Chapter 8

METHODS FOR OPTIMIZING PRODUCT PLATFORMS AND PRODUCT FAMILIES

Overview and Classification

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1. THE ROLE OF OPTIMIZATION IN PRODUCT FAMILY DESIGN

Optimization has been used for many years during product design to help determine the values of design variables, $\mathbf{x}$, that minimize (or maximize) one or more objectives, $\mathbf{f}(\mathbf{x})$, while satisfying a set of constraints, $\{\mathbf{g}(\mathbf{x}), \mathbf{h}(\mathbf{x})\}$, and the design variable lower and upper bounds, $\mathbf{x}^l$ and $\mathbf{x}^u$, respectively. The typical notation for formulating the optimization problem is as follows:

$$\begin{aligned} &\text{Find:} && \mathbf{x} \\ &\text{Min:} && \mathbf{f}(\mathbf{x}) \\ &\text{Subject to:} && \mathbf{g}(\mathbf{x}) \leq 0 \\ & && \mathbf{h}(\mathbf{x}) = 0 \\ & && \mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^u \end{aligned} \qquad (1)$$

When optimizing a product family, this formulation must expand to include the values of the design variables for each product in the family such that now a *set* of constraints must be satisfied while trying to achieve a *set* of objectives for the family. Thus, the challenge when optimizing a family of products lies in resolving the tradeoff between commonality and individual product performance in the family: companies desire as much commonality as possible within a family without sacrificing the distinctiveness of the

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individual products in the family as discussed in Chapter 1. In this regard, optimization can be used to help identify the Pareto frontier for this inherent tradeoff. For instance, Simpson, et al. (2001b) examine the tradeoff between different levels of platform commonality within a family of three aircraft, while Nelson, et al. (2001) study the Pareto sets of two derivative products to find a suitable product platform for a family of nail guns. Rai and Allada (2003) present an agent-based optimization framework to capture the Pareto frontier for module-based product families, demonstrating their approach using a family of power screwdrivers and electric knives.

By identifying promising designs along the Pareto frontier, optimization provides useful information to determine the best values for the design variables that define the product platform and the individual products in the family. In some instances, the design variables that define the product platform within the family are known *a priori*, i.e., before performing the optimization, whereas in other instances, determining which variables should be part of the platform and which variables should be unique to each product is a desired output from the optimization. We can thus classify approaches to product family as requiring either *a priori* or *a posteriori* specification of the platform within the family.

Accordingly, we can envision two alternative approaches for optimizing the product platform and corresponding family of products, namely, optimize the platform first and then optimize the individual products or optimize both simultaneously. These two ways of approaching the problem allow us to classify optimization approaches based on the number of stages used. In a *two-stage approach*, for instance, the product platform is designed during the first stage of the optimization, followed by instantiation of the individual products from the product platform during the second stage. In a *single-stage approach*, the product platform and corresponding family of products are optimized simultaneously.

In the next section, an example involving the design of a family of electric motors is introduced to shed light on the merits and pitfalls of both types of approaches and clarify the challenges associated with product platform and product family optimization. Section 3 provides formulations for optimizing the family of motors using two-stage and single-stage approaches and *a priori* and *a posteriori* specification of the platform variables. In Section 4, forty approaches for optimizing product platforms and families of products are classified and reviewed, and closing remarks are offered in Section 5.

2. EXAMPLE: DESIGN OF A FAMILY OF UNIVERSAL ELECTRIC MOTORS

Universal electric motors are so named for their capability to function on both direct current and alternating current. Universal motors deliver more torque for a given current than any single-phase motor (Chapman, 1991). The high performance characteristics and flexibility of universal motors have led to a wide range of applications, especially in household use where they are found in products such as electric drills and saws, blenders, vacuum cleaners, and sewing machines (Veinott and Martin, 1986).

A schematic of a universal motor is shown in Figure 8-1. As shown in the figure, a universal motor is composed of an armature and a field, which are also referred to as the rotor and stator, respectively. The armature consists of a metal shaft and slats (armature poles) around which wire is wrapped longitudinally as many as a thousand times. The field consists of a hollow metal cylinder within which the armature rotates. The field also has wire wrapped longitudinally around interior metal slats (field poles) as many as hundreds of times. For a universal motor, the wire wrapped around the armature and the field is wired in series, which means that the same current is applied to both sets of wire.

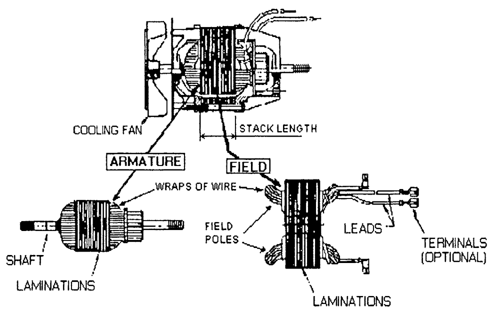

Figure 8-1. Schematic of a universal electric motor (G. S. Electric, 1997).

According to Lehnerd (1987), in the 1970s Black & Decker developed a family of universal motors for its power tools in response to a new safety regulation, namely, double insulation. Prior to that, they used different motors in each of their 122 basic tools with hundreds of variations, from jigsaws and grinders to edgers and hedge trimmers. By redesigning and standardizing the product line, they were able to produce all their power tools using a line of motors that varied only in the stack length and the amount of copper wrapped within the motor. As a result, all of the motors could be produced on a single machine with stack lengths varying from 0.8" to 1.75", and power output ranging from 60 to 650 watts. In addition to significant material and labor savings, new designs were developed using standardized components such as the redesigned motor, allowing products to be introduced, exploited, and retired with minimal extra development cost.

Motivated by Lehnerd's case study, an example problem involving the design of a family of universal electric motors has been created (Simpson, et al., 2001a). The goal in the example is to design a scale-based family of 10 universal electric motors that satisfy a variety of torque requirements based on a single platform. The motor platform consists of the set of common physical dimensions (design variables) that describe the motor while one or more variables are used to 'scale' the motor to satisfy the range of torque requirements. The motor analyses are described next, and specifications for the problem are given in Section 2.2.

2.1 Analyses for the universal electric motor example

The following equations relating the motor design variables to the system responses (i.e., mass, power, torque, and efficiency) are presented in their entirety in (Simpson, et al., 2001a) and are based on analyses from Chapman (1991) and Cogdell (1990). There are eight design variables for each motor:

1. Number of wire turns on the armature, N_c ($100 \leq N_c \leq 1500$)
2. Number of wire turns on each field pole, N_s ($1 \leq N_s \leq 500$)
3. Cross-sectional area of armature wire, A_{wa} ($0.01 \leq A_{wa} \leq 1.0$ mm^2)
4. Cross-sectional area of field wire, A_{wf} ($0.01 \leq A_{wf} \leq 1.0$ mm^2)
5. Radius of the motor, r_o ($0.01 \leq r_o \leq 0.10$ m)
6. Thickness of the stator, t ($0.0005 \leq t \leq 0.10$ m)
7. Current drawn by the motor, I ($0.1 \leq I \leq 6.0$ Amp)
8. Stack length of the motor, L ($0.001 \leq L \leq 0.10$ m)

The *mass* of the motor is the combined weight of the stator (field), the armature, and the windings on both the field and the armature.

$$\text{Mass} = M_{stator} + M_{armature} + M_{windings} \tag{2}$$

where:

$$M_{stator} = \pi L[r_o^2 - (r_o - t)^2]\rho_{steel}$$

$$M_{armature} = \pi L(r_o - t - l_{gap})^2 \rho_{steel}$$

$$M_{windings} = \rho_{copper}\{[2L + 4(r_o - t - l_{gap})]N_c A_{wa} + 2[2L + 4(r_o - t)]N_s A_{wf}\}$$

The *power*, P, output for the motor is the power input minus losses in the copper wiring and brushes; mechanical and core losses are assumed to be small and are thus neglected.

$$P = P_{in} - P_{losses} \tag{3}$$

where: $$P_{in} = VI$$

$$P_{losses} = P_{copper} + P_{brush}$$

with: $$P_{copper} = I^2(R_a + R_s)$$

$$P_{brush} = 2I$$

where: $$R_a = \{\rho[2L + 4(r_o - t - l_{gap})]N_c\}/A_{wa}$$

$$R_s = \{\rho(\#poles)[2L + 4(r_o - t)]N_s\}/A_{wf}$$

The *efficiency,* η, is the ratio of the power output to the power input.

$$\eta = P/P_{in} \tag{4}$$

Finally, the *torque* generated by the motor is the product of the motor constant, K, the magnetic flux, ϕ, and the current, I.

$$T = K\phi I \tag{5}$$

where: $$K = N_c/\pi$$

$$\phi = \Im / \Re$$

$$\Im = N_s I$$

$$\Re = \Re_s + \Re_r + 2\Re_a$$

with $\Re_s = l_c/(2\mu_{steel}\mu_o A_s)$, $\Re_r = l_r/(\mu_{steel}\mu_o A_r)$, and $\Re_a = l_g/(\mu_{steel}\mu_o A_a)$. The μ's are obtained from magnetizing intensity curves in (Chapman, 1991), which requires:

$$H = (N_c I)/(l_c + l_r + 2l_{gap}), \tag{6}$$

where: $$l_c = \pi(2r_o + t)/2.$$

2.2 Problem specifications for the motor example

There are two distinct objectives that must be considered when designing the family of universal motors: minimizing the mass (kg) and maximizing the efficiency (%), which is equivalent to minimizing the negative of the efficiency of each motor. There are six constraints for each motor in the family, which are described as follows.

1. Constraint on torque, T_i, for each of the ten motors ($i = 1, \ldots, 10$):

$$T_i = \{0.05, 0.10, 0.125, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.50\} \text{ Nm} \quad (7)$$

2. Constraint on power, P, for each motor in the family:

$$P = 300 \text{ W} \quad (8)$$

3. Constraint to ensure a feasible geometry for each motor in the family:

$$r_o/t \geq 1 \quad (9)$$

4. Constraint on the magnetizing intensity, H, in each motor in the family:

$$H \leq 5000 \text{ Amp*turns/m} \quad (10)$$

5. Constraint on the maximum mass of the each motor in the family:

$$\text{Mass} \leq 2 \text{ kg} \quad (11)$$

6. Constraint on the minimum efficiency of each motor in the family:

$$\eta > 15\% \quad (12)$$

Optimizing each motor individually involves 8 design variables, 2 objectives, and 6 constraints, but to optimize the family of 10 motors, the optimization problem, Eq. (1), becomes rather large. It is formally stated as:

$$\begin{aligned}
&\text{Find:} && \mathbf{x} = \{N_{c,i}, N_{s,i}, A_{wa,i}, A_{wf,i}, r_{o,i}, t_i, I_i, L_i\} \quad (13)\\
&\text{Min:} && \mathbf{f(x)} = \{\text{Mass}_i, -\eta_i\}\\
&\text{Subject to:} && H_i(\mathbf{x}) \leq 5000 \text{ Amp*turns/m}\\
& && r_{o,i}/t_i \geq 1\\
& && \text{Mass}_i(\mathbf{x}) \leq 2 \text{ kg}\\
& && \eta_i(\mathbf{x}) \geq 15\%\\
& && P_i(\mathbf{x}) = 300 \text{ W}\\
& && T_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\} \text{ Nm}\\
& && x_i^l \leq x_i \leq x_i^u
\end{aligned}$$

where $i = 1, \ldots, 10$ indicates each motor in the family (motor #1 has the lowest torque setting (0.05 Nm) and motor #10 the highest (0.50 Nm)).

All told, there are 80 design variables, 20 objectives, and 60 constraints, which is a challenging problem to solve for many optimization algorithms. Notice, however, that the idea of a platform is nowhere to be found in Eq.

(13); this formulation is simply for the set of 10 motors. Having a platform helps in reducing the size of the optimization problem by splitting the set of design variables, **x**, into two subsets: one that is common for each product in the family and one that is unique for each product in the family. The set of common variables is usually represented as $\mathbf{x}_c$ where *c* stands for *common variables* while the set of *unique variables* is usually represented by $\mathbf{x}_{v,i}$, where *v* stands for each variant (i = 1, …, # products) based on the platform. Sometimes the notation $\mathbf{x}_p$ is used instead of $\mathbf{x}_c$, where *p* stands for platform variables (Gonzalez-Zugasti, et al., 2000), but we avoid that notation to avoid confusion as to whether *p* stands for product or platform.

The designer must now decide how to partition the set **x** into these two subsets, $\{\mathbf{x}_c, \mathbf{x}_{v,i}\}$, which can either be specified before (i.e., *a priori*) or be found during (i.e., *a posteriori*) optimization. This gives rise to the two extreme cases of the tradeoff between commonality and distinctiveness: one in which all variable values are common and one in which all variable values are unique. In the first case, every product is the same, which means that none of them are distinct, whereas in the second case every product is unique, and there is no commonality between them. This latter case is referred to as the *null platform*, an important alternative if individual product distinctiveness is critical to market success (Nelson, et al., 2001; Simpson and D'Souza, 2004). While neither case is very practical, they provide the anchor points for the Pareto frontier that is defined by the competing objectives of commonality and individual product performance, and the optimization is used to find the best solution along this frontier for a given product family. The four different formulations follow.

3. PROBLEM FORMULATIONS AND RESULTS

The following four formulations demonstrate how the number of stages used and the specification of the platform variables in the subset, $\mathbf{x}_c$, affect the resulting solution for the family of motors. In Sections 3.1 and 3.2, the platform variables are specified *a priori* to the optimization while the optimization is solved first using a two-stage approach and then a single-stage approach, respectively. In Sections 3.3 and 3.4, more flexible formulations using a two-stage approach and a single-stage approach, respectively, are presented that do not require the specification of the platform variables *a priori*; instead, the optimization determines which variables should be made common and which should be made unique along with the best value for each variable (i.e., *a posteriori* specification of the platform variables). Section 3.5 provides a comparison of all the solutions.

3.1 Two-stage approach with platform variables specified a priori

The first formulation for the motor family followed the description given in the Black & Decker case study (Lehnerd, 1987), which stated that the axial profile of the motor was common and that the stack length was scaled to realize the family of motors. In (Simpson, et al., 2001a), we used this description to partition $\mathbf{x}$ from Eq. (13) into the platform variables, $\mathbf{x}_c = \{N_c, N_s, A_{wa}, A_{wf}, r_o, t\}$, and the variables for each motor, $\mathbf{x}_{v,i} = \{I_i, L_i\}$. Note that I_i, the current in each motor, is best thought of as a state variable that varies for each motor to achieve the desired power. The resulting formulation is:

$$\begin{aligned}
\text{Find:}\quad & \mathbf{x}_c = \{N_c, N_s, A_{wa}, A_{wf}, r_o, t\} - \textit{Stage 1} \qquad (14)\\
& \mathbf{x}_{v,i} = \{I_i, L_i\} - \textit{Stage 2}\\
\text{Min:}\quad & \mathbf{f}(\mathbf{x}) = \{\text{Mass}_i, -\eta_i\}\\
\text{Subject to:}\quad & H_i(\mathbf{x}) \le 5000 \text{ Amp*turns/m}\\
& r_{o,i}/t_i \ge 1\\
& \text{Mass}_i(\mathbf{x}) \le 2 \text{ kg}\\
& \eta_i(\mathbf{x}) \ge 15\%\\
& P_i(\mathbf{x}) = 300 \text{ W}\\
& T_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\} \text{ Nm}\\
& x_i^l \le x_i \le x_i^u
\end{aligned}$$

where $i = 1, \ldots, 10$.

This formulation was solved using a goal programming approach for the two objectives that utilized targets of 0.5 kg and 70% for the mass and efficiency, respectively, and equally weighted deviations from these targets. In essence, once a motor weighed less than 0.5 kg and had an efficiency of 70% or more, it was "good enough" for the family. This approach provides more flexibility when finding solutions since we are not trying to optimize the performance of each individual motor, just reach a suitable target for each. The optimization was completed in two stages using the Generalized Reduced Gradient (GRG) algorithm in OptdesX (Parkinson and Balling, 2002). The first stage involved determining the best settings for the platform variables, $\mathbf{x}_c$, while the unique variables could take on any feasible value. In the second stage, the best values for the platform variables from the first stage, $\mathbf{x}_c^*$, where held constant, and 10 optimization problems were solved to find the best values of the remaining unique variables, $\mathbf{x}_{v,i}^*$, for each motor. The results are summarized in Table 8-1. When compared to a set of individually optimized motors (with no commonality), we found that the motor family based on this platform weigh 9% more, on average, and are 7%

less efficient, on average. Essentially, this compromise in product performance represents the loss of having increased commonality among the family of motors. We refer the reader to (Simpson, et al., 2001a) for more details and the complete formulation for each stage.

Table 8-1. Universal electric motor family based on initial platform formulation.

Motor No.	Values of Platform Variables, $\mathbf{x}_c$: N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	r_o [cm]	t [mm]	Values of $\mathbf{x}_{v,i}$: I [Amp]	L [cm]	Responses: T [Nm]	P [W]	η [%]	M [kg]
1	1062	54	0.376	0.241	2.59	6.66	3.395	0.865	0.05	300	76.8	0.380
2	↓	↓	↓	↓	↓	↓	3.616	1.53	0.10	300	72.2	0.520
3	↓	↓	↓	↓	↓	↓	3.729	1.79	0.125	300	70.0	0.576
4	↓	↓	↓	↓	↓	↓	3.845	2.02	0.15	300	67.9	0.625
5	↓	↓	↓	↓	↓	↓	4.083	2.39	0.20	300	63.9	0.703
6	↓	↓	↓	↓	↓	↓	4.332	2.66	0.25	300	60.2	0.759
7	↓	↓	↓	↓	↓	↓	4.594	2.83	0.30	300	56.8	0.797
8	↓	↓	↓	↓	↓	↓	4.870	2.94	0.35	300	53.6	0.820
9	↓	↓	↓	↓	↓	↓	5.163	2.99	0.40	300	50.5	0.830
10	↓	↓	↓	↓	↓	↓	5.817	2.95	0.50	300	44.8	0.820

To examine this tradeoff in more detail, we examined motors in commercially available drills, and we determined that motor manufacturers vary more than just stack length when they scale their motors to meet a variety of torque and power ratings. In addition to increasing the stack length of the motor, they also allow the number of turns in the field and armature and the cross-sectional area of the wires in the field and armature to vary from one motor to the next. What this means is that the initial set of platform variables, $\mathbf{x}_c = \{N_c, N_s, A_{wa}, A_{wf}, r_o, t\}$, may have been too restrictive, hence the loss in mass and efficiency due to the platform. If we reformulate Eq. (14) to reflect this, we get:

Find: $\mathbf{x}_c = \{r_o, t\}$ – *Stage 1* (15)

$\mathbf{x}_{v,i} = \{N_{c,i}, N_{s,i}, A_{wa,i}, A_{wf,i}, I_i, L_i\}$ – *Stage 2*

Min: $\mathbf{f}(\mathbf{x}) = \{\text{Mass}_i, -\eta_i\}$

Subject to: $H_i(\mathbf{x}) \leq 5000$ Amp*turns/m

$r_{o,i}/t_i \geq 1$

$\text{Mass}_i(\mathbf{x}) \leq 2$ kg

$\eta_i(\mathbf{x}) \geq 15\%$

$P_i(\mathbf{x}) = 300$ W

$T_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\}$ Nm

$x_i^l \leq x_i \leq x_i^u$

where $i = 1, \ldots, 10$.

Using the same two-stage approach and GRG algorithm, we obtain the results shown in Table 8-2. It turns out that these results are essentially equivalent in terms of their mass and efficiency to the set of individually optimized motors, yet the ten motors have the same axial profile (i.e., r_o and t are the same for all 10 motors) and vary in the amount of wire wrapped around each motor and its stack length just like the Black & Decker example (Lehnerd, 1987). Consequently, we have been able to resolve the tradeoff between commonality and individual product performance in a satisfactory manner for this family of motors using optimization.

Table 8-2. Universal electric motor family based on revised platform formulation.

	Values of Platform Variables, $\mathbf{x}_c$		Values of $\mathbf{x}_{v,i}$						Responses			
Motor No.	r_o [cm]	t [mm]	N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	I [Amp]	L [cm]	T [Nm]	P [W]	η [%]	M [kg]
1	2.59	6.66	970	41	0.306	0.221	3.49	1.18	0.05	300	74.7	0.397
2	↓	↓	981	66	0.306	0.224	3.62	1.37	0.10	300	72.1	0.456
3	↓	↓	986	74	0.306	0.225	3.67	1.44	0.125	300	71.1	0.477
4	↓	↓	990	82	0.306	0.227	3.72	1.51	0.15	300	70.1	0.499
5	↓	↓	999	84	0.307	0.230	3.86	1.81	0.20	300	67.5	0.568
6	↓	↓	1064	80	0.359	0.239	4.03	2.03	0.25	300	64.6	0.646
7	↓	↓	1135	76	0.309	0.257	4.19	2.20	0.30	300	62.2	0.712
8	↓	↓	1166	75	0.282	0.268	4.35	2.42	0.35	300	59.9	0.774
9	↓	↓	1195	72	0.280	0.277	4.51	2.60	0.40	300	57.7	0.833
10	↓	↓	1242	67	0.286	0.293	4.85	2.91	0.50	300	53.8	0.941

3.2 Single-stage approach with platform variables specified a priori

Although the two-stage approach was successful in optimizing the platform and corresponding family of products, we were not certain as to what extent the tradeoff between commonality and individual product performance associated with Eq. (14) was caused by the selection of the platform variables versus the use of the two-stage approach. Consequently, we modified the formulation in Eq. (14) and solved it using a single-stage approach as shown in Eq. (16). This required a different optimization algorithm due to the increased problem size. In particular, Physical Programming (Messac, 1996) was used to formulate and solve the optimization problem in a single stage. The results are summarized in Table 8-3, and details can be found in (Messac, et al., 2002b) along with the complete formulation. When compared to the set of individually optimized motors mentioned earlier, this family of motors weigh 7% more, on average, and are 4.5% less efficient on average. Compared to the two-stage solutions given in Table 8-1, this represents a 2% improvement in mass, on average, and a 2.5% gain in efficiency, on average. While this may not seem like much, it translates into weight reductions in 7 of the 10 motors and increased efficiency in 8 of the 10 motors—results any manufacturer would enjoy.

Find: $\mathbf{x}_c = \{r_o, t\}$, $\mathbf{x}_{v,i} = \{N_{c,i}, N_{s,i}, A_{wa,i}, A_{wf,i}, I_i, L_i\}$ – *Stage 1* (16)

Min: $\mathbf{f(x)} = \{\text{Mass}_i, -\eta_i\}$

Subject to: $H_i(\mathbf{x}) \leq 5000$ Amp*turns/m

$r_{o,i}/t_i \geq 1$

$\text{Mass}_i(\mathbf{x}) \leq 2$ kg

$\eta_i(\mathbf{x}) \geq 15\%$

$P_i(\mathbf{x}) = 300$ W

$T_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\}$ Nm

$x_i^l \leq x_i \leq x_i^u$

where $i = 1, \ldots, 10$.

Table 8-3. Motor family using single-stage approach and initial platform formulation.

Motor No.	Values of Platform Variables, $\mathbf{x}_c$ N_c	N_s	A_{wf} [mm²]	A_{wa} [mm²]	r_o [cm]	t [mm]	Values of $\mathbf{x}_{v,i}$ I [Amp]	L [cm]	Responses T [Nm]	P [W]	η [%]	M [kg]
1	1273	61	0.271	0.271	2.673	7.745	3.432	0.617	0.05	300	76.0	0.395
2	↓	↓	↓	↓	↓	↓	3.618	1.110	0.10	300	72.1	0.513
3	↓	↓	↓	↓	↓	↓	3.713	1.317	0.125	300	70.3	0.562
4	↓	↓	↓	↓	↓	↓	3.810	1.501	0.15	300	68.5	0.606
5	↓	↓	↓	↓	↓	↓	4.010	1.806	0.20	300	65.1	0.678
6	↓	↓	↓	↓	↓	↓	4.219	2.040	0.25	300	61.8	0.734
7	↓	↓	↓	↓	↓	↓	4.438	2.213	0.30	300	58.8	0.775
8	↓	↓	↓	↓	↓	↓	4.668	2.333	0.35	300	55.9	0.803
9	↓	↓	↓	↓	↓	↓	4.912	2.408	0.40	300	53.1	0.821
10	↓	↓	↓	↓	↓	↓	5.451	2.444	0.50	300	47.9	0.830

3.3 Two-stage approach with platform variables determined during optimization

The primary goal when specifying the platform variables *a priori* is to reduce the problem size and resulting computational burden of solving the product family optimization; however, this is when the designer knows the least about which variables have the largest impact on product performance. Selecting the appropriate set of common variables, $\mathbf{x}_c$, for the platform and unique variables, $\mathbf{x}_{v,i}$, for the individual variants within a product family is not an intuitive or trivial task, and we saw the adverse impact that this can have on the overall performance of the product family in Section 3.1. If n is the number of variables that are possible candidates for being made common to a platform (with the remainder being unique among each product variant), then the number of platform alternatives is:

$$\begin{matrix}\#\,platform \\ alternatives\end{matrix} = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{2} + \binom{n}{1} + \binom{n}{0} = 2^n \qquad (17)$$

where $\binom{n}{c}$ is "*n* choose *c*", namely, the number of possible combinations of *n* items (i.e., design variables) taken *c* at a time (i.e., made common). Note that the alternative $c = 0$ is the *null platform* discussed in Section 2.2.

Ideally, an algorithm for product family design optimization would explore varying levels of commonality to determine the best platform for the family rather than require specifying the common and unique variables *a priori*. Toward that end, we developed a two-stage optimization approach that incorporates a Product Family Penalty Function (PFPF) into the Physical Programming formulation to help determine which variables have the largest impact on performance to drive commonality (Messac, et al., 2002a). The PFPF is used to minimize the variations of the design variables within the family by minimizing the percent variation, $pvar_j$:

$$pvar_j = \frac{\mathrm{var}_j}{\overline{\mathrm{x}}_j} \tag{18}$$

where:

$$\mathrm{var}_j = \sqrt{\frac{\sum_{i=1}^{p}\left(\mathrm{x}_{ij} - \overline{\mathrm{x}}_j\right)^2}{p-1}} \quad \text{and} \quad \overline{\mathrm{x}}_j = \frac{\sum_{i=1}^{p}\mathrm{x}_{ij}}{p} \tag{19}$$

x_{ij} is the value of the j^{th} design variable for the i^{th} product of the *p* products in the family. The PFPF is an additional objective function that is computed by summing the percent variation of all *n* design variables within the family:

$$\mathrm{PFPF} = \sum_{j=1}^{n} pvar_j \tag{20}$$

Lower values of PFPF mean more commonality while higher values indicate less. The PFPF is added to the "Min:" statement of Eq. (13) to yield:

$$\begin{aligned}
&\text{Find:} && \mathbf{x} = \{\mathrm{N}_{c,i}, \mathrm{N}_{s,i}, \mathrm{A}_{wa,i}, \mathrm{A}_{wf,i}, \mathrm{r}_{o,i}, \mathrm{t}_i, \mathrm{I}_i, \mathrm{L}_i\} \\
&\text{Min:} && \mathbf{f}(\mathbf{x}) = \{\mathrm{Mass}_i, -\eta_i, \mathrm{PFPF}\} \\
&\text{Subject to:} && \mathrm{H}_i(\mathbf{x}) \leq 5000 \text{ Amp*turns/m} \\
& && \mathrm{r}_{o,i}/\mathrm{t}_i \geq 1 \\
& && \mathrm{Mass}_i(\mathbf{x}) \leq 2 \text{ kg} \\
& && \eta_i(\mathbf{x}) \geq 15\% \\
& && \mathrm{P}_i(\mathbf{x}) = 300 \text{ W} \\
& && \mathrm{T}_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\} \text{ Nm} \\
& && \mathrm{x}_i^{l} \leq \mathrm{x}_i \leq \mathrm{x}_i^{u}
\end{aligned} \tag{21}$$

where $i = 1, \ldots, 10$.

The two-stage optimization approach, which is described in detail in (Messac, et al., 2002a) along with the complete formulation of the optimization problem, uses the PFPF to identify which variables have the largest impact on product performance during the first stage of the optimization, and these variables are selected as the unique variables, $\mathbf{x}_v$, while the remaining variables are taken as platform variables, $\mathbf{x}_c$. The second stage involves finding the best settings for the variables in $\mathbf{x}_c$ and $\mathbf{x}_v$ using Physical Programming as described in the previous section. In this example, the unique variables were limited to any one variable plus the current, and the results are listed in Table 8-4. Compared to the set of individually optimized motors mentioned earlier, this family of motors weigh only 3% more, on average, and are only 3% less efficient on average, a marked improvement over the results given in Table 8-1, which also scale the platform around a single variable.

Table 8-4. Motor family using two-stage approach and scaling the platform by radius.

	Values of Platform Variables, $\mathbf{x}_c$						Values of $\mathbf{x}_{v,i}$		Responses			
Motor No.	N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	t [mm]	L [cm]	I [Amp]	r_o [cm]	T [Nm]	P [W]	η [%]	M [kg]
1	1319	68	0.256	0.256	9.22	2.12	3.18	1.46	0.05	300	82.0	0.312
2	↓	↓	↓	↓	↓	↓	3.40	1.83	0.10	300	76.6	0.422
3	↓	↓	↓	↓	↓	↓	3.52	1.98	0.125	300	74.1	0.472
4	↓	↓	↓	↓	↓	↓	3.64	2.11	0.15	300	71.6	0.518
5	↓	↓	↓	↓	↓	↓	3.90	2.33	0.20	300	67.0	0.595
6	↓	↓	↓	↓	↓	↓	4.16	2.48	0.25	300	62.7	0.653
7	↓	↓	↓	↓	↓	↓	4.43	2.59	0.30	300	58.9	0.693
8	↓	↓	↓	↓	↓	↓	4.71	2.65	0.35	300	55.4	0.719
9	↓	↓	↓	↓	↓	↓	4.99	2.68	0.40	300	52.2	0.732
10	↓	↓	↓	↓	↓	↓	5.58	2.69	0.50	300	46.8	0.734

While we expected stack length to have the largest impact on the individual product performance, we were somewhat surprised by these results when we learned that stack length was part of the platform and that the radius was the unique variable used to 'scale' the motors. In talking with practicing motor designers, we confirmed that our finding was true: by varying the torque requirement as we do, it is more effective to scale the radius than the stack length; however, it is much more cost effective to manufacture motors that are scaled along the stack length, which influenced Black & Decker's decision. Furthermore, as we saw in Section 3.1, as long as we also vary the amount of wire wrapped around each motor, we obtain an equivalent set of motors if the axial profile is fixed. Our findings were confirmed in parallel work by Nayak, et al. (2002), who used a commonality goal within their two-stage goal programming formulation. They also found that the platform should be scaled around the motor radius (as well as N_s, A_{wa}, A_{wf} as the selection of the scaling variable is not limited to one variable) not the stack length, to get the best performance within the motor family.

3.4 Single-stage approach with platform variables determined during optimization

We are currently investigating a single-stage approach that uses genetic algorithms (Goldberg, 1989) to examine varying levels of platform commonality during product family optimization (D'Souza and Simpson, 2003; Simpson and D'Souza, 2004). As outlined in (Simpson and D'Souza, 2004), our approach utilizes a set of commonality controlling genes a genetic algorithm (GA) to evaluate varying levels of platform commonality. As shown in Figure 8-2, the chromosome string in the GA concatenates the individual chromosomes strings for each product into one long string and then augments this string with n genes that control the commonality within the individual chromosome strings. The resulting length of the chromosome string is $n + np$, where n is the number of design variables and p is the number of products. Note that if any of these first n genes take the value of 1, then that particular design variable is made common among all of the products in the family; a value of 0 makes that design variable unique within the family. It follows then that if these first n genes are all 1's, there is one hundred percent commonality among the products in the family while a string of all 0's indicates no commonality among the products within the family. As such, varying levels of platform commonality are considered in a single stage process, where the results from the optimization indicate:

1. which variables should be made common (i.e., platform variables),
2. the values that they should take, and
3. the values that the remaining unique variables should take.

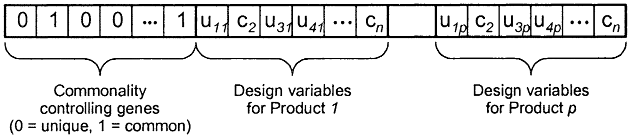

Figure 8-2. GA representation for searching varying levels of platform commonality.

For the universal electric motor family, the resulting chromosome string is 88 genes long, and an example is shown in Figure 8-3. Note that this particular chromosome string represents the motor family listed in Table 8-1. The platform variables, as indicated by the commonality controlling genes, are the first six variables, which equates to $\mathbf{x}_c = \{N_c, N_s, A_{wa}, A_{wf}, r_o, t\}$, while the last two variables are unique to each product, which equates to $\mathbf{x}_{v,i} = \{I_i, L_i\}$. The values for the corresponding variable for each motor are the same values listed in Table 8-1. This example is only one potential solution that the GA would consider during optimization, as each population in each generation will have different values for the commonality controlling genes as well as the individual variables for each product in the family.

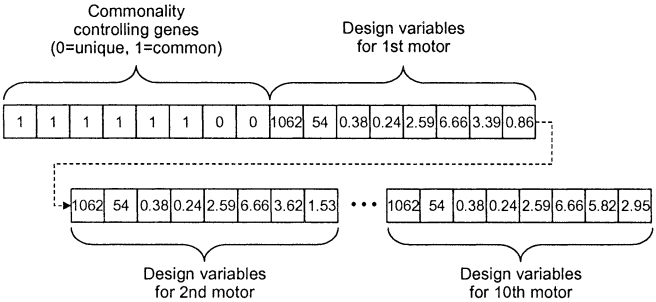

Figure 8-3. Example of GA representation for the universal electric motor family.

Formulation of the product family optimization problem is similar to that of Eq. (21) when using the genetic algorithm. One difference is that the values of the commonality controlling genes are added to the optimization as they dictate how $\mathbf{x}$ is partitioned into $\mathbf{x}_c$ and $\mathbf{x}_{v,i}$. We denote these genes as $\mathbf{x}_{cc}$, where $x_{cc,j}$ indicates the value of the j^{th} commonality controlling gene, which is either 0 or 1, where $j = 1, \ldots, n$ (= 8). To solve the problem, we use the NSGA-II, which is available online from the Kanpur Genetic Algorithm Lab in India: http://www.iitk.ac.in/kangal/soft.htm. The NSGA-II is a multi-objective genetic algorithm that can handle multiple fitness functions and constraints (Srinivas and Deb, 1995), and we use three fitness functions (i.e., minimize mass, minimize negative efficiency, and minimize PFPF) to optimize the motor family. The resulting formulation is as follows.

Find: $\mathbf{x} = \{N_{c,i}, N_{s,i}, A_{wa,i}, A_{wf,i}, r_{o,i}, t_i, I_i, L_i\}$ & $\mathbf{x}_{cc} = \{x_{cc,j}\}$ (22)

Min: Fitness function 1, 2, & 3 = $\sum_{i=1}^{10} \text{Mass}_i$, $-\sum_{i=1}^{10} \eta_i$, & PFPF

Subject to: $H_i(\mathbf{x}) \leq 5000$ Amp*turns/m
$r_{o,i}/t_i \geq 1$
$\text{Mass}_i(\mathbf{x}) \leq 2$ kg
$\eta_i(\mathbf{x}) \geq 15\%$
$P_i(\mathbf{x}) = 300$ W
$T_i(\mathbf{x}) = \{0.05, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5\}$ Nm
$x_{cc,j} = \{0,1\}$
$x_i^l \leq x_i \leq x_i^u$

where $i = 1, \ldots, 10$, and $j = 1, \ldots, 8$.

Two representative sets of results are listed in Table 8-5 and Table 8-6. The motor family listed in Table 8-5 results from the commonality controlling genes, $\mathbf{x}_{cc}$, taking the values {1,1,1,1,1,1,0,0}, which is equivalent to the platform defined in Eq. (14). Consequently, the resultant motors listed in Table 8-5 are very similar to those listed in Table 8-1 in terms of their mass and efficiency. For the family listed in Table 8-6, $\mathbf{x}_{cc}$ = {0,0,0,0,1,1,0,0}, which equates to the platform defined in Eq. (15) and motor family listed in Table 8-2. Comparing these solutions to those from Table 8-2, we have much less variability in the values of the $\mathbf{x}_{v,i}$ variables as well as slight improvements in both mass and efficiency. This demonstrates the power and flexibility of the GA-based method in that both of these motor families come from the same generation; two separate optimization problems do not have to be solved to find them. Moreover, they are obtained using a single-stage approach that does not require *a priori* specification of the platform.

Table 8-5. Universal motor family from GA equivalent to initial platform.

	Values of Platform Variables, $\mathbf{x}_c$						Values of $\mathbf{x}_{v,i}$		Responses			
Motor No.	N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	r_o [cm]	t [mm]	I [Amp]	L [cm]	T [Nm]	P [W]	η [%]	M [kg]
1	1057	55	0.348	0.238	2.54	7.08	3.39	0.878	0.05	300	77.4	0.364
2	↓	↓	↓	↓	↓	↓	3.61	1.542	0.10	300	72.6	0.500
3	↓	↓	↓	↓	↓	↓	3.73	1.806	0.125	300	70.4	0.554
4	↓	↓	↓	↓	↓	↓	3.84	2.043	0.15	300	68.3	0.602
5	↓	↓	↓	↓	↓	↓	4.08	2.412	0.20	300	64.3	0.678
6	↓	↓	↓	↓	↓	↓	4.34	2.689	0.25	300	60.5	0.735
7	↓	↓	↓	↓	↓	↓	4.59	2.860	0.30	300	57.2	0.770
8	↓	↓	↓	↓	↓	↓	4.86	2.975	0.35	300	54.0	0.793
9	↓	↓	↓	↓	↓	↓	5.15	3.028	0.40	300	51.0	0.804
10	↓	↓	↓	↓	↓	↓	5.79	3.005	0.50	300	45.3	0.800

Table 8-6. Universal motor family from GA with radius and thickness as platform.

	Values of Platform Variables, $\mathbf{x}_c$		Values of $\mathbf{x}_{v,i}$						Responses			
Motor No.	r_o [cm]	t [mm]	N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	I [Amp]	L [cm]	T [Nm]	P [W]	η [%]	M [kg]
1	2.54	7.03	1057	55	0.366	0.236	3.38	0.88	0.05	300	77.3	0.365
2	↓	↓	1051	55	0.356	0.236	3.61	1.547	0.10	300	72.5	0.499
3	↓	↓	1057	55	0.357	0.238	3.73	1.828	0.125	300	70.3	0.560
4	↓	↓	1057	55	0.367	0.239	3.84	2.044	0.15	300	68.5	0.606
5	↓	↓	1057	55	0.367	0.237	4.08	2.408	0.20	300	64.3	0.679
6	↓	↓	1057	57	0.359	0.238	4.30	2.632	0.25	300	61.1	0.727
7	↓	↓	1057	55	0.368	0.236	4.59	2.855	0.30	300	57.0	0.769
8	↓	↓	1057	56	0.359	0.236	4.84	2.973	0.35	300	53.9	0.793
9	↓	↓	1057	55	0.351	0.238	5.15	3.028	0.40	300	51.0	0.805
10	↓	↓	1057	55	0.353	0.238	5.79	2.995	0.50	300	45.4	0.799

The GA-based method reports motor families for 64 different platforms, which are based on different feasible combinations of {0,1} for $\mathbf{x}_{cc}$. Among these are solutions based on the 'null platform', $\mathbf{x}_{cc}$ = {0,0,0,0,0,0,0,0}, and an example is shown in Table 8-7. The performance of the family based on this null platform is very similar to the two families listed in Table 8-5 and Table 8-6 due to the use of the PFPF as a third fitness function in the GA, i.e., all of the solutions are driven to nearly the same region within the design space, and there is not much variation in the values of $\mathbf{x}_{v,i}$ as seen in the table. In fact, some might argue that this is not really a 'null' platform since many values are common across several, but not all, of the motors in the family. This gives rise to the question: is platforming an "all or nothing" proposition? The answer is no, which is exactly how variant components come about that are shared between some, but not all, of the products in the family. This is exactly the problem that Hernandez, et al. (2002) tackle, using the electric motor family as an example. Meanwhile, work continues with the GA-based method to improve solution diversity to spread out points in the design space and determine the best settings for the GA parameters to generate more diverse solution sets (Akundi, et al., 2005).

Table 8-7. Universal motor family from GA based on the null platform.

	Values of $\mathbf{x}_{v,i}$								Responses			
Motor No.	N_c	N_s	A_{wf} [mm^2]	A_{wa} [mm^2]	r_o [cm]	t [mm]	I [Amp]	L [cm]	T [Nm]	P [W]	η [%]	M [kg]
1	1056	55	0.348	0.234	2.54	6.61	3.38	0.880	0.05	300	76.6	0.365
2	1056	55	0.356	0.236	2.54	6.96	3.61	1.547	0.10	300	72.4	0.501
3	1056	55	0.356	0.236	2.54	6.99	3.73	1.808	0.125	300	70.2	0.554
4	1056	55	0.356	0.235	2.54	6.99	3.84	2.039	0.15	300	68.0	0.600
5	1056	56	0.357	0.236	2.52	6.99	4.08	2.408	0.20	300	64.3	0.670
6	1056	57	0.359	0.237	2.54	6.99	4.29	2.632	0.25	300	61.0	0.726
7	1056	55	0.355	0.236	2.54	6.99	4.59	2.855	0.30	300	56.9	0.768
8	1055	57	0.354	0.236	2.54	6.99	4.83	2.926	0.35	300	54.2	0.784
9	1056	55	0.351	0.236	2.54	6.99	5.15	3.027	0.40	300	50.6	0.803
10	1056	55	0.356	0.235	2.54	6.99	5.79	2.995	0.50	300	44.8	0.795

3.5 Comparison of motor families

Figure 8-4 provides a graphical comparison of the motor families based on how well they achieve their mass and efficiency targets of $\leq$ 0.5 kg and $\geq$ 70%, respectively, which is labeled the 'Utopia Region'. The results are plotted in the order in which they were presented, progressing from the *a prior* formulations that use two stages (● = Table 8-1 and ◎ = Table 8-2) and a single stage (○ = Table 8-3) to the *a posteriori* formulations that use two stages (■ = Table 8-4) and a single stage (□ = Tables 8-5 to Table 8-7). The results from the single-stage *a posteriori* GA-based method are nearly identical even though the two platforms differ; therefore, only one solution set is plotted in the figure with the □ symbol. The set of individually optimized motors from (Simpson, et al., 2001a) is also included for comparison as indicated by the ◆ symbol.

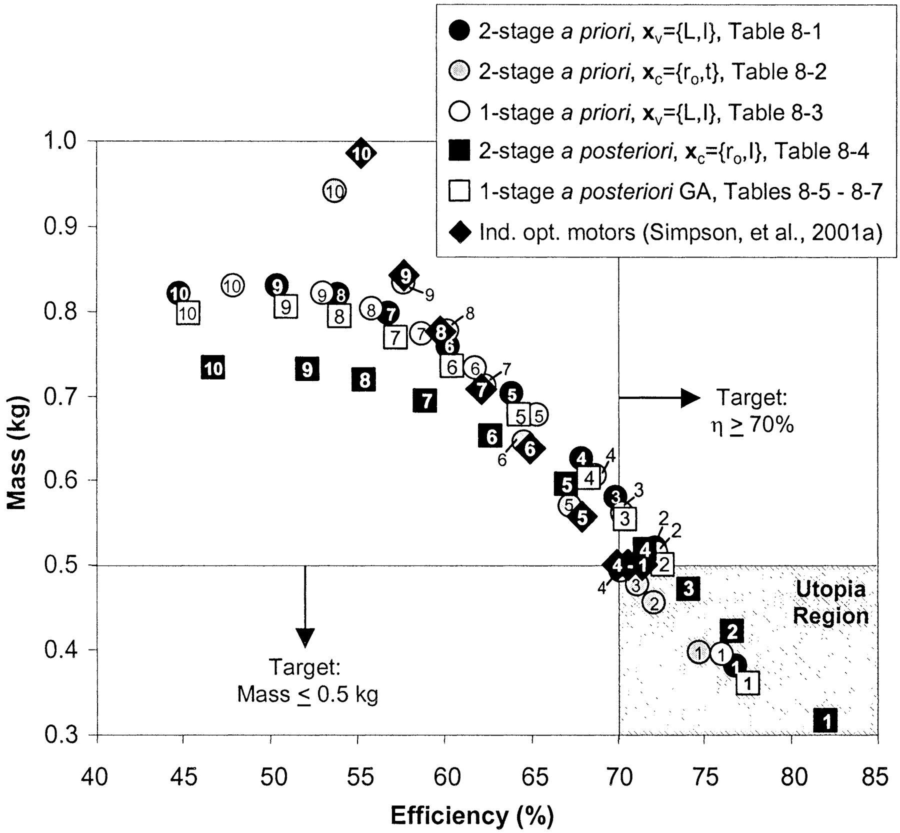

Figure 8-4. Graphical comparison of universal electric motor families.

To facilitate comparison, each symbol is numbered to correspond to a particular motor in the family where a 1 denotes the motor with the lowest required torque setting (0.05 Nm) and a 10 indicates the motor with the highest required torque setting (0.5 Nm). Based on Figure 8-4, it is much easier to visualize the tradeoff between mass and efficiency within the motor family and how the amount of commonality exacerbates this tradeoff. For instance, we can clearly see how the ◎ family of motors is nearly equivalent to the set of individually optimized motors: both have four motors in the 'Utopia Region' and the higher torque motors fall very close to one another except for motor #10, which is slightly less efficient but weighs less. Conversely, we can see the extent of the performance loss for the initial platform family (●) when compared to the set of individually optimized motors (◆). If a company wanted to use that platform, we would suggest the family obtained from the GA-based method (□). We can also see the improvement in the results of the single-stage *a priori* formulation (○) to the two-stage formulation (●): most of these motors offer improved efficiency at equivalent mass or offer more efficiency at less mass. The two-stage *a posteriori* motor family (■) yields the best motor family, as these motors tend to have the least mass while having equivalent or higher efficiency.

Several researchers have used the universal electric motor example to benchmark their optimization approaches against published results. For instance, recent work has investigated the use of ant colony optimization (Kumar, et al., 2004) and preference aggregation (Dai and Scott, 2004a) to improve the performance of the motor family. Meanwhile, others have designed the family of motors by solving it as a problem of access in a geometric space (Hernandez, et al., 2002) and by using sensitivity and cluster analysis (Dai and Scott, 2004b) to help identify the platform. A classification and review of many different approaches to product platform and product family optimization is given next.

4. CLASSIFICATION AND SUMMARY OF OPTIMIZATION APPROACHES

Several optimization approaches have been developed within the engineering design community during the past decade to facilitate product family design and optimization. Table 8-8 classifies 40 approaches from the literature based on the following categories:

1. *Module- or scale-based product family?* – does the problem formulation focus on module- or scale-based families or both? In the universal electric motor example, the emphasis was on scaling the motor around one or more design variables, but the motor could just as easily be taken as a module within a larger problem in designing a family of power tools, for instance. In Table 8-8, 'M' indicates module-based, 'S' scale-based, and 'MS' both.
2. *Single or multiple objectives* – how many objectives are used when formulating the problem? In some cases, only a single objective is used whereas multiple objectives are often considered as evidenced in the universal electric motor example. A 'S' in the table indicates that only a single objective is used while a 'M denotes multiple objectives are considered in the problem formulation.
3. *Model market demand?* – is market demand explicitly modeled and used in the problem formulation? A 'Y' under this heading indicates yes, and a blank indicates that market demand is not being considered. Although not part of the universal electric motor example, this is an important aspect of the problem that should be considered whenever possible.

Table 8-8. Summary of engineering optimization approaches for product family design.

Reference	Details of Formulation: Module- or scale-based family	Single or multiple objectives	Model market demand	Model manufacturing costs	Consider uncertainty	Specify platform *a priori*?	Number of stages	Optimization Algorithm(s)	Product Family Example (# of products in the family)
(Allada and Jiang, 2002)	M	S	Y		Y	Y	2+	DP	Generic modular products (3)
(Akundi, et al., 2005)	S	M					1	GA	Universal electric motors (10)
(Blackenfelt, 2000)	M	S		Y	Y	Y	1	OA	Lift tables (4)
(Cetin and Saitou, 2004)	M	S					1	GA, SA	Welded auto. structures (2)
(Chang and Ward, 1995)	M	S			Y	Y	1	OA	Automotive A/C units (6)
(D'Souza and Simpson, 2003)	S	M				Y	1	GA	General Aviation Aircraft (3)
(Dai and Scott, 2004a)	S	S				Y	1,2	SQP	Universal electric motors (10)
(Dai and Scott, 2004b)	S	S					2	SQP	Universal electric motors (10)
(de Weck, et al., 2003)	M	S	Y	Y			2	SQP	Automotive vehicles (7)
(Farrell and Simpson, 2003)	S	S	Y			Y	2	GRG	Flow control valves (16)
(Fellini, et al., 2000)	M	M				Y	2	NLP	Automotive power train (3)
(Fellini, et al., 2002a)	S	M					2	SQP	Automotive vehicle frames (2)
(Fellini, et al., 2002b)	S	M					2	SQP	Automotive vehicle frames (2)
(Fujita, et al., 1998)	M	S	Y	Y		Y	1	SQP	Commercial aircraft (2)
(Fujita, et al., 1999)	M	S		Y			1	SA	TV receiver circuits (6)
(Fujita and Yoshida, 2001)	B	S	Y	Y			1	SQP, GA, B&B	Commercial aircraft (4)
(Fujita and Yoshioka, 2003)	S	M				Y	1	GA	Auto. lift gate dampers (6)
(Gonzalez-Zugasti, et al., 2000)	M	M				Y	1	NLP	Interplanetary spacecraft (3)
(Gonzalez-Zugasti and Otto, 2000)	M	S		Y			1	GA	Interplanetary spacecraft (3)
(Gonzalez-Zugasti, et al., 2001)	M	S	Y	Y	Y	Y	2	NLP	Interplanetary spacecraft (3)
(Hernandez, et al., 2001)	S	M		Y		Y	2	SA	Absorption chillers (8)
(Hernandez, et al., 2002)	S	S					2+	PaS	Universal electric motors (10)
(Hernandez, et al., 2003)	M	S		Y			2+	ExS	Pressure vessels (16)
(Jiang and Allada, 2001)	M	S	Y	Y	Y		2	SLP	Vacuum cleaners (3)
(Kokkolaras, et al., 2002)	M	M				Y	2	NLP	Auto. vehicle frames (2)
(Kumar, et al., 2004)	S	M				Y	1	Ant	Universal electric motors (10)
(Li and Azarm, 2002)	M	S	Y	Y	Y	Y	2	GA	Cordless screwdrivers (3)
(Messac, et al., 2002a)	S	M					2	NLP	Universal electric motors (10)
(Messac, et al., 2002b)	S	M				Y	1	NLP	Universal electric motors (10)
(Nayak, et al., 2002)	S	M					2	SLP	Universal electric motors (10)
(Nelson, et al., 2001)	M	M				Y	2	NLP	Nail guns (2)
(Ortega, et al., 1999)	S	M		Y		Y	1	SLP	Oil filters (5)
(Rai and Allada, 2003)	M	M	Y	Y			2	NLP	Screwdrivers (3), knives (4)
(Hassan, et al., 2004)	M	M					1	GA	Commercial satellites (3)
(Seepersad, et al., 2000)	S	M	Y	Y	Y	Y	1	SA	Absorption chillers (8)
(Seepersad, et al., 2002)	S	S	Y	Y	Y	Y	2	SA	Absorption chillers (12)
(Simpson, et al., 1999)	S	M				Y	1	SLP	General Aviation Aircraft (3)
(Simpson, et al., 2001a)	S	M				Y	2	GRG	Universal electric motors (10)
(Simpson and D'Souza, 2004)	S	M					1	GA	General Aviation Aircraft (3)
(Willcox and Wakayama, 2003)	S	S				Y	1	SQP	Blended-wing-body aircraft (2)

4. *Model manufacturing cost?* – is manufacturing or production cost explicitly modeled and used in the problem formulation? A 'Y' under this heading in the table indicates yes, and a blank indicates that manufacturing cost is not being considered. As with market demand, it is important to model and include this aspect of the problem whenever possible as it is often an important decision criterion as noted in Section 3.3 for the electric motor example.
5. *Consider uncertainty?* – does the problem formulation take uncertainty into account in either the design, manufacturing, and/or market demand aspects of the problem? A 'Y' in the table under this heading indicates that one or more sources of uncertainty is being considered; a blank indicates that no uncertainty is being incorporated into the problem formulation. While this was not considered in the electric motor example, many researchers have explored the implications of uncertainty as noted in the table.
6. *Specify platform a priori?* – does the designer have to specify the platform variables *a priori* or is the problem formulated so as to identify both the platform and the family during optimization (i.e., *a posteriori*)? A 'Y' under this heading in the table indicates that the platform variables must be specific *a priori* whereas a blank indicates that they do not. Examples of both cases were given for the universal electric motor example in the previous section.
7. *Number of stages* – how many stages are used to solve the optimization problem? A '1' under this heading in the table indicates that a single stage is used, a '2' indicates that two stages are used, and '2+' indicates that more than two stages are used. Examples of two-stage and single-stage approaches were given for the universal electric motor example in Section 3.
8. *Optimization algorithm* – what optimization algorithm is used to solve the problem once it is formulated? One or more of the following acronyms is listed under this heading to indicate the type of algorithm used: B&B = Branch and Bound, DP = Dynamic Programming, ExS = Exhaustive Search, GA = Genetic Algorithm, GRG = Generalized Reduced Gradient, NLP = Non-Linear Programming, OA = Orthogonal Array, PaS = Pattern Search, SA = Simulated Annealing, SLP = Sequential Linear Programming, and SQP = Sequential Quadratic Programming. Many of these algorithms have been applied to the universal electric motor example as noted in the table.
9. *Product family example* – the last column in the table lists the type(s) of product family that is used as an example or test case in the cited work. The number of products in the family is also listed to provide an indication as to the size of the problem being solved.

In looking at the table, the approaches are split evenly between module-based and scale-based product families, while the work by Fujita and Yoshida (2001) specifically addresses both (see also Chapter 10). More than half of the approaches use multi-objective optimization, and three assumptions are often made when using multi-objective optimization:

1. maximizing each product's performance maximizes its demand,
2. maximizing commonality among products minimizes costs, and
3. resolving the tradeoff between (1) and (2) yields the most profitable product family.

Without explicitly modeling *market demand* and associated *manufacturing costs*, however, these assumptions may lead to sub-optimal product families.

The universal electric motor example in the previous sections provides an example of when this can occur. The initial formulation scaled the motors around the stack length of the motor (see Section 3.1), but maximizing commonality in the family using two different approaches revealed that the motor platform should be scaled by the radius to maximize performance. As discussed in Section 3.3, the best choice is stack length, and through discussions with experienced motor designers, we found that production costs, not performance, primarily drove the use of stack length as the scaling variable (Simpson, et al., 2001a). In the table, note that only about half the approaches integrate manufacturing costs directly within the formulation while fewer than one-third incorporate market demand. Also, note that the majority of approaches that include production costs or market demand in their formulation use single objective optimization, rather than multi-objective, where the objective is to either maximize profit or minimize cost.

Although not specifically noted in the table, most of the approaches that incorporate uncertainty in the formulation model it in the market demand and future sales of the products in the family. Uncertainty in customer requirements has also been used to develop robust product platforms. Chang and Ward (1995) were among the first to use robust design techniques to develop a family of products that were insensitive to design changes. Simpson and his co-authors use robust design techniques to develop scale-based platforms for General Aviation Aircraft (Simpson, et al., 1999), electric motors (Simpson, et al., 2001a), and absorption chillers (Hernandez, et al., 2001). Blackenfelt (2000) uses robust design techniques to maximize profit and balance commonality and variety within a family of lift tables.

More than half of the approaches require specifying the platform *a priori* in order to reduce the design space and make the optimization problem more tractable. This is not ideal, however, since most designers use optimization to explore varying levels of platform commonality within the product family as noted in Section 3.3.

Note that single-stage and two-stage approaches are employed almost equally in the literature. While both approaches are effective at determining the best design variable settings for the product platform and product family, single-stage approaches will yield better families of products as discussed in Section 3.2 since the optimization is not partitioned into two or more stages. The dimensionality of single-stage optimization problems, however, is considerably higher than in two-stage approaches, which can lead to computational challenges (Messac, et al., 2002a). A modification to the two-stage approach is introduced by Nelson, et al. (2001) and used by Fellini, et al. (2002a; 2002b; 2000): the first stage involves individually optimizing each product while the second stage involves optimizing the product family with constraints on performance losses due to commonality (see also Chapter 9). Only two multi-stage approaches have been developed. First, Hernandez, et al. (2002; 2003) develop a multi-stage optimization approach by viewing the product platform design problem as a problem of access in a geometric space. Second, Allada and Jiang (2002) introduce a dynamic programming (DP) model for configuring module instances within an evolving family of products. An alternative classification of optimization approaches based on the extent of the optimization (i.e., module attributes, module combinations, or both) is discussed in (Fujita, 2002).

Based on the variety of optimization algorithms listed in the table, there does not appear to be a preferred algorithm for product family design. Both linear and non-linear programming algorithms (e.g., SLP, SQP, NLP, GRG) are employed in many formulations, as are derivative-free methods such as genetic algorithms (GA), simulated annealing (SA), pattern search (PaS), and Branch and Bound (B&B) techniques. When the design space is small, exhaustive search (ExS) techniques (Hernandez, et al., 2003) or orthogonal arrays (Blackenfelt, 2000; Chang and Ward, 1995) can be used to enumerate different combinations of parameter settings and modules. However, very few problems involve so few options that such an approach can be taken, and many researchers advocate the use of GAs for product platform design due to the combinatorial nature of the product family design problems as noted earlier. Finally, algorithm choice is often mandated by the selected framework, e.g., Decision-Based Design (Li and Azarm, 2002), Target Cascading (Kokkolaras, et al., 2002), 0-1 integer programming (Fujita, et al., 1999), Physical Programming (Messac, et al., 2002b), and the Compromise Decision Support Problem (Simpson, et al., 1999).

Finally, these optimization approaches have been tested on a variety of product families as noted in the last column of the table. These product families range from 2-16 products and include *consumer products* such as drills, vacuum cleaners, and automobiles; *industrial products* such as chillers and flow control valves; and *complex systems* such as aircraft and spacecraft.

Detailed analyses for the universal electric motor problem can be found in (Simpson, et al., 2001a); it has been used to benchmark a variety of optimization approaches as noted in the table. The commercial aircraft problem found in (Fujita, et al., 1998; Fujita and Yoshida, 2001) uses aircraft analyses available in the literature in combination with their own models for design and development, facility, and production costs and a profit model for the manufacturer. The nail gun (Nelson, et al., 2001), vacuum cleaner (Jiang and Allada, 2001), and power screwdriver and electric knife (Rai and Allada, 2003) examples are pretty comprehensive as well. The automotive example used in (Fellini, et al., 2002a; Kokkolaras, et al., 2002) is based on a detailed vehicle body structural model that is currently unavailable to the public; simpler models of the automotive vehicle frame can be in (Cetin and Saitou, 2004; Fellini, et al., 2002b). Other analyses are not publicly available.

5. CLOSING REMARKS

As evidenced by the multitude of approaches listed in Table 8-8, formulations for solving product family optimization problems vary widely. They have been applied successfully to a wide variety of problems as well, but we must bear in mind that optimization primarily supports one aspect of product platform and product family design, namely, parameter (detail) design. New and innovative ways are needed to propagate the use of these techniques into the early stages of design when decision support is critical. Moreover, few, if any, of these approaches have found their way into industrial applications or day-to-day use within industry, and we should strive to educate practicing engineers with the power and potential of these approaches. Finally, we believe that research in this promising area of product platform and product family design will stagnate if test problems and benchmarks are not established and propagated within the community at large. We challenge interested researchers to consider this when devising new and improved approaches for product family optimization.

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