Chapter 22
Single Mode Resonators

22.1 Axial Mode Spectrum of Lasers

Without additional frequency selecting elements, most lasers utilizing linear resonators emit at several axial modes. A resonator operating in a single transverse mode, exhibits a frequency spectrum which consists of multiple lines. If we neglect the frequency pulling induced by the gain profile, these emission lines are spectrally separated by $c_0/(2L)$, where L is the optical resonator length and c_0 is the speed of light in a vacuum. In inhomogeneously broadened lasers, all axial modes are observed that experience a gain high enough to overcome the resonator losses (Fig. 22.1a). Since no interaction between different axial modes occurs, the number of oscillating axial modes increases as the gain is increased. Well above laser threshold, the number of observed axial modes n can be approximated by:

$$n = \frac{\Delta\nu \, 2L}{c_0} \tag{22.1}$$

where $\Delta\nu$ is the gain bandwidth of the active medium (see Table 4.2 for bandwidths of common lasers). Inhomogeneous line broadening is found in low pressure gas lasers (HeNe, Ar, CO_2) and in semiconductor lasers at high injection currents. In both laser types, typically on the order of 10 axial modes oscillate simultaneously.

The majority of lasers utilize active media with homogeneous line broadening. In a homogeneously broadened laser, each axial mode has access to the whole gain spectrum. The axial mode that first reaches the laser threshold saturates the gain at all frequencies. This automatically ensures single mode operation because the other axial modes are kept below threshold (Fig. 22.1b). However, in a standing wave resonator, the gain saturation leads to a longitudinal modulation of the gain referred to as spatial hole burning. This may result in the simultaneous operation of several axial modes (Fig. 22.1c) because different modes are amplified in different gain regions. Furthermore, small-scale variations of the optical resonator length due to mechanical vibrations or varying pumping conditions lead to random jumping between modes. In general, the bandwidth of the emission is smaller than the gain bandwidth of the active medium. In free-running Ti:sapphire lasers, for instance, the bandwidth of the laser emission is only a few nm, although the gain profile would allow laser oscillation over a range of 300nm. Spatial holeburning is particularly strong in solid state and liquid lasers because the gain modulation cannot be smoothed out by a fast movement of the active atoms.

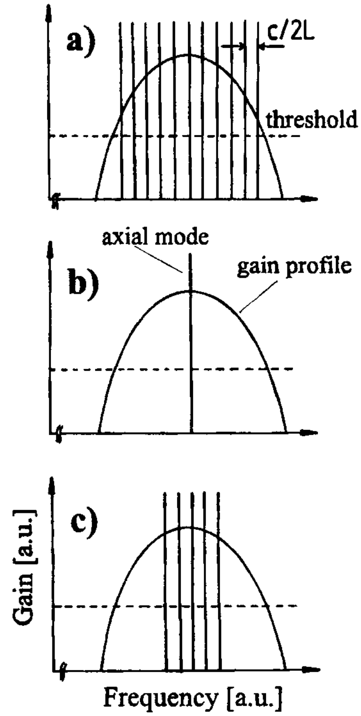

Fig. 22.1 Unsaturated gain profile and axial mode spectrum for a) inhomogeneous line broadening, b) homogeneous line broadening without spatial holeburning, c) homogeneous line broadening with spatial holeburning.

Due to the difference in gain saturation, different mode selecting techniques have to be applied to generate single axial mode operation in homogeneously and inhomogeneously broadened lasers. In the latter systems, losses have to be generated for the other axial modes, whereas in homogeneously broadened lasers, single axial mode operation can also be attained by preventing the spatial holeburning.

Single axial mode operation is needed in applications where a low bandwidth, a high temporal coherence, or a low noise is required. Examples are the measurement of atomic/molecular absorption spectra, the determination of small-scale shifts using Michelson interferometers (e.g. gravitational wave detector), and laser radar. Narrow linewidth lasers are also of interest for the development of length and wavelength standards. For all these applications, a relative frequency stability of better than 10^{-12} is required. In order to apply active frequency stabilization techniques it is necessary that the laser resonator oscillates in a single axial mode. Without frequency stabilization, the linewidth of a laser operating at one axial mode is on the order of MHz due to frequency variations caused by environmental fluctuations. The reduction of the mechanical instabilities by using rigid structures and the control of the pumping conditions can reduce the linewidths to values of several kHz. With active frequency stabilization techniques which lock the laser frequency to a reference frequency using a feedback loop, linewidths down to the mHz level have been attained [5.244,5.246,5.265-5.268,5.272].

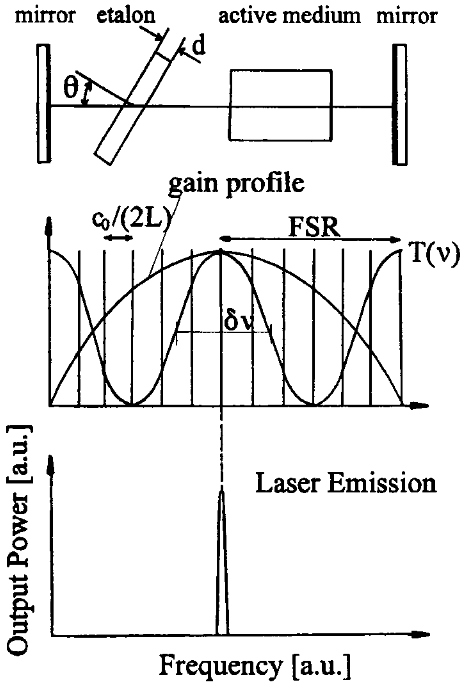

Fig. 22.2 Realization of single axial mode operation with an intracavity etalon with spectral transmission $T(\nu)$. The free spectral range (FSR) of the etalon is adapted to the gain bandwidth and the bandwidth $\delta\nu$ of the etalon is on the same order of magnitude as the axial mode separation. $c_0/(2L)$.

22.2 Axial Mode Selection with Intracavity Elements

In both homogeneously and inhomogeneously broadened lasers, single axial mode operation can be attained by generating losses for the other axial modes. This can be accomplished by using intracavity elements that exhibit a frequency dependent transmission. The most common technique is the insertion of an etalon into the resonator as illustrated in Fig. 22.2. An etalon of thickness d with an angle of θ between the surface normal and the optical axis and a reflectance R of both surfaces exhibits a spectral transmission of (we neglect any absorption losses and assume that the tilt angle θ is small):

$$T(\nu) = \frac{1}{1 + F\sin^2\delta} \tag{22.2}$$

with:

$$F = 4R/(1-R)^2$$

$$\delta = 2\pi\frac{d}{\lambda_0}\sqrt{n^2-\sin^2\theta}$$

where n is the index of refraction and λ_0 is the vacuum wavelength. The maximum transmission of $T=1.0$ is obtained at the frequencies:

$$\nu_q = q \frac{c_0}{2d\sqrt{n^2 - \sin\theta^2}} = q\,\Delta\nu \tag{22.3}$$

The frequency separation $\Delta\nu$ is called the free spectral range of the etalon. The spectral width (FWHM) of the transmission maxima is given by:

$$\delta\nu = \Delta\nu \frac{1-R}{\pi\sqrt{R}} \tag{22.4}$$

The etalon has to be designed such that the spectral transmission $T(\nu)$ forces all axial modes except one below the laser threshold. If $G(\nu)$ is the gain profile and V is the loss factor (=1-loss) per transit of the resonator (without etalon, but including output coupling), single mode oscillation occurs if $TGV>1$ for one axial mode and $TGV<1$ for all other modes. This is certainly the case, if the free spectral range $\Delta\nu$ of the etalon is larger than the gain bandwidth and the bandwidth $\delta\nu$ is smaller than twice the axial mode spacing. However, depending on the gain factor, the gain profile, and the homogeneous linewidth, etalons having a smaller FSR and a larger bandwidth may also work.

Example:
An Argon laser has a gain bandwidth of 5GHz (see Table 9.2). For a resonator length of 0.5m, the axial mode separation is 300MHz. In order to realize single mode operation, we use an etalon with a FSR $\Delta\nu$ of 5GHz and a bandwidth $\delta\nu$ of 600MHz. For a glass etalon (n=1.5) inserted at an angle of 5°, Eq. (22.3) yields a thickness of 2cm. According to (22.4), the bandwidth of 600MHz is attained for a reflectance of $R=69\%$.

According to (22.4), it is difficult to realize etalons that combine a large free spectral range and a small spectral width. For lasers with a wide gain profile (>30GHz), it is common to use two etalons having different spectral ranges (Fig. 22.3). The etalon with the smaller FSR suppresses the axial modes adjacent to the center mode and the second etalon discriminates against the axial modes located in the outer area of the gain profile. For a Nd:YAG laser with a resonator length of 1m (gain bandwidth: 120GHz, axial mode spacing: 150 MHz), single axial mode operation can be achieved with the following glass etalons [S.26]:

	d[mm]	R[%]	$\Delta\nu$[GHz]	$\delta\nu$[GHz]
etalon 1	20	60	7.5	1.2
etalon 2	0.5	60	300	49

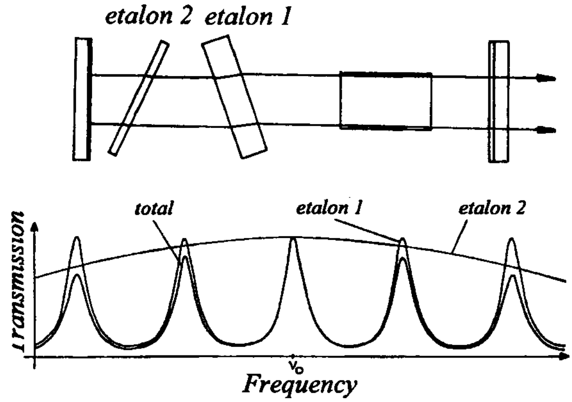

Fig. 22.3 Resonators with single axial mode selection using two etalons with different free spectral ranges. The lower graph shows the spectral transmission of each etalon and the resulting transmission when the etalons are combined [S.27].

Single frequency TEM_{00} mode operation operation with output powers of up to 35W have been attained with Yb:YAG disk lasers with two etalons inserted [5.274,5.275].

Axial mode selection can also be realized with intracavity Fabry Perot interferometers or Lyot filters [5.219,5.223,5.228]. The Lyot filter consists of a birefringent crystal placed between parallel polarizers. For a crystal length L and an index difference Δn between the ordinary and the extraordinary wave, the spectral transmission reads:

$$T(\nu) = T_0 \cos^2\left[\frac{\pi \Delta n L}{c_0}\nu\right] \tag{22.5}$$

where T_0 is the transmission of the crystal. The spectral width of the transmission maxima is equal to half the free spectral range of $\Delta\nu = c_0/(\Delta n L)$. The spectral width can be decreased by combining several Lyot filter with different lengths. For lasers with a wide gain profile (e.g. dye lasers, tunable solid state lasers), several etalons or Lyot filters have to be used in combination with prisms or gratings to achieve single mode operation.

22.3 Axial Mode Selection in Coupled Resonators

An alternate way to select one axial mode, used in gas and semiconductor lasers, is the coupling of two resonators. The simplest coupling scheme is the three mirror cavity depicted in Fig. 22.4. The axial mode spectrum of such a coupled resonator can be evaluated by using the matrix method presented in Sec. 4.3. After calculating the reflectance coefficient for light entering the system from the left, the resonance frequencies are determined by an infinite reflectance of the resonator [5.240]. Instead of going through these calculations, let us try to gain a basic understanding of the axial mode discrimination by considering the coupled resonator as a single resonator with length L_2 having an FPI with length L_1 as the output coupling mirror.

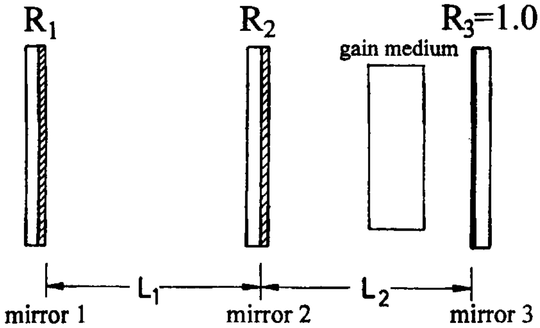

Fig. 22.4 Three mirror resonator.

As discussed in Sec. 4.2.1, the reflectance R of the FPI formed by mirror 1 and mirror 2 assumes a maximum when odd multiples of a quarter wavelength fit into the mirror spacing L_1 (anti-resonance). The maximum reflectance reads:

$$R_{max} = \left| \frac{\sqrt{R_1} + \sqrt{R_2}}{1 + \sqrt{R_1 R_2}} \right|^2 \tag{22.6}$$

and the frequency spacing of the reflectance maxima is given by the free spectral range:

$$\Delta\nu = \frac{c_0}{2L_1} \tag{22.7}$$

We can now tune the length L_1 until one axial mode of the resonator (mirror 2 and 3) is matched to the anti-resonance condition of the FPI. The wavelength λ of the matched axial mode fulfills the two conditions:

$$q\frac{\lambda}{2} = L_2 \quad , \qquad (2p-1)\frac{\lambda}{4} = L_1 \qquad p,q > 0 \qquad \begin{matrix}(22.8)\\(22.9)\end{matrix}$$

This axial mode experiences an effective output coupling reflectance given by (22.6). All other axial modes within the free spectral range of the FPI have a lower reflectance and, consequently, a higher laser threshold. Depending on the gain, the resonator geometry and the mirror reflectances, this loss difference may be sufficient to suppress the adjacent axial modes (see also Fig. 22.5.c). Due to the coupling between the two segments, the mode frequency will shift if one of the two lengths L_1,L_2 is varied.

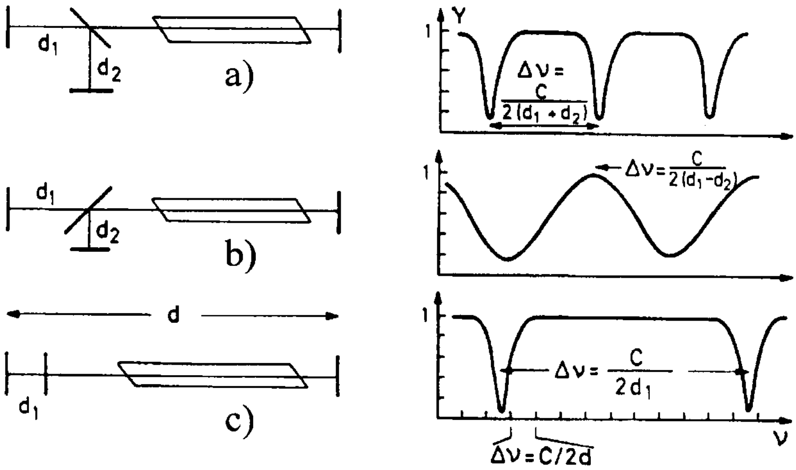

Fig. 22.5 Three mirror resonator schemes used for axial mode selection. The right curves are the round trip losses as a function of the light frequency [5.260]. a) Fox-Smith interferometer, b) Michelson interferometer, c) etalon mirror resonator. d_1, d_2, d are the geometric distances, c is the effective speed of light taking into account the refractive index of the medium.

As the length of the FPI section is scanned, the axial mode of the resonator will try to stay in the reflectance maximum by changing its wavelength. However, if the length L_2 of the resonator is not adjusted according to (22.8), this wavelength shift will generate losses and a jumping to the next order axial mode will occur. Tunability over a frequency range wider than the axial mode spacing $c_0/(2L_2)$, therefore, can only be realized if both resonator lengths are controlled.

Three mirror resonators are widely used in semiconductor lasers to generate single mode operation. In the cleaved coupled cavity approach, two diode lasers are attached with their cleaved end faces forming the resonator mirrors [5.235,5.238,5.262]. Axial mode selection and tunability is provided by controlling the length and the gain of each laser. In the external cavity technique, an external mirror and one endface of the semiconductor laser form the FPI [5.232-5.234,5.242,5.259,5.263,5.270]. The external mirror is moved by means of a piezoelectric transducer to select the axial mode. Some multi-mirror resonator designs used for axial mode control and frequency tuning in gas lasers are shown in Fig. 22.5 [5.224,5.226,5.240].

22.4 Resonators for Homogeneously Broadened Lasers

The basic principle of the resonator concepts discussed in the previous two sections is the generation of frequency dependent losses to control the axial mode spectrum. These resonator schemes can be applied to both inhomogeneously and homogeneously broadened lasers. Additionally, axial mode control in homogeneously broadened lasers can be achieved by preventing spatial holeburning or reducing its effect on the mode spectrum. At low to medium output powers, this second class of resonators is capable of providing single axial mode output without additional dispersive elements. At higher gain levels, however, etalons may have to be inserted to suppress the oscillation of adjacent axial modes. In the following, the different resonator schemes are briefly discussed:

a) Microchip Laser

For some solid state materials, the gain bandwidth is small enough to realize single axial mode operation with a short resonator. For a Nd:YAG laser with an optical resonator length of 3mm, the spectral mode separation $c_0/(2L)$ of 50GHz is comparable to the gain bandwidth of 120GHz. This means that no more than two axial modes can be supported. This concept of axial mode control is inherent to laser diode pumped microchip lasers. The resonator is formed by the HR coated endface of the crystal and an external output coupling mirror. The external mirror can be moved by piezoelectric transducers to maintain single mode operation. Nd:YAG and Nd:YVO_4 microchip lasers having crystal lengths of less than 1mm and optical resonator lengths between 3mm and 5mm provide single axial mode operation with output powers in the 100mW range and optical-to-optical efficiencies of better than 30% [5.247,5.256,5.257,5.272]. For materials with a larger bandwidth (e.g. Nd:glass, Er:Yb:glass), uncoated etalons are inserted to provide axial mode control and frequency tuning [5.269]. In 1996, an actively frequency stabilized Nd:YVO_4microchip laser was used to attain a linewidth of less than 1mHz [5.272].

b) Resonator with a Thin Gain Medium

In standing wave resonators, all axial modes have a common node at the resonator mirrors. Over a short distance from the mirrors (on the order of $L/100$ where L is the optical resonator length), the peaks of the standing wave intensity patterns of different axial modes will therefore nearly coincide. If one mirror is attached to a thin homogeneously broadened gain medium, the nearly identical spatial holeburning of different modes will lead to single axial mode operation. The axial mode with the highest small-signal gain (in the center of the gain profile) will oscillate first and saturate the gain to its threshold value. The other axial modes are suppressed since they are amplified in the same gain areas and have a higher threshold. This scheme has been used in dye lasers and diode pumped solid state lasers. In [5.252], a 2mm thick Nd:YVO_4 crystal was end-pumped by a 200mW laser diode array to generate a single frequency output power of 35mW in a 20mm long resonator. The scheme works in this case because Nd:YVO_4 exhibits a very small absorption length of 90µm. At higher pump powers, however, the gain deposited farther inside the crystal became high enough to support several axial modes. This is due to the increasing spatial dephasing of the standing wave patterns at larger distances from the mirror. To sustain single mode emission, etalons have to be inserted into the cavity. This is also true for thin disk lasers, where the disk thickness of several 100 microns is too large to sustain single mode lasing. Single mode output powers of up to 35W have been achieved with Nd:YAG disk lasers using two etalons (e.g. 0.1mm and 1.0mm thick fused silica) [5.274,5.275]

In some commercial diode pumped solid state lasers, the thin gain medium is placed in the middle of a symmetric resonator. In cw operation, these lasers oscillate on two axial modes. Inside the medium, the standing wave pattern of one mode is shifted by $\lambda/4$ with respect to the second pattern, leading to a homogeneous axial gain saturation.

c) Twisted Mode Resonator

In a standing wave resonator, spatial holeburning can be prevented by using orthogonal polarization states for the back and the forth traveling wave. In the twisted mode resonator [5.220, 5.285], this is accomplished by placing the active medium between two quarter wave plates. The fast axes of the plates are rotated by +45° and -45° with respect to the pass direction of a polarizer placed in front of one mirror. Inside the active medium, the counterpropagating waves are circularly polarized (see Sec. 8.2). As a result the average intensity is homogeneous along the optical axis. This resonator concept, however, has not found widespread application.

d) Unidirectional Ring Resonators

In solid state and dye lasers, ring resonators with unidirectional beam propagation are usually used to achieve single frequency output [5.176,5.186,5.225,5.227,5.231, 5.239,5.260,5.271] (see Chapter 21). The unidirectionality prevents spatial holeburning, thereby enabling one axial mode to saturate the entire gain. The conventional ring geometry is more versa tile than the resonator schemes presented above because there are no constraints on the size of the active medium or the resonator length. Furthermore, additional tuning elements like prisms, gratings, Lyot filters, or etalons can be easily incorporated into the resonator. This is of particular importance for lasers having a large gain bandwidth like dye lasers and tunable solid state lasers. Typical output powers of single mode dye ring lasers are in the Watt range. Five times higher single frequency output power has been achieved with Ar laser pumped Ti:Al_2O_3 ring oscillators with a tuning range of 650-1200nm in pulsed operation. The single frequency output powers of diode pumped solid state ring lasers are as high as 20W with optical-to-optical efficiencies of 50% and more. With internal frequency doubling, a single mode output powers of greater than 8.5W at 532nm have been achieved in [5.273] using a diode pumped Nd:YVO_4 ring oscillator with LBO (20W optical pump power). Recently the output power of this single mode ring resonator has been increased to 18W at 532nm (Verdi V18). Very high stability and narrow linewidths can be obtained with diode pumped monolithic nonplanar solid state laser oscillators (Nd:YAG, Nd:GGG) (see Sec. 21.3) [5.239,5.248,5.255]. Due to the rigid structure, linewidths of less than 3kHz can be attained in free-running mode. With active frequency stabilization, linewidths in the sub-Hertz level have been measured [5.261,5.265].

22.5 Micro-Optical Resonators

Micro-optical resonators have dimensions comparable to the wavelength of light. The eigenmodes have, in principle, the same structure as those of macroscopic resonators, as long as real eigenfrequencies exist. In most cases closed cavities are used as shown in Figs. 22.6 and 22.7. The eigenmodes depend on the geometry. For a cubic cavity (Fig. 22.8) with length a in the three directions and refractive index n, the normalized amplitude reads:

$$f(x) = \sin(2\pi x/\lambda_x)\,\sin(2\pi y/\lambda_y)\,\sin(2\pi z/\lambda_z) \tag{22.10}$$

with the resonance condition for metallic walls:

$$a = q\,\lambda_{x,y,z}/(2n) \tag{22.11}$$

The active media of micro-lasers are quantum dots (semiconductor emitters), as indicated in Fig. 22.7. The wavelength of these InAs-dots is about 1 μm and they excite the gallery modes of the cylindrical resonator modes travelling around the cylinder envelope. The dots are optically pumped by a Ti-sapphire laser.

The interesting feature of micro-resonators is the possibility to control the spontaneous emission, which means that the spontaneous emission can be enhanced or suppressed. A review of these effects was given by Haroche and Kleppner [5.276].

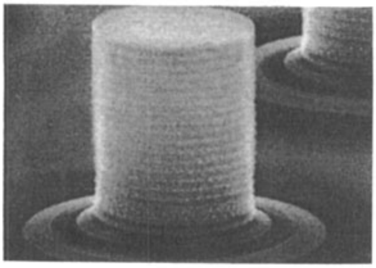

Fig. 22.6 A cylindrical micro-resonator. The height is approximately 4μm (courtesy of M. Bayer, Universität Würzburg, Germany) [5.283].

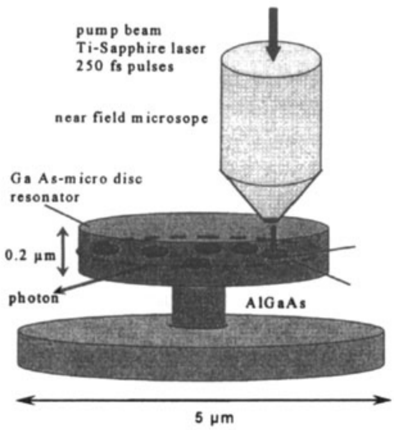

Fig.22.7 A GaAs micro-disk resonator (Michler and Becher). Universität Würzburg, Germany) [5.282]. The quantum dots represent the active medium.

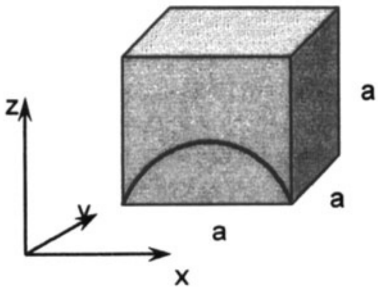

Fig. 22.8 The cubic micro-cavity with the fundamental mode in the x-direction.

Spontaneous emission as well as the photon character of light are a result of the quantization of the electromagnetic field. This theory had already been developed in 1930 by Wigner and Weisskopf [5.277], and Purcell was the first to discuss the influence of a cavity on spontaneous emission in 1946 [5.280]. With the advent of micro-resonators, this theory can now finally be proven. One theoretical prediction is the exponential decay of excited atoms, even if they do not interact with a field. The number n_2 of atoms in the upper level decays according to:

$$n_2(t) = n_2(0)\exp(-\Gamma t) \tag{22.12}$$

Fermi's golden rule can be applied to the decay rate Γ: the decay rate of an excited system is proportional to the density $D(\omega_E)$ of the lower states. As shown in many textbooks [5.278, 5.279], the decay rate is given by:

$$\Gamma = \frac{2\pi}{3\epsilon_0}\frac{|\mu_{12}|^2\omega_E}{\hbar V} D(\omega_E)\, |f(r_E)|^2 \tag{22.13}$$

with:	μ_{12}	:	dipole moment of the transition
	V	:	volume of the cavity
	$f(r_E)$	:	amplitude of the normalized eigenmode (22.10)
	r_E, ω_E	:	emitter position and emitter frequency

For free space, the density of modes is given by Rayleigh's formula:

$$D(\omega_E) = \frac{\omega_E^2\, V}{\pi^2\, c^3} \tag{22.14}$$

Insertion of (22.14) into (22.13) yields the spontaneous emission rate of free atoms:

$$\Gamma_0 = \frac{1}{\tau} = \frac{|\mu_{12}|^2\, \omega_E^2}{2\pi\epsilon_0\, \hbar c^3} \tag{22.15}$$

where τ is the upper state lifetime due to spontaneous emission. For a small resonator with a single resonance in the range of the gain profile, the density of states is given by the resonator response. It can be approximated by a Lorentzian line profile (see Fig. 22.9):

$$D(\omega_E)_{resonator} = \frac{2}{\pi\Delta\omega_C}\frac{1}{1+(\omega_E-\omega_C)^2/(\Delta\omega_C/2)^2} \tag{22.16}$$

where $\Delta\omega_C$ is the bandwidth of the resonator and ω_C is the resonance frequency. Insertion into (22.13) delivers the decay rate [5.279]:

$$\Gamma_C = \frac{1}{\tau_C} = \Gamma_0\frac{4c^3}{\Delta\omega_C\omega_E^2 V}\frac{1}{1+(\omega_E-\omega_C)^2/(\Delta\omega_C/2)^2} \tag{22.17}$$

The decay time $\tau_C = 1/\Gamma_C$ can be longer or shorter than the natural decay time $\tau = 1/\Gamma_0$ depending on the frequency and position of the emitters and the bandwidth of the cavity. An example is shown in Fig. 22.10. The ratio of cavity decay time τ_C to free space decay time τ is plotted versus detuning . In resonance the decay time is reduced by a factor of three. For the detuned resonator the decay time is much longer than the natural decay time. The effect strongly depends on the resonator bandwidth. A high-Q (coated resonator) produces a much stronger effect than a low-Q resonator.

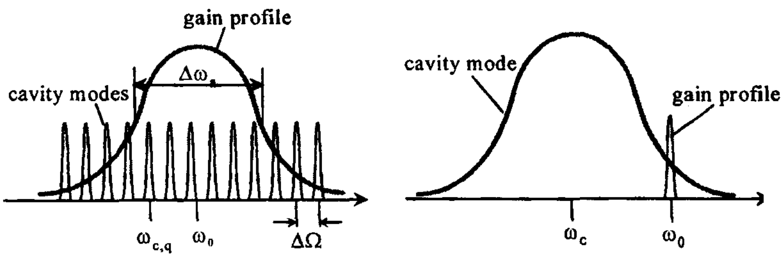

Fig.22.9 Gain profile and the cavity modes for a large (left) and a small (right) resonator.

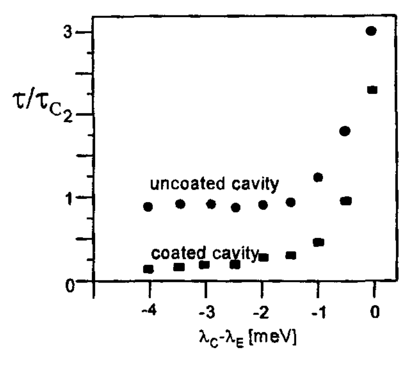

Fig.22.10 Ratio of spontaneous decay time τ to spontaneous emission time τ_C of a coated and an uncoated cavity as a function of the wavelength mismatch between emission and cavity resonance [5.284].

Part VI

Measurement Techniques