can find them in Pham Huu Chung book published at the beginning of the 1700's.

it can be said that before the colonial era, there was neither teaching nor training of Mathematics at school.

2.2 The colonial era

In the second half of the 19th century Vietnam had to face up to French Colonial Aggression; whereas the Monarchy and the Mandarins compromised with the Enemy, the Confucian *lettrés* (the learned elite) roused the farmers and organised a form of resistance. However bloody defeats inflicted by a foe using modern weapons spelt the end of Confucianism. Meanwhile Chinese translations of Rousseau's and Montesquieu's works brought in notions of science and democracy unknown to Confucian philosophy.

Around the beginning of the 20th century teaching in Chinese was abolished. All the schools of Indochina must use French. New generations of students were moulded in the French schools; they learnt Physics, Algebra, Biology, Geography as well as the Republican Constitution and electoral laws. But they could not play the part of the Confucian *lettrés*, because they were not closely connected enough with the common people. In those days, Education was limited and obscurantist; more than 90% of Vietnamese were illiterate. Elementary schools were few: from 5 to 6 per district or province. The number of *collèges*[8] found in some large towns and provinces amounted to 20. The whole country did not number more than 5 *lycées*[9]: two in the North, one in the Centre and two in the South, out of which two were reserved to students from French origin, or from Mandarin and wealthy families.

The syllabus for Mathematics was the French one, more or less adapted for primary and secondary schools. The textbooks came from France.

The Indochinese Communist Party was created in 1930; Marxism came to Vietnam as a tool for independence, after the failure of the anti-feudal and anti-colonial struggles led by the Confucian *lettrés* and an elite belonging to the middle-class. Thus Marxism would succeed Confucianism while acknowledging it and the work of the past elite as a positive national legacy to be integrated into the new society.

Higher Education was only opened in 1940, since French students could no longer go to France, because of the war. In those days only a few bright grant holders, reached a University level in Mathematics. The number of Vietnamese students who had graduated in Maths hardly reached 10, and only 2 of them obtained a Masters. At the end of their *cursus*, they became teachers in secondary Education. Among them, Pr. Hoàng Xuân Han,

8 Lower secondary schools
9 Upper secondary school

student at the Paris *Ecole Polytechnique*, regarded as the founder of the Vietnamese scientific culture, suggested a translation of scientific terms into Vietnamese. In 1942, he published a glossary *Danh Tu Khoa Hoc* which would allow the textbooks in Mathematics, physics and chemistry at all levels to be written in Vietnamese in the future.

2.3 The time of independence (after 1945)

2.3.1 The French influence on the evolution of Vietnamese teaching of mathematics

From April to June 1945, Pr Hoàng Xuân Han, Minister of Education and Arts supervised the curriculum of secondary schools in Vietnamese. This curriculum bears his name. The Math syllabus was common to both sections A (Classical literature) and B (Modern literature). In fact, it took up the syllabus of the *lycée* of colonial times, though modified on some points.

After the revolution of August 1945, the new curricula were modified to face the needs and difficulties of the time and eliminate what was regarded as "against democratic institutions". In wartime in 1950, school attendance was reduced from 10 to 9 years. Concerning Mathematics, it led to the suppression of several subjects of the French syllabus: Probabilities, Analytic Geometry, Analysis, Arithmetic, Descriptive Geometry, Trigonometry, Mechanics and Astronomy. This light syllabus would remain in force until 1956.

2.3.2 The influence of the Soviet system of education

From the fifties, the Soviet Union and the communist countries supported the training of students and scientists in various subjects, among others in Mathematics. They offered Vietnamese students numerous grants for higher Education (BAs, Masters, PhDs).

In 1956, Education in North Vietnam was reformed for the second time to make it closer to the Soviet curriculum prevailing at the time[10]. In Vietnamese high schools the teaching of Mathematics was inspired by the Soviet system and the textbooks were influenced by the Russian experiences. Since schooling in the Soviet Union only lasted 10 years the syllabus in Mathematics was lighter than the French one. For instance, Arithmetic was studied in forms 5 and 6, but was absent in the top form (which is not the

[10] This reform did not reach South Vietnam over American influence at that time.

case in France). Furthermore, the secondary school curriculum did not include Analysis, Mechanics or Astronomy. However the Vietnamese maintained the elements of Analysis such as functions, limits, derivatives and primitives in the top form (10). In the North, this 10-year curriculum close to the Soviet one would remain unchanged from 1956 to 1980.

Similarly, the teaching of Mathematics at university level, introduced in 1951 at the time of the Vietnamese resistance, whose aims had been widened after Geneva's agreement in 1954, was now largely influenced by Soviet directions and experiences.

2.3.3 Years of reunification (after 1975)

In 1975, the war was over and Vietnam was reunited. Mathematicians from the South trained in western countries such as France, USA, Germany, along with those trained in the Soviet Union and in Eastern countries now built together a Vietnamese culture in Mathematics.

3. BRIEF COMPARATIVE STUDY OF ORGANIZATION OF SCHOOL AND OF MATHEMATICAL PROGRAMS

3.1 Two centralized systems

In France, as in Vietnam, the teaching is organized at a national level, in particular for each subject matter the number of hours as well as the programme of studies is decided at a national level and is followed in all schools across the country.

In the two countries, the school system is now divided into three parts

- Elementary school: 6 year old to 11 year old students, grade 1 to 5
- Lower secondary school (called *Collège* in France): 11 year old to 15 year old students, grade 6 to 9
- Upper secondary school (called *Lycée* in France): 15 year old to 18 year old students, grade 10 to 12.

In France, there is only one unified type of class for all students from the age of 6 to the age of 16, called *tronc commun*, so mathematics teaching is the same for all students until they have completed grade 10. The number of hours varied over time and is currently smaller than it used to be. Teachers complain about the lack of time for allowing a deep learning even if the content to be taught has been rather decreasing in size.

In Vietnam, the government promotes the students "gifted" in sciences and particularly in Mathematics from the beginning of lower secondary schools (11 year old), where selection is carried out from marks obtained during primary *cursus*: when they attend upper secondary schools, they can be recruited, on a competitive selection basis, in gifted classes opened in the best upper secondary schools, in which the teaching staff is carefully selected and the number of students reduced compared to normal classes.

It is only when attending grade 11 that French students start following different tracks: they may choose then between Literature and Language orientation, Economic and Social orientation, and Scientific orientation, in the general *lycées*, or between four tracks, Tertiary Science and Technology, Industrial Science and Technology, Laboratory Science and Technology, Medical and social Sciences in the technological *lycées* or between several tracks in vocational *lycées*.

In France as in Vietnam, secondary school ends with a national examination called in France *Baccalauréat* (success rate was 78.3% in 1999 in France and 90% in 2002 in VN). But if, in France, only the general or technological *Baccalauréat* is needed to enter university, in Vietnam, students need to pass competitive exams to enter universities and the success rate is very low: about 18.6%.

3.2 The two national programs

3.2.1 France [11]

The programme of studies in France has been changing at a regular pace (about every ten years) since the modern mathematics reform at the beginning of the seventies. This programme is designed by a group of experts: mathematics teachers, mathematics teacher-educators, inspectors, as well as mathematics university lecturers.

As Laborde (2002) writes: "A specific epistemological approach and learning hypothesis underlies the national curriculum (...): mathematics by essence is abstract, and to acquire mathematical concepts, students must themselves achieve these abstraction processes by manipulating, experimenting and observing. Mathematical objects will be constructed by students

[11] d'après C. Laborde (2002)

as emerging from these experiments in a process of abstraction, eliminating all irrelevant aspects linked to the context of emergence. (...)"

In order to help teachers to improve students learning, the programme of studies is not simply the presentation of the main objectives and orientations of mathematics teaching, and the list of notions to be taught at each grade level. It also provides documents with comments and examples of classroom activities giving indications and trends about ways to organize the teaching in order to favour learning with developed examples.

3.2.2 Vietnam

In Vietnam, the system of Education recently underwent the two following reforms: in 1990, Scientific, Techno-scientific and Literary final forms were created at the beginning of upper secondary school; vectorial method in geometry and introduction to computational science were introduced in form 10 and analytical geometry, basics of combinatorics, of integral calculus, and mathematical statistics and probability in form 12.

In 1998, the distinctions between these three sections were abolished and a return was made to a final unique form before University. However this latest reform is partly formal, since competitive examinations giving access to the University are based upon the former distinctions; thus upper secondary schools distribute the students among pseudo-sections, with a common curriculum but applied at different levels.

A new curriculum is currently being developed: it plans a return to the division of pupils into two streams: science and humanities from 10^{th} grade.

The aim of this third reform is to lighten theoretical input by removing contents and techniques considered to be too complex and replace them with activities and problem solving.

In class, the teacher uses a single textbook, supplemented by a teacher's guide and a workbook of related exercises with corrections.

This reform seems to focus more on teaching content and organisation than on the transformation of teaching practices.

4. TWO COMPARATIVE STUDIES AT THE SECONDARY LEVEL

4.1 Introduction and methodology

In spite of specific genesis, the present Vietnamese mathematics syllabus seems similar to those of Western Countries, but the arrangement of the content and the approach to particular topics could be very different because of this genesis.

If we consider curriculum as a process which goes from official text (Official Curriculum) to the implementation in the classrooms and effective learning by students (Real Curriculum), the main factors which influence this process are those attached to the educational system actors: teachers, pedagogical advisors, who act directly on student learning. So we are interested in analysing the means provided for this process development and the "products" of process.

Therefore we will try to propose this comparative study not only in terms of educational strategy, but also and mostly in terms of produced and observed teaching and learning in the classroom. We can make the hypothesis that the distortions which appear between real curriculum and official curriculum are the key to certain problems and questions which must be identified and understood.

However, it is not enough to gather data in the field to understand the origin of the problems encountered. Our methodology is based on the conducting and linking of three studies connected to the process of didactic transposition (Chevallard 1985):

- **An epistemological study** of the mathematical concept at stake. This study allows the characterisation of the problems which give meaning to the concept, the relative position of elements of this knowledge in the wider conceptual field in which it is contained, and also the variability of this data in relation to periods and institutions. Its aim is to determine the conditions which allowed the passage from one stage to another in the historical evolution of the knowledge or on the other hand, the obstacles which slowed its development.

- **A comparative study of the historical evolution of curricula and textbooks.** This study draws out and discusses the stages of the processes which culminated in the state of the knowledge to be taught in the two educational systems (France and Vietnam) in the period studied. Successive, often local, modifications and reorganisations, were essentially due to constraints within the teaching institution.

- **An experimental study** of students' and teachers' personal relationship to the knowledge at stake, designed to test the hypotheses which were formulated through the two preceding studies, about the institutional relationship to knowledge in each country, as well as foreseeable difficultties. These personal relationships are due both to constraints inherent to the institution studied, and the interactive process of the negotiation of the knowledge between the teacher and the students, a process which is conditioned by the culture and traditions of the countries studied.

The research below has been conducted[12] in this context:

- Didactical and Epistemological Study about Teaching Vector at Upper Secondary School, in France and in Vietnam (Lê Thi Hoai Chau[13], 1997)
- A didactic study of links between functions and equations in French and Vietnamese Upper Secondary Education (Lê Van Tien[14], 2001)

We now develop some of this Studies' results and articulate them with history and underlying values of Vietnamese society.

4.2 Introduction of vectorial geometry in form 10, and the subsequent results on student's performances

4.2.1 Context of the study

Very briefly, we shall say that the French approach to Mathematics is to divide the subject between "pure" and "applied"; emphasis is laid more on its formative value than on its social usefulness.

Mathematics in secondary schools should basically train the students "to learn a scientific approach through experimental abilities, power of reasoning, imagination and acute critical analysis (...) abilities, organisations and

[12] These studies have been conducted in the context of a co-operative activity between Laboratory Leibniz (University of Grenoble) and the mathematics department of the University of Ho Chi Minh.

[13] PhD (Grenoble and Vietnam Universities) under the co-direction of Tran V. T. and C. Comiti .

[14] PhD (Grenoble and Vietnam Universities) under the direction of Nguyen B. K. and A. Bessot

communications (...)"[15]. Regarding the usefulness of Mathematics, it is simply mentioned casually as if it were taken for granted.

In Vietnam, absolute abstraction unconnected with reality goes against the principles of Vietnamese Education. According to uncle HO's maxim "training and practice are always linked", the Vietnamese schools always attempt to link theory and reality.

The choices for introducing the study of vectors in form 10 seem to have been due to this pragmatic approach. It took place in 1990 when the curricula were greatly modified and vectors appeared for the first time in the first year of the upper secondary school as a new Mathematical object.

We shall use this example of vectors being studied in form 10 to compare the weights of different cultural and Mathematical traditions in both countries. The real question will be to account for the difficulties encountered by French and Vietnamese students tackling vectors for the first time: are they due to differences in culturally different teaching methods or "more simply" induced by the intrinsic difficulty of the subject?

4.2.2 Comparing the French and Vietnamese approaches to introducing the teaching of vectors in the 1990's

In France, the starting point is translations (which are not introduced as a transformation of the whole plane, but only through its effects on specific geometrical figures). The object 'vector' is inextricably linked with information made up of three inseparable components (direction, orientation and length) through which this translation is entirely defined. In this case, the equality vectorAB = vectorCD means that the first point of each pair (A,B) and (C,D) has the second for its image through the same translation. Therefore, right from the beginning, the vector does not appear through only one of its representations. This choice allows the algebraic nature of the geometrical vector to be taken into account while it diminishes the risk of confusing a vector and one of its representations, an essential step towards the idea of class of equivalence.

The authors of the Vietnamese syllabus and textbooks dismiss the French approach to the teaching of vectors on the following ground: "The French definition of vectors is too far from the notions of Force and Speed in Physics. Therefore, the students will be confronted with difficulties when having to apply vectors in Physics"[16]. The vector is basically regarded as an oriented segment, both ends of which are defined. According to the authors:

[15] Extract from the official comments of the french curriculum.

[16] Van Nhu Cong and al., p.10

"Even if this definition is imperfect, it does not harm the following knowledge of geometry whereas it favours the understanding of such notions of Physics as force and speed", the main aim of vectors being thus: "to build up a tool to study geometry without trying to deepen this knowledge as a Mathematical object"[17].

Yet, in order for skills on vectors to be an effective tool in studying geometry, it is necessary that one be able to calculate with vectors which have different starting points. With the given definition, there is the problem of the determination of the result of vectorial operations. Therefore, it is clear that if one wants to introduce vectorial methods in geometry, it is indispensable to introduce free vectors (or classes of equivalence of equipollent bi-points or directed line segments).

In spite of these different approaches, each country maintains the three basic features of a vector: direction, orientation, length.

In French textbooks and syllabus, a vector direction is clearly connected with the direction of two parallel lines or one straight line (slope). Furthermore, the vector is linked with a translation, and the vectorial equality is expressed by a parallelogram; that helps the students to distinguish between the "orientation"[18] of a vector, and the meaning of "orientation" (*sens*) in common language.

On the contrary, Vietnamese students are not taught what a vector direction is. When meaning that two vectors AB end CD have the same "orientation" they are reduced to saying that: "Lines AB and CD are parallel or merged, and the orientation from A to B is similar to the one from C to D". To justify this choice, the Vietnamese authors put forward the following reasons: "there is no need to mention the 'orientation of a vector' because 'orientation', 'same orientation', 'opposite orientation' are familiar to students. These notions belong to their daily experience: two friends go together from home to school and come back from school to home. They have met those situations in common life as well as in lower secondary school problems"[19].

4.2.3 Are the students' performances influenced by these different approaches?

We shall compare the results of the same test dealing with vectors equality, between Vietnamese students (form 10) and French students (form 10).

[17] ibid. p.11
[18] Called *sens* in French
[19] Van Nhu Cong and al., p.12

Both groups seem reluctant to leave the metrical approach when confronted with the first question; nearly 25% of the students tested in any Group[20] uses length equality as the only vector criterion. However, it is more obvious with the Vietnamese students than with the French ones (comparing vector lengths): numerous Vietnamese students who supply a correct answer to this first question come back to a logic entirely based on lengths when the following questions of the test become more complex.

The historical study of the concept of vector and of the growth of vectorial calculus shows how metrical geometry hindered the genesis of vectorial calculus. It can be guessed that this type of error has epistemological roots: whatever the different types of teaching, it is not easy to move from scalar magnitudes to the notion of orientation of vectorial entities, and the difficulty cannot be avoided. Nevertheless the Vietnamese choice (introducing the notion of vector from the oriented segment) seems to worsen the mental process out of the metrical model.

Results show that 65% of the Vietnamese students and 32% of the French ones have not properly mastered the criteria of direction and orientation; for instance, they compare the orientation of two vectors whose directions are different, deciding on the equality of 2 vectors because: "they have the same orientation", this orientation being as well 'from left to right' as 'from top to down' or 'that of the hands of a watch, or confuse the two criteria in their answers – for example, proving the non-equality of two vectors by: "because they have neither the same orientation nor the same direction".

The epistemological study[21] carried out in the thesis shows how it is possible to move from the metrical model to the vectorial model in two stages. The first one will lead the student from the metrical model to a uni-directional oriented model including two orientations on the same direction. This process is similar to moving from the natural numbers model to the Integers one. The second stage will be shifting from an oriented uni-directional model to the vectorial model, it requires giving up the image of the straight line and accepting the concept of several directions oriented and their links in a plane or in space.

Thus, the concept of "orientation" should come first, before the concept of "direction". To give up the metrical model, it is necessary to "comprehend" the oriented straight line first. Then the oriented uni-directional model

[20] 28% of Vietnamese students n the standard forms and 11% in the "Maths specialised forms" versus 36% of French students which will access to non scientific form 11, but 0% we shall find next year in Scientific form 11.

[21] Lê T.H.C. 1997, chapitre B1, pp.79-108.

must be overcome to reach the vectorial model including all the oriented directions. Integrating both direction and orientation in a single concept raises very complex epistemological problems.

Again, it can be said that epistemology accounts for the difficult transfer from the scalar magnitudes to the features of orientation of the vectorial magnitudes. However, the study shows the importance of the didactic choices, linked to cultural and mathematical tradition in each country as we see above, which help to overcome this difficulty, since in the selected samples, the number of Vietnamese students in trouble was (at the time of our study) twice as high as the one found among French students.

4.3 Graph in analysis in secondary schools in France and Vietnam

Teaching in both countries can be analysed through two reforms: 1980 in France, 1990 in Vietnam.

4.3.1 The French "counter-reform"

The French "counter-reform" shows a sharp break with the 1970 reform inspired by Bourbaki. Mathematics is no longer regarded as a world of structures, but as a field where problems are solved. In terms of teaching it means promoting the experimental feature of Maths first, and stressing the need for the linking of an experimental and a theoretical stage, regarded as distinct. The experimental stage highlights the student's activity regarded as essential, in agreement with the constructivist theories of the moment. To the minds of the people who initiated the counter-reform, experimental activity must be grounded in problem solving and observation of numbers and/or graphs; it must emerge into terms expressing conjectures which will therefore have to be demonstrated later on. Such an approach changes the part played by drawings[22] in the course of teaching: in the 70's they were scarce and restricted to an ancillary role of illustration, interpretation, or synthesis of theoretical results. With the counter-reform, they are massively brought in at the level of the part they are expected to play in experimental activity.

In this context, solving equations appears to be a privileged ground for approximation methods which assign a major role to function graphs and

[22] As graphical representations of functions in Calculus or drawings in Geometry.

numbers. Then Calculus is understood by the leaders of the counter-reform as the realm of approximation and techniques of *majorations – minorations*[23].

However, coming into this Calculus supposes a break from the Algebraic approach and serious difficulties are to be foreseen before carrying out such a teaching. In this connection, Legrand (1993) mentions an epistemological obstacle. French curricula seem to take these difficulties into account:

- some themes involving *majorations – minorations* have been suppressed and approximation complexity has been reduced in upper secondary schools,
- the notion of experimentation is altered by the growing importance of numerical and graph records, having lost their conjectural nature.

We shall now try to see how these observations affect the practice of both students and teachers when they tackle equation solving and examine the part assigned to the graph in this operation.

4.3.2 What are the Vietnamese orientations?

The basic epistemological choices of the reform of the years 1990[24] are neither those of the French reform of modern Mathematics, nor those of the counter-reform. Its aim is to promote the methodical feature of the Mathematical universe founded on rational rules: "To form and develop among the students the abilities of logical, algorithmic, and dialectical thought through providing them with basic knowledge of methods and rules of reasoning, demonstration, (algebraic) calculus, resolution of equations, transformations of algebraic expressions, graphic representations of functions, as well as some laws of dialectic…"[25]

With such a context Calculus is greatly influenced by Algebra. The institutional relation to the concept of equation is built on the algebraic resolution of fundamental equations. In such a Calculus the graph plays a minor part in the study of elementary functions; it provides a synthesis of results obtained theoretically and helps to visualise the properties of the function studied.

[23] Setting an upper bound - setting a lower bound.

[24] Lê V. T. analysed official documents as syllabus, pedagogic guide of handbook, the book of Nguyen B.K. et al. (1994) for 4th form of upper training schools. He also interviewed M. Tran Van Hao, mathematician and leader of a team of handbooks' authors during the 1990 reform.

[25] Extract from the comments of 1990 curriculum

However, problems including graphic resolutions of equations are now found in the competitive exams for upper secondary school entrance (though the subject does not appear in 9th year textbooks); therefore we may predict its unofficial teaching at the college level.

The study of both school systems leads us to wonder about the part played by graphs in student's and teacher's real practice of Calculus. We will simply restrict the question to the study of the role of the graph in demonstrating the existence of the solutions of an equation[26].

4.3.3 Results of a survey conducted among teachers and students of forms 11 and 12

The survey shows clear differences about the influence of graphs on the student theoretical justifications between French and Vietnamese schools.

France

Most French teachers reject the graph as a sole means to validate a demonstration; however the majority of students (74%) regard the graph when it is present in the terms of the paper as an opportunity to use it to demonstrate or justify. It is obvious that some students' answers demonstrate that the "graph" method and the "theoretical" one ("bijection theorem") are equally valid in their eyes.

One of the possible explanations for this observation could be the alteration of the didactic notion of experimentation. In accordance with institutional attempts, the French teacher regards the graph as a tool for illustration or conjecture, but never uses it for a demonstration; he trains the students to be wary of the graph as a proof when he has the opportunity of doing so. But the present curriculum offers few opportunities to grant results obtained by graphs the nature of mere conjectures. Experimental activity is reduced to the benefit of graphic records and theoretical activity is reduced to the benefit of generalisation of terms; therefore, students are more formally and less strongly taught to beware of graph results as a means of demonstration.

[26] In France, the study took place after teaching the notion of derivative and the "bijection theorem": "If a function f is derivable and strictly monotonically decreasing or monotonically increasing in a closed interval [a, b] and if f(a).f(b) <0, then it exists one and only one number c in]a,b[at which f(c)=0". The notion of continuity is missing in Syllabus. In Vietnam, the study took place after teaching the notion of continuity and the "intermediate value theorem": "If a function f is defined and continuous in a closed interval [a, b] and if f(a).f(b) <0, then there is at least one number c in [a,b] at which f(c)=0."

Vietnam

Graphs are much less used to prove the existence of equation solutions than in France. Many students make a link between graph records and demonstrations. However, the influence of graphs on justifications is increasing (though it is much less than in France: 52% against 97%), when the graph is available, and the equation to solve is of the form $f(x) = k$ ($k \neq 0$) instead of the usual form $f(x) = 0$.

When the graph is not available and when the "intermediate value theorem" is unable to provide a complete solution, 31% of the students use the graph: it means that they regard the graph as an available substitute for missing or latent theoretical data.

The use of graphs as a validating tool shows a contradiction between Vietnamese teachers and students, which is not the same as in France.

Whereas the majority of Vietnamese teachers are willing to accept the graph as a total or partial proof of the existence of solutions of an equation in a standard situation, Vietnamese students will be reluctant to use the graph as a demonstration.

This paradox can be accounted for by the minor part played by graphs in Vietnamese curricula, especially in the study of the link between functions and equations. In fact, graph resolution of equations is taught in lower secondary schools; in upper secondary schools the students are familiar with them, and they have made the acquaintance of the graphs. But in Vietnam, lower secondary school and upper secondary school teachers are trained in different training schools. Thus there is a possibility for upper secondary school teachers to know little about the use of graphs in lower secondary teaching (applied to the resolution of equations). They are silent about a break they are not aware of. The students may translate that silence about the graph as an official prohibition banning it as a validating tool.

REFERENCES

Artaud, M., 1998, Conditions et contraintes de l'existence des mathématiques dans l'enseignement général, *Petit x*, n°50, IREM de Grenoble, pp.23-38.

Brousseau, G., 1997, *Theory of Didactical Situations in Mathematics*, Kluwer.

Belhoste, B. and al., 1996, *Les sciences au lycée. Un siècle de réforme des mathématiques et de la physique, en France et à l'étranger*. Vuibert, Paris.

Chevallard, Y., 1985, *La transposition didactique*, éd. La Pensée Sauvage, Grenoble.

Dorier, J-L., 1997, *L'algèbre en question*. Coll. Travaux et thèses en Didactique des Mathématiques, éd. La Pensée Sauvage, Grenoble..

Hoàng, Xuân, H., 1942, *Danh Tu Khoa Hoc.*

Laborde C., 2002, The Mathematics Curriculum in France at the Beginning of the New Century, Chongqing Conference, China.

Lê Thi, H.C., 1997, *Didactical and Epistemological Study about Teaching Vector at Upper Secondary School, in France and in Vietnam.* PhD, University Joseph Fourier and University of Viêt-nam.

Lê, T.H.C., 2001, Difficultés d'apprentissage de la notion de vecteur pour des élèves de première année de lycée en France et au Viêt-nam. *Recherche en Didactique des Mathématiques*, 21(1.2.), éd. La Pensée Sauvage, Grenoble, pp.157–188.

Lê, V.T., 2001, *A didactic study of links between functions and equations in French and Vietnamese Upper Secondary Education.* PhD, University Joseph Fourier and University of Viêt-nam.

Ngô Thuc, L., in press, *Vai net ve lich su giao duc toan hoc o Vietnam.*

Nguyen, K.V., 1962, Confucianisme et marxisme au Viêt-nam. *Expériences vietnamiennes.* Paris, pp.201-232.

Pham, H.C., beginning of the 1700's, *Cuu chuong lap thanh tinh phap.*

Nguyen, B.K., & al., 1994, *Méthodologie de l'enseignement des mathématiques – Enseignement des contenus mathématiques fondamentaux.* Maison de l'Education, Hanoi, Viêt-nam.

Legrand, M., 1993, Débat scientifique en cours de mathématiques et spécificité de l'analyse. *Repères* – IREM, **10**:9–12.

Van Nhu, C., et al., 1990, *Géométrie de 10^{e}.* Maison de l'éducation, Hanoi, Viêt-nam.

Vergnaud, G., 1990, La théorie des champs conceptuels, *Recherches en Didactique des Mathématiques,* 10 (2.3), éd. La Pensée Sauvage, Grenoble, pp.133-170.