Chapter 6

Resonators on the Stability Limits

6.1 Resonators with $g_1g_2=1$

If the g-parameters of the stable resonator approach the stability boundaries at $g_1g_2=0$ or $g_1g_2=1$, the beam radii at the mirrors of the Gaussian beam go to zero or infinity. On the stability boundaries, the Gaussian beam is not an eigensolution of the resonator. The field distributions of the eigenmodes are still solutions of the diffraction integral equation (5.66), but the mode structure and the diffraction losses have to be calculated numerically. A helpful property of resonators on the stability limits is their equivalency since they all exhibit the same absolute value of the equivalent g-parameter $G=2g_1g_2-1=\pm 1$. If the properties are known for one resonator as a function of the aperture radius, the mode structure and the losses of all other resonators on the stability limits can be determined.

Let us first discuss resonators that are located on the two hyperbolas $g_2=1/g_1$ in the stability diagram. They either have two plane mirrors or the resonator length is given by the sum of the radii of curvature of the two mirrors:

$$L = \rho_1 + \rho_2 \tag{6.1}$$

The centers of curvature are on top of each other.

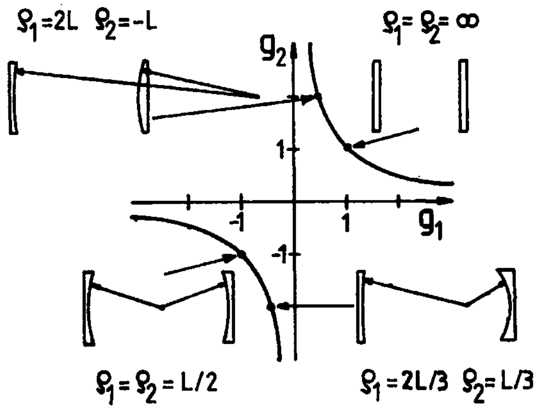

Fig. 6.1 Resonators with $g_1g_2=1$ in the stability diagram.

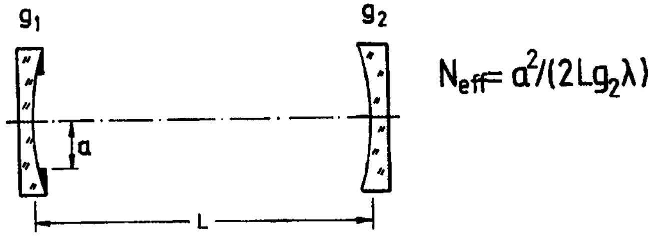

Fig. 6.2 Resonator with $g_1g_2=1$ and one aperture limited mirror. Loss and mode structure depend only on the absolute value of the effective Fresnel number N_{eff}.

If one of the mirrors is limited by the aperture (we choose mirror 1 again), the intensity distribution at this mirror and the diffraction loss per round trip is obtained by applying the integral equation used in Sec. 5.3 to calculate the properties of stable resonators with one aperture. The mode structure and the losses depend only on the absolute values of both the equivalent g-parameter G and the effective Fresnel number N_{eff} (Fig. 6.2). Since the equivalent g-parameter is $G=1$ for all resonators on the two stability hyperbolas, we only have to investigate the resonator properties as a function of the effective Fresnel number to deal with all resonators having $g_1g_2=1$.

Similar to stable resonators, these resonators exhibit an infinite set of transverse eigenmodes whose beam radii increase with the mode indices $p\ell$ and mn. This leads to the oscillation of transverse modes of higher order as the aperture radius is increased, resulting in a decrease of the beam quality and a decrease of the diffraction losses. Figures 6.3-6.5 show the intensity distributions of the TEM_{00} mode at the aperture limited mirror, the diffraction loss factor per round trip, and the beam parameter product as a function of the absolute value of the effective Fresnel number, respectively. By using these graphs, the properties of any resonator with $g_1g_2=1$ can be determined. The beam radius of the TEM_{00} mode at mirror 1 can, to a good approximation, be calculated by using the relation:

$$w_{00} = 2\sqrt{2Lg_2\lambda} \tag{6.2}$$

In order to realize fundamental mode operation the effective Fresnel number N_{eff} has to be less than 3. The beam propagation factor M^2 as a function of the effective Fresnel number can be approximated by (see Fig. 6.5 with $N_{eff}=N/2$):

$$M^2 = \frac{w\theta}{\lambda/\pi} \approx 2\sqrt{N_{eff}} \tag{6.3}$$

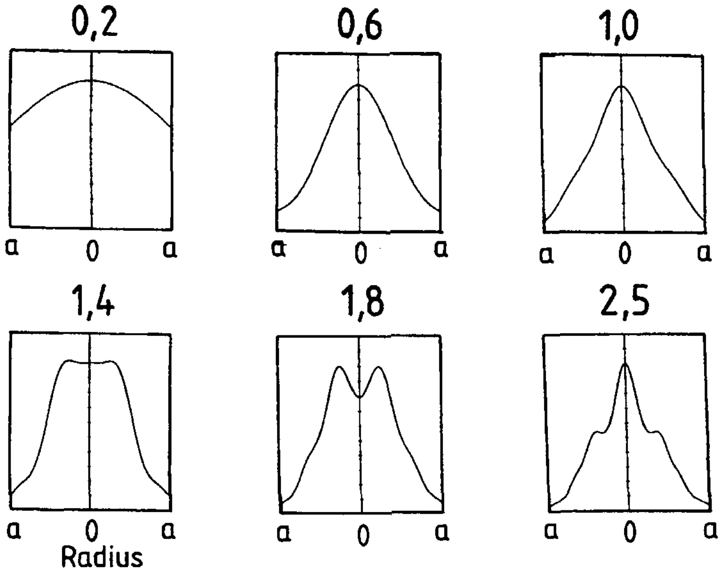

Fig. 6.3 Radial intensity distributions of the TEM_{00} mode for resonators with $|G|=1$ as a function of the effective Fresnel number N_{eff}.

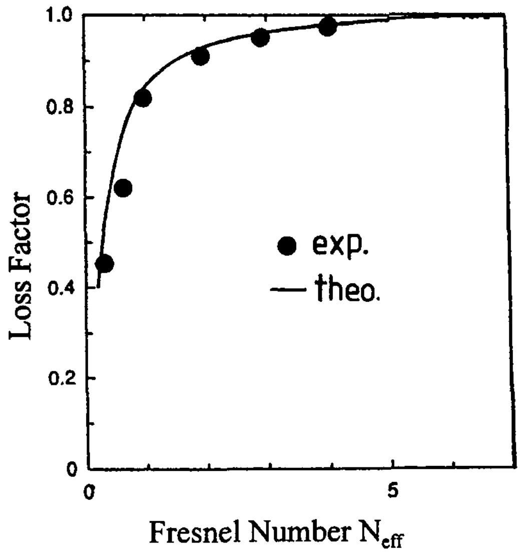

Fig. 6.4 Calculated and measured loss factor per round trip for resonators with $g_1g_2=1$ as a function of the absolute value of the effective Fresnel number N_{eff} (round aperture with radius a).

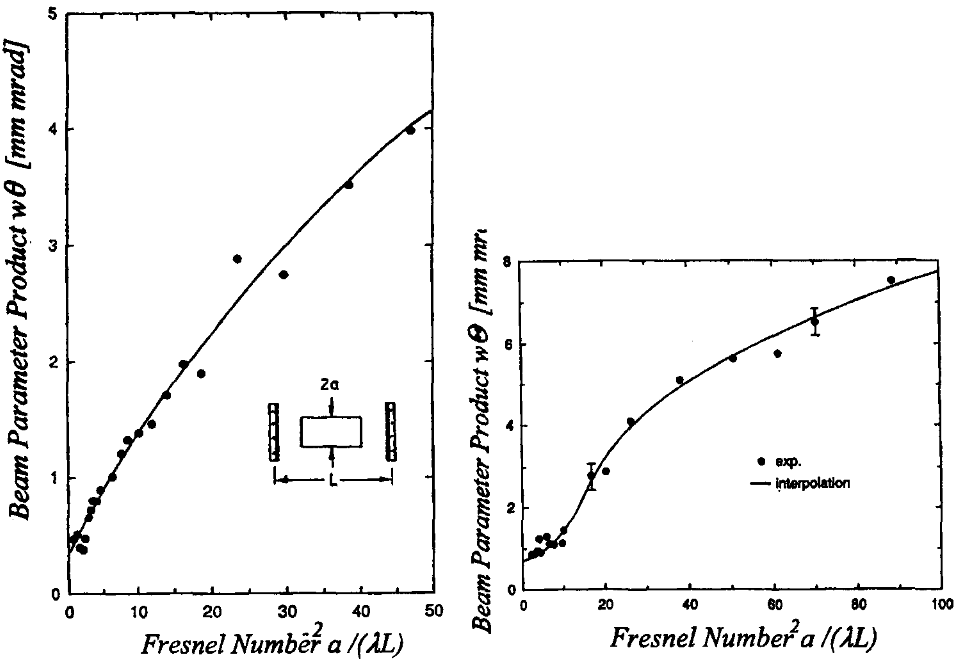

Fig. 6.5 Measured beam parameter products wθ of symmetric plane-plane resonators (g_1=g_2=1, *w:* waist radius, *θ*: half angle of divergence) as a function of the Fresnel number $N=a^2/(\lambda L)$. Left: round aperture with radius *a*, right: square aperture with side *2a*, pulsed Nd:YAG laser (λ=1.064μm) in single shot operation. Note that the effective Fresnel number is given by N_{eff}=N/2.

Examples:

1) Nd:YAG laser (λ=1.064μm) with rod radius of 5mm, ρ_1=2m, ρ_2=-1m, L=1m
The rod is placed close to mirror 1 which means that the aperture radius *a* is 5mm. Figs. 6.3-6.5 provide the following properties:

Resonator parameters:	g_1=0.5, g_2=2.0, N_{eff}=5.8
Loss of the TEM_{00} mode per round trip:	ΔV = 1%
Beam propagation factor:	M^2 = 4.0
Beam parameter product:	wθ= 1.33 mm mrad

2) CO_2 laser (λ=10.6μm) with tube radius of 10mm, ρ_1=1m, ρ_2=1m, L=2m
Again, the gas tube represents the aperture that limits one mirror.

Resonator parameters:	g_1=g_2=-1, N_{eff}= -2.358
Loss of the TEM_{00} mode per round trip:	ΔV = 7%
Beam propagation factor:	M^2 = 2.5
Beam parameter product:	wθ= 8.35 mm mrad

6.2 Resonators with One Vanishing g-Parameter

If one of the two g-parameters is equal to zero, which means that the radius of curvature of the corresponding mirror is equal to the mirror spacing, the resonator is located on an axis of the stability diagram. If the axis is approached from the stable region, the Gaussian beam radius at the mirror with the vanishing g-parameter becomes infinite, whereas the beam radius goes to zero at the other mirror. This means that the Gaussian beam is not an eigensolution. However, we can conclude that the beam propagation in a resonator on an axis of the stability diagram exhibits beam radii at the mirrors that are different by several orders of magnitude. In the following we choose mirror 1 as the mirror with a zero g-parameter, as shown in Fig. 6.6. In order to attain the best beam quality possible, it is advantageous to place the aperture or the active medium as close as possible to mirror 1. We are thus dealing with resonators on the axis $g_1=0$ with mirror 1 being limited by an aperture with radius a.

Since the resonators on the g-axes are equivalent to the resonators on the hyperbolas $g_2=1/g_1$, we can use Figs 6.3-6.5 and Eq. (6.3) to determine their mode properties. Again, fundamental mode operation requires an effective Fresnel number N_{eff} of less than 3. The beam radius at mirror 1 is given by the aperture radius a, on mirror 2 the beam radius can be calculated to a good approximation with:

$$w^{(2)} = \frac{\lambda \rho_1}{\pi a} \sqrt{M^2} \tag{6.4}$$

with the propagation factor M^2 according to (6.3). In fundamental mode operation, this beam radius is very small (typically on the order of 100μm for λ=1μm). This can lead to damage of the mirror's surface.

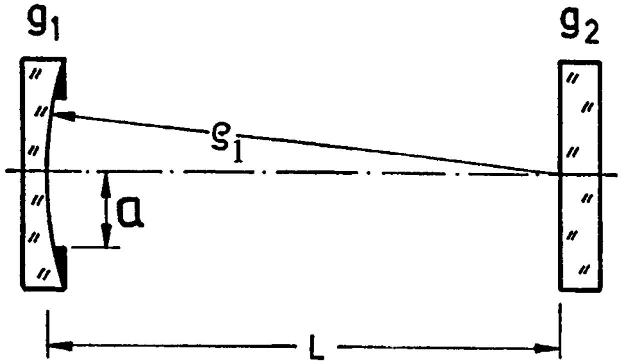

Fig. 6.6 Resonators on one axis of the g-diagram (here $g_1=0$) have one mirror with a radius of curvature equal to the mirror spacing.

Example:

A resonator with a length of $L=0.5m$ has to be designed for a pulsed Nd:glass laser (λ=1.054μm) with a 50mm long and 6mm thick rod. The laser should provide a high output energy in fundamental mode operation, and a low sensitivity to mirror tilt is required. The repetition frequency is well below 0.3 Hz which means that thermal effects can be neglected.

What we need is a resonator that exhibits a large TEM_{00} mode with a radius on the order of 2-2.5mm. A stable concave-convex resonator (see Sec. 5.2.3) provides the large Gaussian beam radius but it exhibits a high sensitivity to mirror misalignment. A much better choice is a resonator on an axis of the g-diagram. We choose a radius of curvature of mirror 1 of $\rho_1=0.5m$ ($g_1=0$) and place the glass rod as close as possible to this mirror. For fundamental mode operation we need an effective Fresnel number of less than 3. The g-parameter of mirror 2 is given by:

$$g_2 = \frac{a^2}{N_{eff} 2L\lambda}$$

which yields $g_2=3.4155$ for $N_{eff}=2.5$. The radius of curvature of mirror 2 has to be chosen to be $\rho_2=-0.207m$. According to (6.4) with $M^2=1$, the beam radius at this mirror is $w^{(2)}=56\mu m$. The 10%-angle of the misalignment sensitivity (10% loss increase due to mirror tilt, see Sec. 5.4.1) are:

$$\alpha_{10\%,1} = \infty \quad , \qquad \alpha_{10\%,2} = 2.4\ mrad$$

This example indicates that resonators with one vanishing g-parameter provide a good means to simultaneously achieve a high fundamental mode volume and a low sensitivity to mirror misalignment. However, the small beam radius on the unconfined mirror may cause damage in high power lasers. This problem can be avoided if the unconfined mirror is chosen as the output coupler since the damage threshold of optical coatings decreases with the reflectivity.

In practice, it is difficult to set-up the resonator so that it ends up on the g-axis right away. Due to tolerances of the mirror curvatures and the resonator length it is useful to first set the resonator length shorter than intended. The resonator will then operate in the stable region which makes the initial alignment much easier. By monitoring the beam radius at mirror 2 or the beam quality while slowly backing off one mirror, the resonator can be tuned to a vanishing g-parameter.

6.3 The Confocal Resonator

The symmetric confocal resonator has g-parameters that lie at the origin of the stability diagram and is characterized by the fact that both radii of curvature are equal to the resonator length [3.44,3.45]. This means that the centers of curvature are located on the mirror surfaces (Fig. 6.7). In contrast to the resonators on the axes of the stability diagram, the confocal resonator exhibits a Gaussian beam as an eigensolution. It is for this reason that the confocal resonator is sometimes referred to as a stable resonator. In the limit g_1,g_2-->0, Eqs. (5.10) and (5.18) yield for the beam radii:

waist radius: $$w_0 = \sqrt{\frac{\lambda L}{2\pi}} \tag{6.5}$$

beam radii at the mirrors: $$w_{00}^{(1)} = w_{00}^{(2)} = \sqrt{\frac{\lambda L}{\pi}} \tag{6.6}$$

The waist is located in the middle of the resonator and the Rayleigh range is given by:

$$z_0 = \frac{L}{2} \tag{6.7}$$

If the resonator is limited by apertures with radii $a_1=a_2=a$ at both mirrors, the beam propagation factor can be calculated using the relation:

$$M^2 \cong \pi\ N = \pi\ \frac{a^2}{\lambda L} \tag{6.8}$$

where N is the Fresnel number.

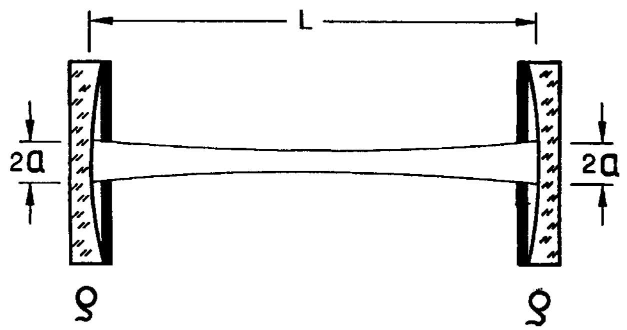

Fig. 6.7 The symmetric aperture-limited confocal resonator.

Fundamental mode operation is obtained for Fresnel numbers N of less than 0.5. Unfortunately, the mode volume of the fundamental mode is quite low resulting in a low output power. For a wavelength λ of 600nm and an effective resonator length L of 1m, for instance, the waist radius w_0 is only 0.31mm. However, the confocal resonator exhibits some interesting properties that make it possible to increase the fundamental mode volume by several orders of magnitude by choosing different aperture sizes at the mirrors.

Let us first consider a confocal resonator with only mirror 1 limited by an aperture (Fig. 6.8). Any field distribution with radial symmetry starting at mirror 1 will be reproduced after the round trip inside the resonator. The aperture is imaged onto itself since the ray transfer matrix for the round trip reads (see Sec. 1.3):

$$\boldsymbol{M} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

All field distributions that are circularly symmetric or exhibit symmetry along the x-axis and the y-axis are eigensolutions of the resonator. As a consequence, any Gaussian beam with q-parameter q at the aperture limited mirror is an eigensolution. For a given aperture radius a, we can thus always find a modified Gaussian beam with beam radius $w_{00}^{(1)}=a$ and radius of curvature ρ at mirror 1 that is reproduced after each round trip. If the aperture radius a, and thus the Gaussian beam radius $w_{00}^{(1)}$, is decreased the beam radius $w_{00}^{(2)}$ is increased in such a way that the following relation holds (Fig. 6.9):

$$w_{00}^{(1)}\, w_{00}^{(2)} = \frac{\lambda L}{\pi} \tag{6.9}$$

The distance z of the beam waist from mirror 1 is given by:

$$z = \frac{L}{1 + \left[\dfrac{\lambda L}{\pi [w_{00}^{(1)}]^2}\right]} \tag{6.10}$$

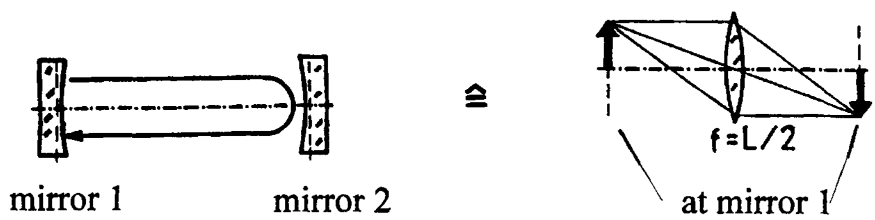

Fig. 6.8 In a confocal resonator each point at mirror 1 is imaged by the round trip into a point opposite the optical axis.

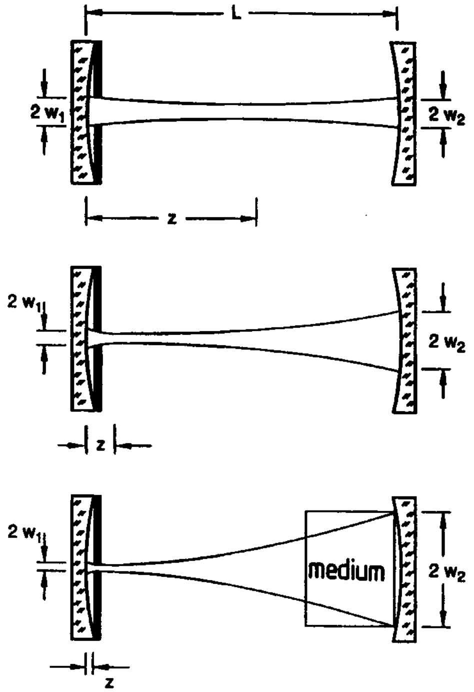

Fig. 6.9 For any aperture radius at mirror 1, one can find a Gaussian beam that exhibits a radius of curvature equal to that of the mirrors and a beam radius that is adapted to the aperture radius. z denotes the distance of the beam waist from mirror 1.

If the active medium is positioned at the infinite mirror 2, a high fundamental mode volume in the medium can be attained by decreasing the aperture radius a.

Example:
The symmetric Gaussian beam in a confocal resonator with $L=1m$ and $\lambda=500nm$ exhibits beam radii at the mirrors of $w_{00}^{(1)}=w_{00}^{(2)}=0.4mm$. If mirror 1 is limited by an aperture with $a_1=0.1mm$, the beam radius at mirror 2 is increased by a factor of four to be $w_{00}^{(1)}=1.6mm$. We can thus fill a cylindrical active medium with a diameter of about 4mm with the fundamental mode.

This special property of the confocal resonator is generated by the fact that the field distributions at the two mirrors are related to each other via a Fourier transform. The field at mirror 2 is the Fourier transform of the field at mirror 1. Decreasing the aperture radius a_1 will therefore lead to an increase of the beam radius at mirror 2. This becomes clearer if we apply the diffraction integral to the field propagation inside the resonator (Fig. 6.10). By assuming a circularly symmetric field distribution on mirror i:

$$E_i(r,\Phi) = u_{pi}(r)\,\exp[i\ell\Phi] \tag{6.11}$$

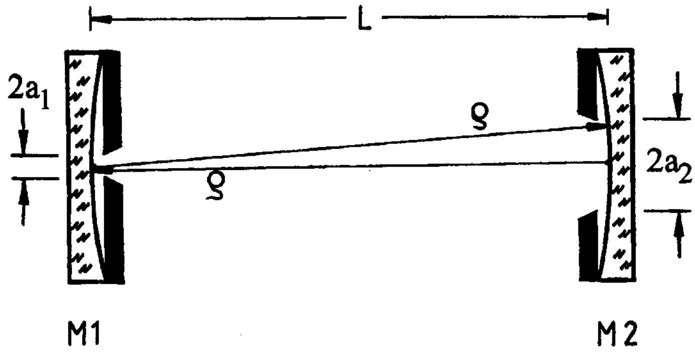

Fig. 6.10 Confocal resonator with different aperture radii at the mirrors.

and scaling the fields with the square root of the Fresnel number $N_i = a_i^2/(\lambda L)$:

$$E'_{p\ell}(r,\Phi) = E_{p\ell}(r,\Phi)\,\sqrt{N_i} \tag{6.12}$$

we find the following relations between the radial parts u'_{p1} and u'_{p2} of the fields on the two mirrors:

$$u'_{p2}(r_2) = (-i)^\ell\, 2\pi\, N \exp[-ikL] \int_0^1 u'_{p1}(r_1) J_\ell(2\pi N r_1 r_2) r_1 dr_1 \tag{6.13}$$

$$\gamma_{p\ell} u'_{p1}(r_1) = (-i)^\ell\, 2\pi\, N \exp[-ikL] \int_0^1 u'_{p2}(r_2) J_\ell(2\pi N r_1 r_2) r_2 dr_2 \tag{6.14}$$

with	J_ℓ	:	Bessel function of order ℓ
	$N = \sqrt{N_1 N_2}$	:	Fresnel number
	r_i	:	normalized radial coordinate on mirror i
	$V_{p\ell} = \gamma_{p\ell}\gamma^*_{p\ell}$	:	loss factor per round trip of the $TEM_{p\ell}$ mode

The similarity of (6.13) and (6.14) indicates that the field distributions on the two mirrors have the same shape since the two integrals are identical. This means that if one aperture is completely filled by the oscillating modes, the same filling is obtained at the second one. The mode structure and the losses depend only on the Fresnel number N. By decreasing the Fresnel number, the number of oscillating modes will decrease until we get fundamental mode operation at low Fresnel numbers N. Figure 6.11 presents loss factor and intensity distributions of the fundamental mode as a function of the Fresnel number N, calculated with (6.13) and (6.14). This figure indicates that the Fresnel number has to be chosen lower than 0.8 to prevent higher modes from oscillating.

If we place the active medium at mirror 2 (the aperture radius a_2 now represents the radius of the active medium), we can decrease the aperture at mirror 1 until the fundamental mode condition $N<0.8$ is fulfilled. Maximum output power is attained for a Fresnel number of about 0.6. The optimum aperture radius a_1 is thus given by:

$$a_1 = 0.6 \, \frac{\lambda L}{a_2} \tag{6.15}$$

As the aperture is decreased a continuous increase in beam quality is observed but the output power will remain fairly constant since the active medium is always filled by the modes. If Fresnel numbers lower than 0.5 are reached, the increased diffraction losses experienced by the fundamental mode will result in a sudden decrease of the output power.

Figures 6.12 and 6.13 show measured output energies per pulse and the corresponding beam parameter products of an Nd:YAG laser with a 1m long confocal resonator. These graphs indicate that the asymmetric confocal resonator is capable of providing high output power and excellent beam quality simultaneously. Furthermore, the misalignment sensitivity is lower compared to other stable resonators in fundamental mode operation (see Sec. 5.4.2). The misalignment sensitivity is characterized by the tilt angle at which the diffraction losses have increased by 10%. For the resonator set-up shown, misalignment of the unconfined mirror 2 resulted in a 10%-angle of 150μrad at a ratio of aperture radius to Gaussian beam radius of $a_1/w=0.7$. For the tilt of mirror 1, a 10%-angle of more than 5mrad was observed.

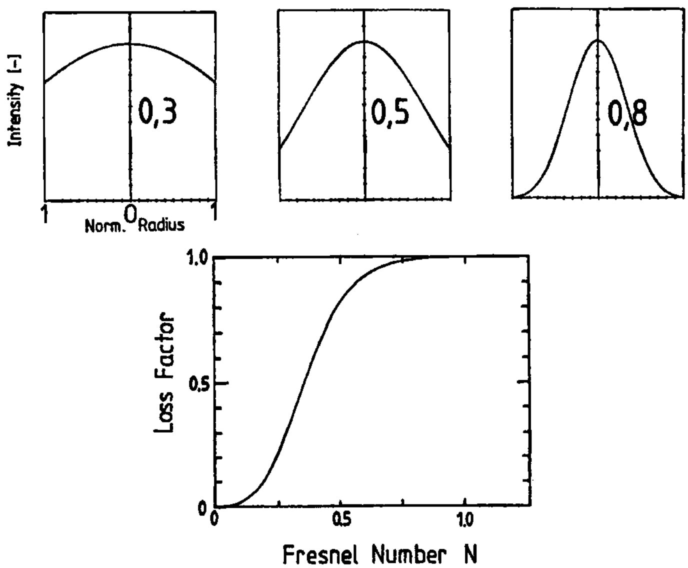

Fig. 6.11 Calculated loss factor per round trip of the fundamental mode as a function of the Fresnel number $N=a_1a_2/(\lambda L)$; The radial intensity distributions in the aperture for different Fresnel numbers N are shown as well (top).

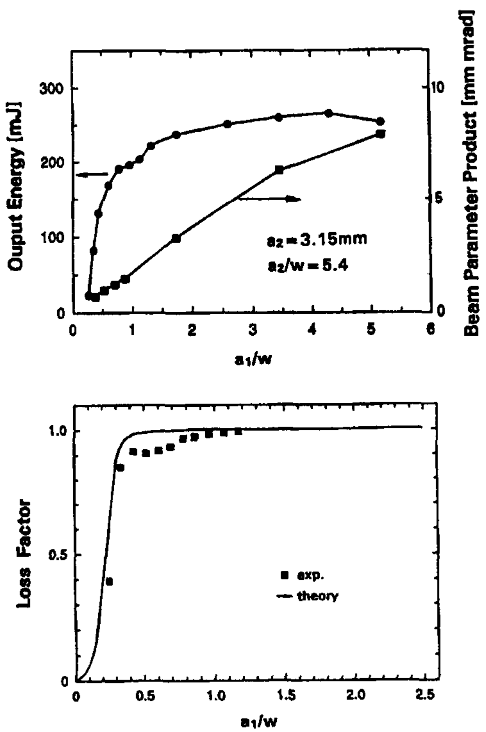

Fig. 6.12 Measured output energy and beam parameter product for a confocal resonator as a function of the aperture radius a_1. The Gaussian beam radius on the mirror w is *0.582mm*; small-signal gain $g_0l=1.0$; $R=80\%$; $L=1.0m$; $a_2=3.15mm$ (Nd:YAG laser, $\lambda=1.064\mu m$). With a stable resonator in multimode operation, the maximum output energy was 320mJ for optimum output coupling [3.53] (© OSA 1993).

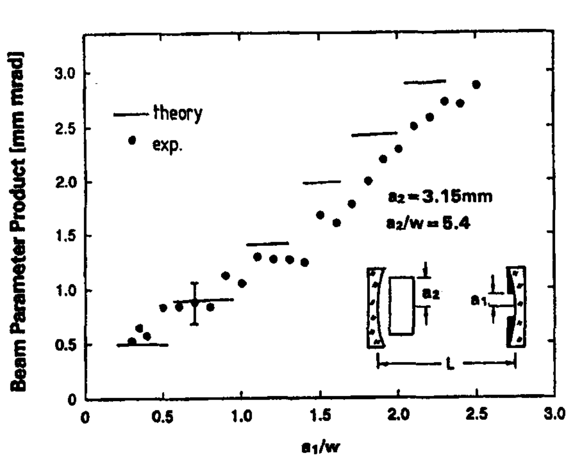

Fig. 6.13 Measured beam parameter products for the experiments presented in Fig. 6.12. The solid lines mark the theoretical values obtained by solving the diffraction integrals (6.13) and (6.14) for $\ell=0$ including the amplification of the field by the active medium [3.53] (© OSA 1993).

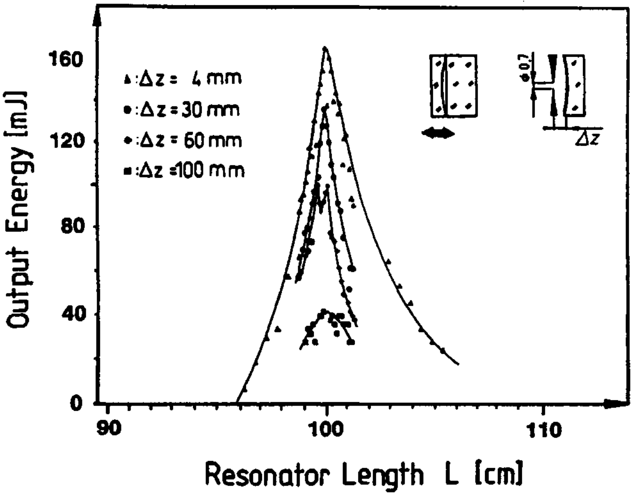

Fig. 6.14 Measured output energy for the resonator of Fig. 6.12 as a function of the resonator length for a Fresnel number of $N=1.0$. The curve parameter is the distance Δz between the aperture and mirror 1 [3.53] (© OSA 1993).

Unfortunately, the price we have to pay for the low misalignment sensitivity and the high fundamental mode volume is a fairly high sensitivity to variations in the resonator length. Small length variations will destroy the Fourier transform relationship between the fields and the resonator will go stable. Only one Gaussian beam is an eigensolution of the stable resonator and this Gaussian beam exhibits a beam radius at mirror 1 that is much larger than the aperture radius a_l. A slight mismatch of the resonator length will therefore lead to a sudden increase of the diffraction losses. As the experimental data in Fig. 6.14 indicate, the length of the confocal resonator has to be controlled with an accuracy of $\pm 1\%$ in order to maintain an output power of at least 90%. In a single-shot operation, this required length stability does not cause any problems, but as far as high power operation is concerned, the confocal resonator can be used only for one fixed input power. Most active materials exhibit a lensing effect which is brought about by a combination of heat generation due to the absorption of pump and laser radiation and the flow of heat to the outer periphery due to cooling. This thermal effect is small in gas lasers but it poses a serious problem in solid state lasers where refractive powers on the order of several diopters are observed. The refractive power increases linearly with the average pumping power. In order to operate the resonator at the confocal point, the refractive power has to be compensated by an appropriate choice of the mirror curvatures. This requires a constancy of the refractive power of better than 1%, and any decrease in the pumping efficiency due to degradation will considerably decrease the output power. Furthermore, any nonparabolic refractive index profile that cannot be compensated for by the mirror curvatures will prevent the resonator from working efficiently.