Chapter 15

Misalignment Sensitivity of the Output Power

15.1 General Properties

The misalignment of the resonator mirrors results in a change of the output power of a laser resonator due to two mechanisms (we intentionally use the term change instead of decrease since in unstable resonators the power can also increase with mirror tilt); the diffraction losses are increased leading to a higher laser threshold and the cross sectional area of the laser beam in the medium is decreased resulting in a lower mode volume (Fig. 15.1). The resonator scheme determines which of these two effects dominates. In stable resonators with multimode operation, the decrease of the power is caused by the decreasing mode volume since additional diffraction losses are only generated if low order transverse modes get clipped by an aperture. In fundamental mode operation, on the other hand, the change in mode volume does not play any role and the output power drops due to the increased losses. We discussed the sensitivity of the losses to misalignment in Sec. 5.4 and Sec. 7.4. The reader who is not familiar with the properties of passive misaligned resonators should first go through Sec. 5.4.

Independent of the resonator type, the output power of a misaligned resonator can be investigated by using the standard model for the output power. The functional dependencies of the resonator parameters on the tilt angle have to be inserted into expression (10.7) for the output power of homogeneously broadened lasers.

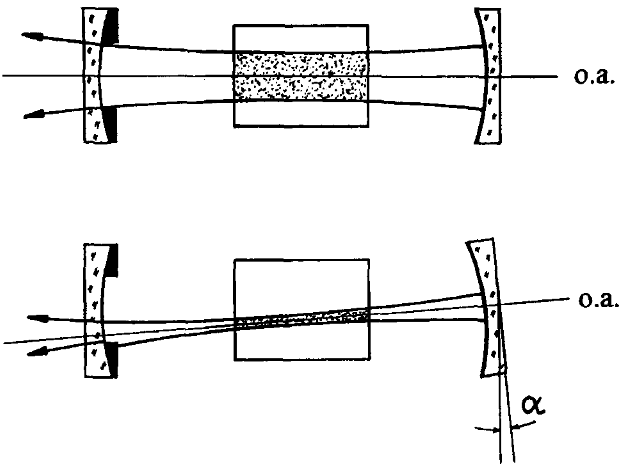

Fig. 15.1 Decrease of the output power of a misaligned stable resonator due to increased diffraction losses and a lower mode volume (o.a.: optical axis).

If we assume that the diffraction losses occur at the high reflecting mirror, the modified expression (10.7) for the output power of a misaligned stable resonator reads:

$$P_{out}(\alpha) = A_b(\alpha)\, I_S \frac{1-R}{1-RV(\alpha)+\sqrt{RV(\alpha)}[1/V_S-V_S]} \left[g_0\ell - |\ln\sqrt{RV_S^2V(\alpha)}|\right] \tag{15.1}$$

where $A_b(\alpha)$ is the cross sectional area of the laser beam, $V(\alpha)$ is the diffraction loss factor per round trip (=1-loss), $g_0\ell$ is the small-signal gain, I_S is the saturation intensity, and V_S is the loss factor per transit of the active medium. This equation can also be applied to unstable resonators if the diffraction loss factor is set to 1.0 and the reflectance R is replaced by the output coupling loss factor V(α). The corresponding expression reads:

$$P_{out}(\alpha) = A_b(\alpha)\, I_S \frac{1-V(\alpha)}{1-V(\alpha)+\sqrt{V(\alpha)}[1/V_S-V_S]} \left[g_0\ell - |\ln\sqrt{V_S^2V(\alpha)}|\right] \tag{15.2}$$

The dependence of the loss factor $V(\alpha)$ on the tilt angle α was discussed in Sec. 5.4 for stable resonators and in Sec. 7.4 for unstable resonators. For single transverse mode operation, the loss factor decreases parabolically with the tilt angle of a resonator mirror:

$$V(\alpha) = V(0)\left[1 - 0.1\left(\frac{\alpha}{\alpha_{10\%}}\right)^2\right] \tag{15.3}$$

where $\alpha_{10\%}$ denotes the tilt angle at which the losses have increased by 10%, a parameter used to define the misalignment sensitivity of the resonator. In a similar way, we can define a 10%-angle $\beta_{10\%}$ at which the output power has dropped by 10% and use this quantity as a figure of merit for the tilt sensitivity of the output power. With the results presented in Sec. 5.4 and 7.4 we can already calculate the 10%-angle $\alpha_{10\%}$ for the losses of stable resonators in fundamental mode operation and of unstable resonators. Typically mirrors can be tilted by an angle on the order of 50-100μrad before the diffraction losses increase by 10%. For such small tilt angles, the change of the mode volume is too small to have a noticeable effect on the power. Therefore, we can use a constant cross sectional area A_b in (15.2) and (15.3). This simplification enables us to determine the ratio $\beta_{10\%}/\alpha_{10\%}$ directly from these equations which means we compare the sensitivity of the output power to the known sensitivity of the losses.

For stable resonators in multimode operation, the losses do not increase for small tilt angles. Only if low order transverse modes get limited, do the losses increase. However, tilt angles on the order of 10mrad are necessary to make this happen. For smaller tilt angles the decrease of the output power is only caused by the decreasing mode volume. We can thus investigate the misalignment sensitivity by expressing the mode volume as a function of the tilt angle. The 10%-angle at which the mode volume has decreased by 10% defines the misalignment sensitivity.

15.2 Stable Resonators in Multimode Operation

15.2.1 Without Thermal Lensing

The tilt of one or both resonator mirrors leads to a rotation of the optical axis. The optical axis is defined by the line connecting the center of curvatures of the two resonator mirrors (Fig. 15.2). With both mirrors aligned the modes fill the entire medium since it represents the only mode-selecting aperture in the resonator. As the optical axis is rotated due to misalignment, the mode structure gets cut at one edge of the medium (lower left corner in Fig. 15.2) resulting in a decrease of the mode volume [4.136,4.140]. However, the optical axis remains the center of gravity of the mode structure which means that the beam gets clipped on both sides although only one side is affected by an edge. This effect can be easily observed experimentally by moving a knife edge into a stable multimode resonator, as shown in Fig. 15.3.

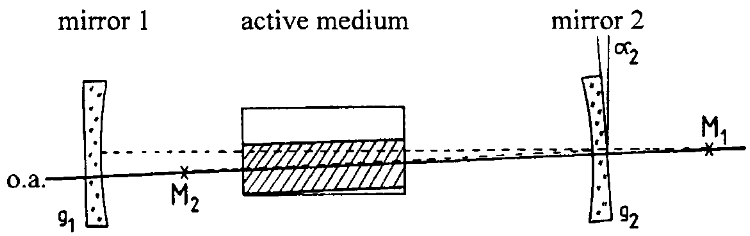

Fig. 15.2 The beam in misaligned stable multimode resonators stays symmetric to the optical axis (a.o.). The optical axis is defined by the line going through both centers of curvature of the mirrors.

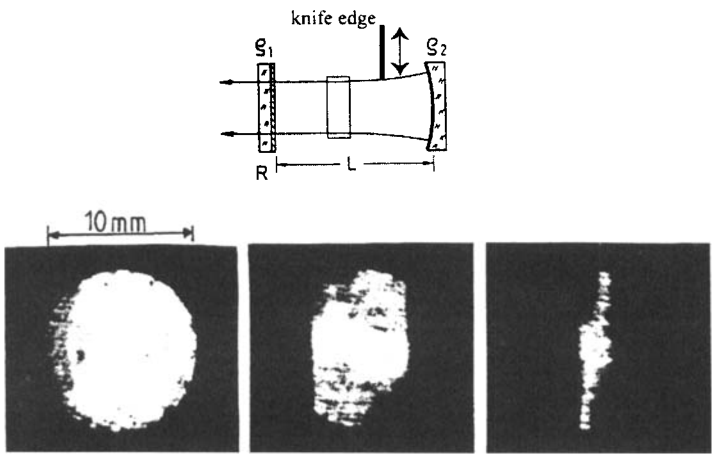

Fig. 15.3 A knife edge moved into a stable multimode resonator clips the beam symmetrically to the optical axis. The intensity profile on the high reflecting mirror 2 is shown (Nd:YAG laser in single shot operation, rod diameter : 9.5mm, $g_1=1$, $g_2=0.75$, $L=0.5m$).

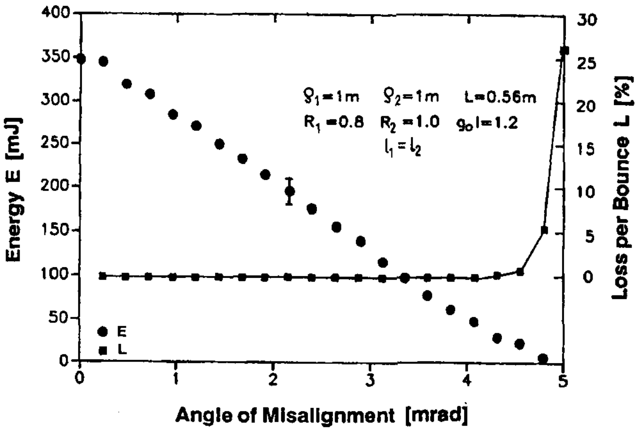

Fig. 15.4 Measured output energy per pulse of a Nd:YAG laser (rod radius: 3.17mm, length: 75mm) with a symmetric stable resonator in multimode operation ($\rho_1=\rho_2=1m$, geometrical resonator length: 0.56m) as a function of the tilt angle of one mirror. The lower curve is the measured diffraction loss per bounce. The diffraction losses increase only at large tilt angles at which the fundamental mode gets clipped. For smaller tilt angles the decrease of the output energy is caused by the decreasing mode volume [4.140] (© Taylor & Francis 1992).

This behavior is to be expected since the knife edge generates losses for the highest order modes and forces these modes to stop oscillating. Since all transverse modes exhibit mirror symmetry with respect to the optical axis, the beam must stay symmetric. The cross sectional area $A_b(\alpha)$ of the beam in the misaligned resonator can now be easily calculated. For a radius b of the active medium and small angles of misalignment one gets (Fig. 15.4):

$$A_b(\alpha) = \pi b^2 \left[1 - \frac{4\,\max(h_1,h_2)}{\pi\, b}\right] \tag{15.4}$$

where the function *max* chooses the maximum of the shifts h_1 and h_2. Since both shifts are proportional to the angle of misalignment, (15.4) predicts that the output power decreases linearly with the mirror tilt. This is in agreement with measurements as shown in Fig. 15.4. Therefore, we can define the 10%-angle $\beta_{10\%}$ at which the output power has decreased by 10% using the following linear expression:

$$A_b(\alpha) = \pi b^2 \left[1 - 0.1\left(\frac{\alpha}{\beta_{10\%}}\right)\right] \tag{15.5}$$

Since both mirrors can be tilted we assign 10%-angles for each mirror, $\beta_{10\%,1}$ if mirror 1 is misaligned and $\beta_{10\%,2}$ for mirror 2.

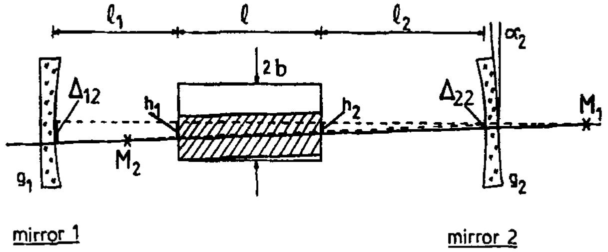

Fig. 15.5 Schematic of a misaligned stable resonator showing all the parameters used in Eq. (15.6) [4.140] (© Taylor & Francis 1992).

The two shifts h_1 and h_2 can be calculated geometrically by using the expressions for the shifts Δ_{ij} of the intersection points of the optical axis on the mirrors (see Sec. 5.4). The combination of (15.4) and (15.5) yields for the 10%-angle $\beta_{10\%,i}$ at which the mode cross section and the output power have decreased by 10% when mirror i is tilted (Fig. 15.5):

$$\beta_{10\%,i} = \frac{0.025\ \pi\ b}{L_{eff}} \frac{|1 - g_1 g_2|}{|1 - x_j/\rho_j|}\ , \qquad i,j=1,2;\ i \neq j \tag{15.6}$$

with:

$x_j = \ell_j - \ell(n-1)/(2n)$ if $\rho_j > \ell_j + \ell/(2n)$

$x_j = \ell_j + \ell(n+1)/(2n)$ else

$L_{eff} = \ell_1 + \ell_2 + \ell/n$: effective resonator length

n : index of refraction of the medium

Figure 15.6 shows the experimental verification of this relation for a Nd:YAG laser with different resonators. In these graphs, the 10%-angle is plotted as a function of the effective resonator length. In the case where the resonator goes unstable, (15.6) is not valid anymore and the theoretical curves, therefore, are broken in these areas. The good agreement enables one to use (15.6) to find improved resonator designs providing minimum misalignment sensitivity. For a tilt of mirror i, the maximum 10%-angle is obtained if the center of curvature of mirror j is located in the middle of the medium, which means that $\ell_j = \rho_j - \ell/(2n)$ holds. In this case, (15.6) yields for the 10%-angle:

$$\beta_{10\%,i} = \frac{0.05\ \pi\ b}{L_{eff}\ \ell} |\rho_j\ (1 - g_1 g_2)| \tag{15.7}$$

An example of a stable resonator optimized in this manner is presented in Fig. 15.7. For the rod radius of 3.17mm used, 10%-angles in excess of 5mrad become possible. This is one order of magnitude higher compared to the non-optimized resonators in the previous figure.

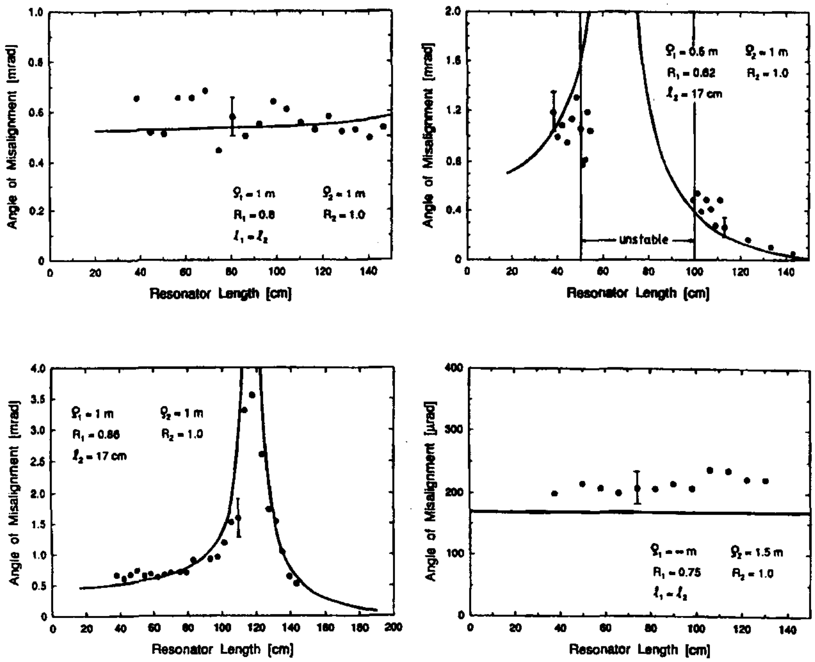

Fig. 15.6 Dependence of the measured 10%-angle $\beta_{10\%,2}$ for four stable resonators on the effective resonator length L_{eff} (pulsed Nd:YAG rod laser in single shot operation, b=*3.17mm*, ℓ=*75mm*). The theoretical curves calculated with (15.6) are marked by solid lines. Resonator data are presented in each graph. In the unstable region (top right graph), Eq. (15.6) does not apply (broken line) [4.140] (© Taylor & Francis 1992).

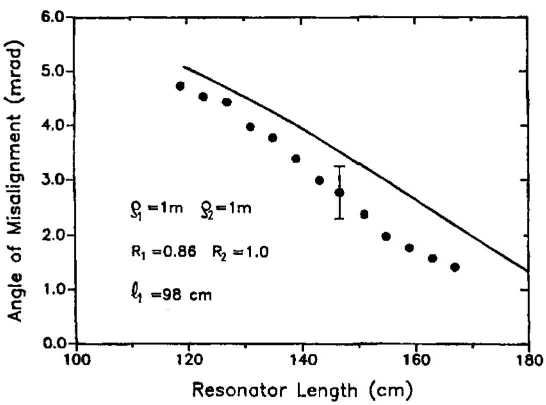

Fig. 15.7 Measured 10%-angles $\beta_{10\%,2}$ for a stable resonator with minimum misalignment sensitivity of mirror 2 according to (14.7) as a function of the effective resonator length L_{eff} (pulsed Nd:YAG rod laser, b=*3.17mm*, ℓ=*75mm*). The center of curvature of mirror 1 is in the middle of the rod. The solid line represents (15.7) [4.140] (© Taylor & Francis 1992).

As a rule of thumb, the 10%-angles $\beta_{10\%,i}$ of stable resonators in multimode operation are on the order of 0.2mrad per mm of rod radius. However, by choosing optimized resonator designs (small g-parameter product and centers of mirror curvature inside the active medium) it is possible to increase this value by one order of magnitude.

15.2.2 With Thermal Lensing

In high power solid state lasers the refractive power D of the active medium affects the rotation of the optical axis if the resonator is misaligned. The expression for the mode cross section derived above, however, is still valid. Thus we have only to investigate how the shifts h_1 and h_2 of the optical axis are changed by the refractive power. In order to simplify the discussion, we use a thick lens, the two principal planes of which are located at a distance d_i from mirror i (Fig. 15.8). In this approximation, the optical axis is shifted parallel by a displacement h between the principal planes. The result of any bending of the optical axis inside the medium is considered by different slopes of the axis outside the medium. These slopes are determined by the fact that the centers of curvature of both mirrors act as virtual sources of the optical axis as indicated in Fig. 15.8 [4.136,4.140]. The calculation of the shift h is then simple, and if mirror i is tilted by an angle α_i the final result reads:

$$h = \left| \frac{\rho_i \, \alpha_i}{(d_i - \rho_i) \, [D - 1/(d_1 - \rho_1) - 1/(d_2 - \rho_2)]} \right| \qquad (15.8)$$

where ρ_i is the radius of curvature of mirror i.

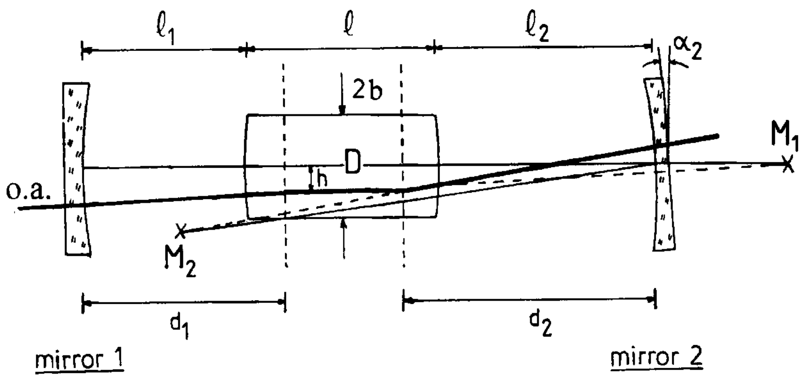

Fig. 15.8 Optical axis in a misaligned resonator with an internal lens. On both sides of the lens the optical axis seems to emerge from a virtual source defined by the centers of curvature M_i of the mirrors [4.140] (© Taylor & Francis 1992).

For a rod with radius b, the 10%-angle $\beta_{10\%,i}$ at which the output power has decreased by 10% when mirror i is tilted becomes:

$$\beta_{10\%,i} = 0.025\ \pi\ b \left| \frac{d_i - \rho_i}{\rho_i} \left[D - \frac{1}{d_1 - \rho_1} - \frac{1}{d_2 - \rho_2} \right] \right| \qquad (15.9)$$

This equation is, of course, only valid if the resonator is in a stable zone in the g-diagram, which means that the refractive power D is within the limits defined by the characteristic refractive powers D_I-D_{IV} (see Sec. 13.1.2). Note that (15.6) is not exactly obtained for D going to zero. This is due to the thick lens approximation used since any difference between the shifts h_1 and h_2 (see Fig. 15.5) has been neglected. Nevertheless, (15.9) can be used to predict the misalignment sensitivity of stable resonators with a thermal lens to a high accuracy as the measurements presented in Fig. 15.9 indicate. No matter what kind of resonator is used, they all have in common the characteristic, that an increasing refractive power D results in a linear increase of the 10%-angle. The thermal lens thus has a stabilizing effect on the resonator.

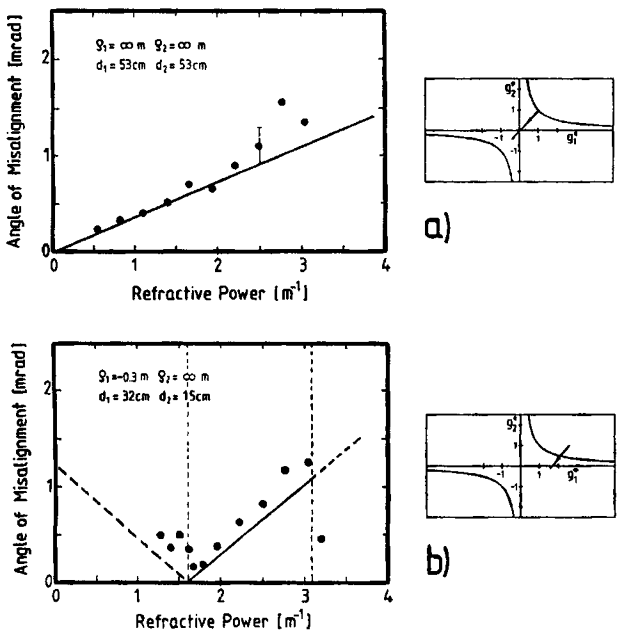

Fig. 15.9 Dependence of the measured 10%-angles $\beta_{10\%,2}$ of a high power Nd:YAG rod laser for two different resonators on the refractive power D of the rod (*b=4.75mm, ℓ=150mm*). The equivalent g-diagram of these resonators is shown as well. The solid lines represent (15.9) [4.140] (© Taylor & Francis 1992).

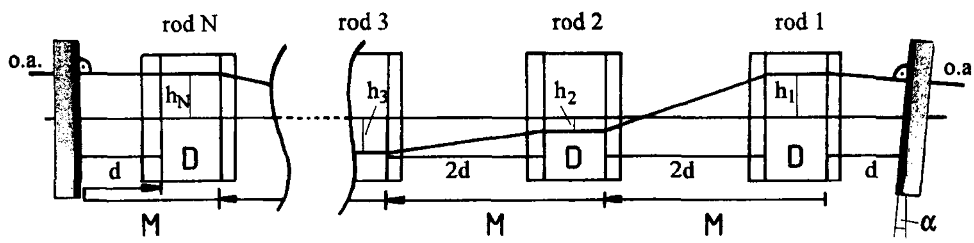

Fig. 15.10 The optical axis in a misaligned symmetric flat-flat resonator with N thermal lenses with refractive power D.

Misalignment in Symmetric Multirod Resonators

The misalignment in symmetric flat-flat resonators with N thermal lenses can be treated in a similar way. The tilt of one mirror by an angle α generates different transverse shifts h_i of the optical axis in the rods (Fig. 15.10). On both mirrors the optical axis is normal to the mirror surface. The shifts can be calculated by repetitively applying the ray transfer matrix M on the ray vector (h_1, α). This ray vector represents the optical axis as it is incident on the first principal plane of rod 1. The ray transformation equations for a total of N rods read:

$$\begin{pmatrix} h_{m+1} \\ \alpha_{m+1} \end{pmatrix} = M^m \begin{pmatrix} h_1 \\ \alpha \end{pmatrix}, \qquad m = 0,\dots,N-1 \tag{15.10}$$

with: $$M = \begin{pmatrix} 1-2dD & 2d \\ -D & 1 \end{pmatrix},$$ D: refractive power per rod

d : distance as shown in Fig. 15.10

By using Sylvester's Theorem (1.46) for periodic optical systems one gets:

$$\begin{pmatrix} h_{m+1} \\ \alpha_{m+1} \end{pmatrix} = \frac{1}{\sin\Phi} \begin{pmatrix} (1-2dD)\sin[m\Phi] - \sin[(m-1)\Phi] & 2d\sin[m\Phi] \\ -D\sin[m\Phi] & \sin[m\Phi] - \sin[(m-1)\Phi] \end{pmatrix} \begin{pmatrix} h_1 \\ \alpha \end{pmatrix}$$

with: $\cos\Phi = 1 - dD$ and $m = 0,\dots,N-1$ (15.11)

At the left mirror, the angle of the optical axis must be equal to zero. To get there we have to apply another matrix M to the last ray. The second row of (15.11) thus provides the shift h_1 if we set m equal *to* N:

$$h_1 = \frac{\sin[N\Phi] - \sin[(N-1)\Phi]}{D \sin[N\Phi]} \alpha \tag{15.12}$$

Insertion of (15.12) into (15.11) results in the final expression for the absolute values of the shifts h_m. In contrast to (15.11), the index m is now going from 1 to N:

$$h_m = \left| \frac{(\sin[N\Phi]-\sin[(N-1)\Phi])(\sin[(m-1)\Phi]-\sin[(m-2)\Phi]) + 2dD\sin[(N-1)\Phi]\sin[(m-1)\Phi]}{D \sin[N\Phi] \sin\Phi} \right| \alpha$$

$$:= \frac{C_{mN}}{D\,|\sin[N\Phi]\,\sin\Phi|} \alpha \tag{15.13}$$

The rod that exhibits the largest shift h_m acts as the limiting aperture for the laser beam, whereas the beam can propagate freely in the other rods. The maximum of the shifts h_m, therefore, determines the reduction of the fill factor in all rods. At the angle $\alpha=\beta_{10\%}$, the mode volume and the output power have decreased by 10%, which means that the fill factor γ is equal to 0.9:

$$\gamma = \frac{\pi b^2 - 4b \max(h_m)}{\pi b^2} = 0.9 \tag{15.14}$$

where the function *max* determines the maximum of N values ($m=1,..,N$). By inserting (15.13) into (15.14) the 10%-angle is obtained:

$$\beta_{10\%} = \frac{0.025\ \pi\ b\ D\ |\sin[N\Phi]\ \sin\Phi|}{\max(C_{mN})} \tag{15.15}$$

For a single rod ($N=1$) this expression is equivalent to (15.9):

$$\beta_{10\%} = 0.025\ \pi\ b\ D \tag{15.16}$$

For two rods, the shift h_2 in the rod closer to the aligned mirror is larger for $D<1/d$. The 10%-angle reads:

$$\beta_{10\%} = 0.025\ \pi\ b\ 2D|(1-dD)|\ , \qquad for\ D < 1/d$$
$$\beta_{10\%} = 0.025\ \pi\ b\ 2D\frac{|1-dD|}{1-2dD}\ , \qquad for\ 1/d \le D \le 2/d \tag{15.17}$$

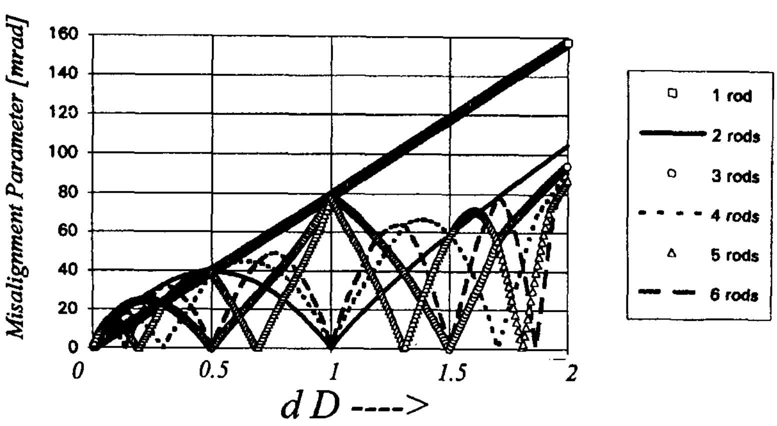

Fig. 15.11 Misalignment parameter $\beta_{10\%}\, d/b$ as a function of dD for symmetric flat-flat resonators with one to six rods (calculated with (15.15)); b: rod radius, d: distance between mirror and the adjacent principal plane of the first rod.

The 10%-angle goes to zero at $D=1/d$ and exhibits a local maximum for $D=1/2d$. The maximum is assumed when the equivalent lens resonator is confocal, a behavior that is to be expected since the confocal resonator generally shows a low misalignment sensitivity. Similar to the single rod, the largest 10%-angle is assumed at the stability limit with $D=2/d$.

For more rods, it is easier to find a numerical solution of (15.15). For the discussion of the misalignment sensitivity, it is more convenient to use the misalignment parameter $\beta_{10\%}d/b$ since, according to (15.13) and (15.15), this parameter is only a function of dD and, therefore, we can use a single graph to read off the 10%-angles of symmetric multirod resonators. This is shown in Fig. 15.11 in which the misalignment parameter is plotted versus dD for one to six rods. The range $0<dD<2$ represents the stable zone of the symmetric flat-flat resonator. The 10%-angle goes to zero if the equivalent resonator hits a stability limit ($g_1^* g_2^*=1$), and minima of the misalignment sensitivity are obtained when the confocal point ($g_1^*=g_2^*=0$) is passed. The refractive powers at which stability limits are reached are given by (see Table 15.1):

$$D = \frac{2}{d}\sin^2\left[\frac{m\,\pi}{2\,N}\right], \qquad m = 0,1,2,...,N \tag{15.18}$$

The equivalent resonator gets confocal at the refractive powers (Table 15.2):

$$D = \frac{2}{d}\sin^2\left[\frac{m\,\pi}{4\,N}\right], \qquad m = 1,3,5,... \text{ and } m\leq N \tag{15.19}$$

Note that the 10%-angle does not assume a maximum for the last passage through the confocal point. According to Fig. 15.11, the average misalignment parameter is about 40mrad. For typical rod radii *b* of 3.5-5mm, and distances *d* between 10-20cm, the corresponding 10%-angles are between 1 and 4mrad (see Fig. 15.12).

Table 15.1 Values of the product *dD* for which a stability limit is reached (*N*: number of rods).

m=	1	2	3	4	5	6
N=1	2.0	-	-	-	-	-
N=2	1.0	2.0	-	-	-	-
N=3	0.5	1.5	2.0	-	-	-
N=4	0.293	1.0	1.707	2.0	-	-
N=5	0.191	0.691	1.309	1.809	2.0	-
N=6	0.134	0.5	1.0	1.5	1.866	2.0

Table 15.2 Values of the product *dD* for which the confocal point is reached (*N*: number of rods).

m=	1	3	5
N=1	1.0	-	-
N=2	0.293	-	-
N=3	0.134	1.0	-
N=4	0.076	0.617	-
N=5	0.049	0.412	1.0
N=6	0.034	0.293	0.741

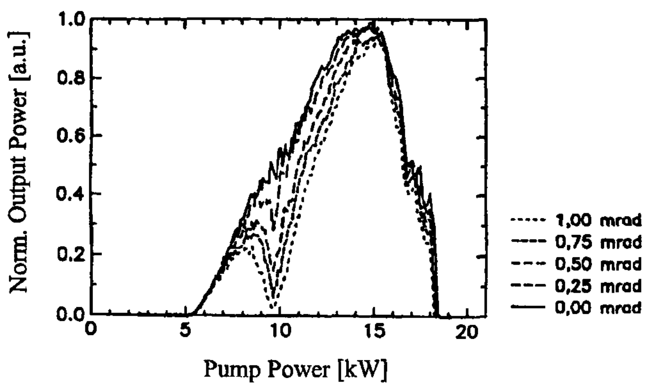

Fig. 15.12 Measured output power of a symmetric flashlamp-pumped dual rod Nd:YAG laser (*b=4mm*, *ℓ=150mm*, *d=0.5m*) with flat mirrors as a function of the total pump power for different tilt angles of one mirror [S.16]. The refractive power of each rod is 0.41 diopters per kW of pump power. The resonator hits the stability limit at a pump power per rod of 4.8kW. Note the high misalignment sensitivity at this point.

15.3 Stable Resonators in Fundamental Mode Operation

In fundamental mode operation the misalignment of the mirrors leads immediately to an increase of the diffraction losses. As mentioned before, the tilt angles at which the losses have increased by 10% typically are between 10 and 50μrad and thus too small to have a noticeable effect on the mode volume. Therefore, we can use the expression (15.1) for the output power with a constant cross sectional area on the beam and insert the tilt dependence of the diffraction loss factor V which is given by:

$$V(\alpha) = V(0)\ [1 - 0.1(\alpha/\alpha_{10\%})^2] \tag{15.20}$$

where $\alpha_{10\%}$ is the angle at which the diffraction losses have increased by 10%. In order to simplify our discussion, we approximate the fraction in (15.1) by $1\text{-}R$. This is a valid approximation for high mirror reflectances ($R>0.9$) and low small-signal gains ($g_0\ell<0.4$). The resulting expression reads:

$$P_{out}(\alpha) = A_b\ I_S\ (1-R)\ [g_0\ell - |\ln\sqrt{RV(\alpha)V_S^2}|] \tag{15.21}$$

Insertion of (15.19) yields the dependence of the output power on the tilt angle:

$$P_{out}(\alpha) = P_{out}(0) \left| 1 - \frac{|\ln\sqrt{1 - 0.1(\alpha/\alpha_{10\%})^2}|}{g_0\ell - |\ln\sqrt{RV(0)V_S^2}|} \right| \tag{15.22}$$

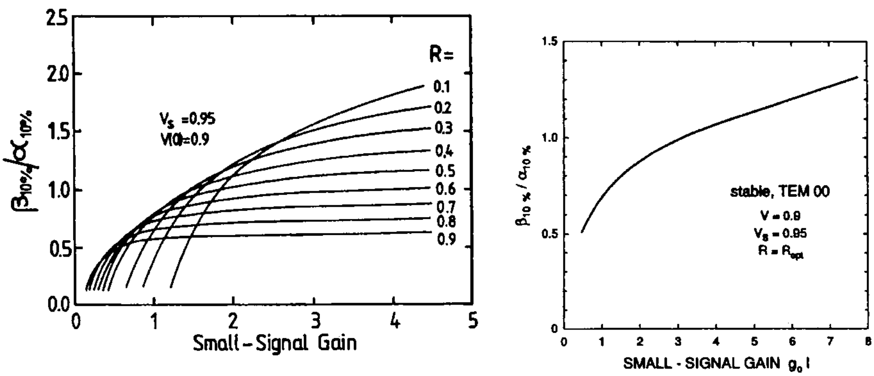

Fig. 15.13 Ratio $\beta_{10\%}/\alpha_{10\%}$ as a function of the small signal gain for $V(0)=0.9$, calculated by combining with (15.1) and (15.20). The angle $\beta_{10\%}$ is the tilt angle at which the output power has dropped by 10%. The left graph shows the angle ratios with the mirror reflectance R being the curve

parameter, the curve in the right graph represents the angle ratios for optimum output coupling.

Thus the output power becomes less sensitive to mirror misalignment as the small- signal gain is increased. The angle $\beta_{10\%}$ at which the power has dropped by 10% is given by:

$$\frac{\beta_{10\%}}{\alpha_{10\%}} = \sqrt{10\ (1\ -\ \exp[-0.2\ (g_0\ell\ +\ \ln\sqrt{RV(0)V_S^2})])} \tag{15.23}$$

This expression indicates that the power is more sensitive to misalignment than the diffraction losses, which means that the ratio $\beta_{10\%}/\alpha_{10\%}$ is less than 1.0. Unfortunately, due to the approximations made above, this equation is only applicable to low gain lasers with low output coupling. In all other cases the correct expression for the output power (15.1) has to be used to calculate the 10%-angle ratio by inserting (15.23) and numerically varying the tilt angle. However, the statement that the power is more sensitive to the tilt than the losses holds also for high gain lasers. The numerically calculated angle ratios are presented in Fig. 15.13 for stable resonators. These resonators have the mode selecting aperture adapted to the Gaussian beam radius (the adaptation generates diffraction losses of about 10% per round trip, see Sec. 11.2). The higher sensitivity of the output power with respect to the loss can also be confirmed experimentally, as shown in Fig. 15.14. The detailed discussion of the 10%-angles $\alpha_{10\%}$ in Sec. 5.4 enables us now to determine the misalignment sensitivity of any stable resonator in fundamental mode operation. Let us calculate an example to clarify this statement.

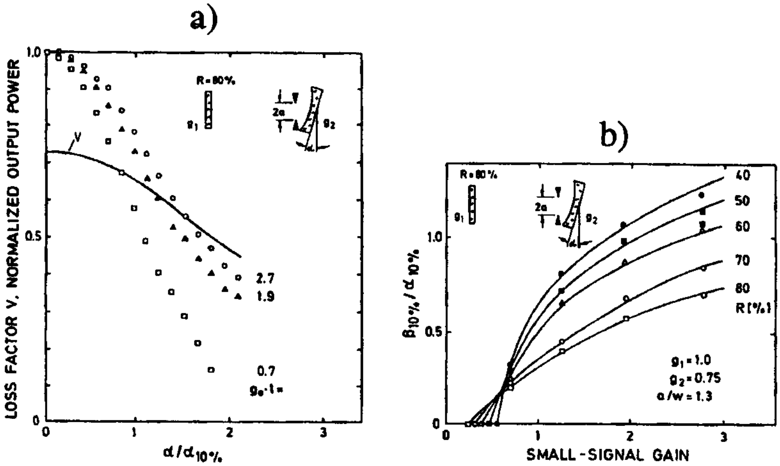

Fig. 15.14 a) Measured output energy per pulse and measured loss factor as a function of the tilt angle for a stable resonator in fundamental mode operation ($g_1=1.0, g_2=0.75, L_{eff}=0.5m, a/w_{00}=1.3$, Nd:YAG laser). The curve parameter is the small-signal gain. b) Measured 10% angle ratios for the resonator of graph a) as a function of the small-signal gain. The curve parameter is the reflectance of the output coupling mirror.

Example:
CO_2-laser (λ=10.6μm), tube diameter $2b = 6mm$, flat mirror 1 (output coupler) and 5m concave high reflecting mirror. The resonator length is 0.5m.

The following resonator properties are obtained:
g-parameters : $g_1 = 1.0$, $g_2 = 0.9$, $G = 2g_1g_2\text{-}1 = 0.8$
Gaussian beam radii : $w_1 = 2.25mm$, $w_2 = 2.37mm$
The tube is located at mirror 2 and the tube radius a is adapted to the Gaussian beam radius with $a/w_2=1.27$. Since the confined mirror is mirror 2, the effective Fresnel number is given by:

$$N_{eff} = \frac{b^2}{2Lg_1\lambda} = 0.849$$

Please do not get confused by the fact that in Sec. 5.4 the aperture is placed at mirror 1. We simply have to swap g_1 and g_2 (as we already did in the expression for the effective Fresnel number). If the unconfined mirror is tilted (output coupler), Fig. 5.49 provides a misalignment parameter $L\alpha_{10\%}/a$ of 45mrad for $G=0.8$ and $N_{eff}=0.849$. This results in a 10%-angle $\alpha_{10\%}$ of 27μrad. If the confined mirror 2 is tilted, the corresponding 10%-angle is $\alpha_{10\%}/g_1$, which means that for this resonator both mirrors exhibit the same 10%-angle. According to Fig. 15.12, the 10%-angle $\beta_{10\%}$ ranges from 13μrad to 25μrad for small-signal gains between 0.5 and 3.0 and optimum output coupling.

15.4 Unstable Resonators

15.4.1 Without Thermal Lensing

In contrast to stable resonators, the mirror misalignment in unstable resonators results in an increase of the output coupling (remember that the loss factor V of unstable resonators corresponds to the mirror reflectance R of a stable resonator). Again, the mode volume is not affected by the misalignment since the tilt angles are too small. In order to investigate the misalignment sensitivity of the output power it is, therefore, sufficient to discuss the dependence of the power on the output coupling (Fig. 15.15). If the aligned resonator is undercoupled, which means that the loss factor is higher than the optimum loss factor V_{opt}, the resonator virtually moves to the left along the power curve and the output power is increased. Accordingly, overcoupled resonators decrease their output power immediately with the mirror tilt. We are, of course, particularly interested in unstable resonators with optimum output coupling. For these resonators, the power decreases as well, but due to the insensitivity of the power to changes in the loss factor, the misalignment sensitivity is relatively low. We get a better understanding of this fact if we expand the output power in a Taylor series around $\alpha=0$:

$$P_{out}(\alpha) = P_{out}(0) + \left.\frac{dP_{out}}{d\alpha}\right|_{\alpha=0}\alpha + \frac{1}{2}\left.\frac{dP^2_{out}}{d^2\alpha}\right|_{\alpha=0}\alpha^2 + \ldots \qquad (15.24)$$

which can be written as:

$$P_{out}(\alpha) = P_{out}(0) + \frac{dP_{out}}{dV}\frac{dV}{d\alpha}\alpha + \frac{1}{2}\left[\frac{dP^2_{out}}{dV^2}\left(\frac{dV}{d\alpha}\right)^2 + \frac{dP_{out}}{dV}\frac{d^2V}{d\alpha^2}\right]\alpha^2 + \ldots \qquad (15.25)$$

Insertion of the loss factor $V(\alpha)$ with:

$$V(\alpha) = V(0)\,[1 - 0.1(\alpha/\alpha_{10\%})^2] \qquad (15.26)$$

results in:

$$P_{out} = P_{out}(0) - 0.1\,V(0)\frac{dP_{out}}{dV}\left(\frac{\alpha}{\alpha_{10\%}}\right)^2 + \left[\frac{0.1\,V(0)}{\sqrt{2}}\right]^2\frac{d^2P_{out}}{dV^2}\left(\frac{\alpha}{\alpha_{10\%}}\right)^4 \qquad (15.27)$$

The first derivative of the power dP_{out}/dV is negative, positive, or zero, depending whether the output coupling is too low, too high, or optimal [4.83]. For optimum output coupling (15.27) becomes:

$$P_{out}(\alpha) = P_{out}(0)\left[1 - 0.1\left(\frac{\alpha}{\beta_{10\%}}\right)^4\right] \qquad (15.28)$$

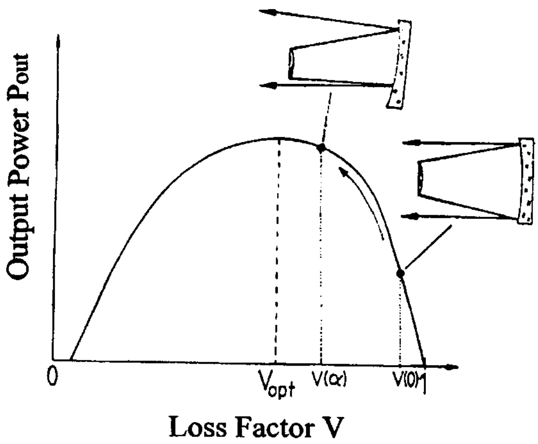

Fig. 15.15 In misaligned unstable resonators the output coupling is increased. The power of under-coupled resonators will thus rise as the mirrors are tilted.

with: $$\frac{\beta_{10\%}}{\alpha_{10\%}} = \left[\frac{200\ P_{out}(0)}{-\ d^2P_{out}/dV^2\ V^2(0)}\right]^{\frac{1}{4}} \tag{15.29}$$

For optimal output coupling, the power decreases only as the fourth power of the angle of misalignment. The 10%-angle $\beta_{10\%}$ at which the power has decreased by 10% is always larger than the 10%-angle of the losses $\alpha_{10\%}$. This can be verified by inserting the expression (15.2) for the output power into (15.29) or (15.27). The resulting analytical expression however, is too complicated so that a numerical solution is more convenient. Numerically calculated angle ratios (15.29) as a function of the small-signal gain are presented in Fig. 15.16. Compared to stable resonators in fundamental mode operation (Fig.15.13), the angle ratio $\beta_{10\%}/\alpha_{10\%}$ is about twice as high. Considering the fact that the 10%-angle of the losses of unstable resonators is already at least twice as high (see Sec. 7.4), the output power of unstable resonators is at least four times less sensitive to misalignment than that of stable resonators in fundamental mode operation with a comparable mode volume. Some experimental results are presented in Fig. 15.17. The right graph shows the increase of the output power for a misaligned, undercoupled, unstable resonator.

Example:
We will use the same CO_2-laser tube as in the last section with a positive branch confocal resonator having a magnification of $M=1.5$ ($\rho_1=-2m$, $\rho_2=3m$, $L=0.5m$). The radius a of the high reflecting zone on the output coupling mirror 1 is adapted to the tube radius b with $a=b/M=2mm$. The resonator parameters are:

g-parameters: $g_1 = 1.25$, $g_2 = 0.833$, $G=1.083$

effective Fresnel number: $N_{eff} = a^2/(2Lg2\lambda) = 0.453$

equivalent Fresnel number: $N_{eq} = N_{eff}\sqrt{G^2-1} = 0.189$

According to Fig. 7.21a, this resonator exhibits a loss factor $V(0)$ of about 0.64, which means that the output coupling is optimum at a small-signal gain of $g_0\ell=3.0$. The misalignment sensitivity of the losses can be calculated from Fig. 7.30. For the tilt of the unconfined mirror 2, the misalignment parameter $L\alpha_{10\%}/a$ is about 80mrad which corresponds to a 10%-angle of $\alpha_{10\%,2}=320\mu rad$. The 10%-angle for the output coupler is given by $\alpha_{10\%,1} = \alpha_{10\%,2}/g_2 = 384\mu rad$. With this information we can use Fig. 15.16 to determine the angles $\beta_{10\%}$ at which the power has decreased by 10%:

if the output coupler is tilted : $\beta_{10\%,1} = 0.8\ mrad$

if mirror 2 is tilted : $\beta_{10\%,1} = 0.67 mrad$

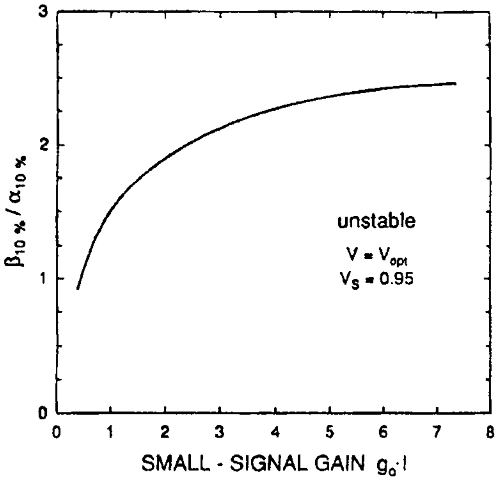

Fig. 15.16 Numerically calculated angle ratios $\beta_{10\%}/\alpha_{10\%}$ for unstable resonators with optimum output coupling as a function of the small-signal gain. Equations (15.2) and (15.26) were used. The 10% angles $\alpha_{10\%}$ and $\beta_{10\%}$ denote the tilt angle at which the losses and the power, respectively, have decreased by 10%.

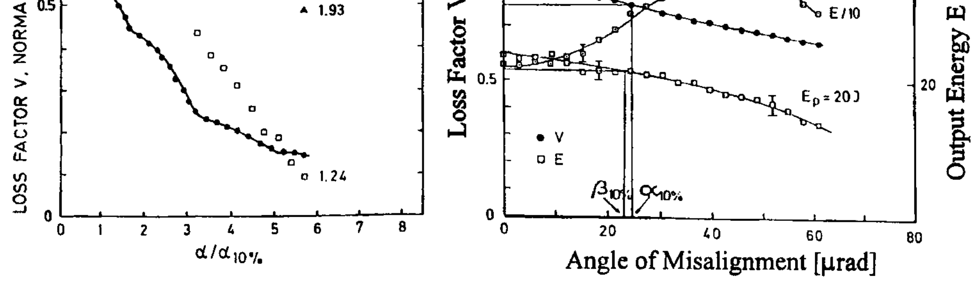

Fig. 15.17 Measured loss factor per round trip and output energy per pulse of a Nd:YAG laser with different unstable resonators as a function of the tilt angle of the unconfined mirror. The curve parameter in both graphs is the small-signal gain. In the right graph, a pump energy of 160J corresponds to a small-signal gain of 3.5.

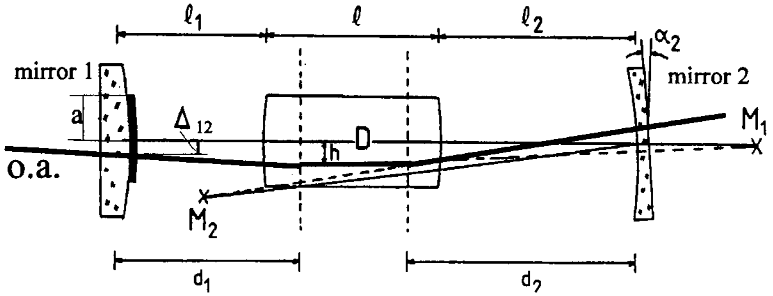

Fig. 15.18 Misaligned unstable resonator with internal variable lens. The tilt of mirror 2 results in a shift Δ_{12} of the intersection point of the optical axis (o.a.) on the output coupler.

15.4.2 With Thermal Lensing

In unstable resonators with an internal variable lens, the additional losses due to misalignment are generated by the shift Δ_1 of the intersection point of the optical axis on the output coupler (Fig. 15.18). For a tilt of mirror i by the angle α_i, the shift is given by:

$$\Delta_{1i} = \left| \frac{\dfrac{(1 - d_1/\rho_1)}{(1 - d_i/\rho_i)}\alpha_i}{\left[D - \dfrac{1}{d_1-\rho_1} - \dfrac{1}{d_2-\rho_2}\right]} \right| \tag{15.30}$$

This equation reveals a symmetry relation between the two mirrors. If mirror 2 is tilted by α_2, the same shift on the output coupler is generated for a tilt of mirror 1 by the angle:

$$\alpha_1 = \alpha_2 \frac{1 - d_1/\rho_1}{1 - d_2/\rho_2} \tag{15.31}$$

Having determined the shift, we can now use the equivalent, empty, resonator with g-parameters g_1^*, g_2^*, and length L^* (see Sec. 15.1). The shift of the optical axis at which the losses have increased by 10% was calculated in Sec. 5.4 for the misalignment of mirror 2:

$$\Delta_{1,10\%} = \frac{2L^*}{|G^* - 1|}\alpha_{10\%} = \frac{2a\ D_2}{|G^* - 1|} \tag{15.32}$$

where $G^* = 2g_1^* g_2^* - 1$ is the equivalent g-parameter and D_2 is the misalignment parameter according to Fig. 7.30. A comparison of (15.32) and (15.30) with $\alpha_i = \alpha_{10\%,i}$ yields the following expression for the 10%-angle when mirror i is tilted:

$$\alpha_{10\%,i} = D_2 \frac{2a}{|G^* - 1|} \left| \left(\frac{1 - d/\rho_i}{1 - d_1/\rho_1} \right) \left[D - \frac{1}{d_1 - \rho_1} - \frac{1}{d_2 - \rho_2} \right] \right| \qquad (15.33)$$

Unfortunately, no analytical expression for the misalignment parameter D_2 exists and we have to get the values from Fig. 7.30. Keep in mind that D_2 is a function G^* and N_{eq}^* and will in general decrease as the refractive power is increased. If we knew the misalignment parameter as a function of the refractive power we could use (15.33) to determine the 10%-angle of the losses and then go into (15.27) or (15.29) to obtain the 10%-angle of the output power $\beta_{10\%}$.

Instead of following this complicated procedure let us make a rough estimate by assuming a constant misalignment parameter of 35mrad (see Fig.7.30). For a positive branch unstable resonator with ρ_1=-0.5m, ρ_2=-0.3m, d_1=0.3m, d_2=0.13m and a refractive power of 3m^{-1}, Eq. (15.33) yields the 10%-angles $\alpha_{10\%,1}$=280μrad and $\alpha_{10\%,2}$=250μrad (Fig. 15.19). For a small-signal gain of 3.0 we expect, according to Fig. 15.16, an angle of misalignment $\beta_{10\%}$ that is about twice as high as the angle $\alpha_{10\%}$. Our final estimate of 0.56mrad and 0.5mrad for the 10%-angles is in agreement with the measurement presented in Fig. 15.19. Note that for a constant small-signal gain the angle $\beta_{10\%}$ stays at the same value as the refractive power is increased (left graph). This agrees with the above statement that the misalignment parameter decreases. In the right graph the small-signal gain increases linearly with the refractive power resulting in an increase of the 10%-angle.

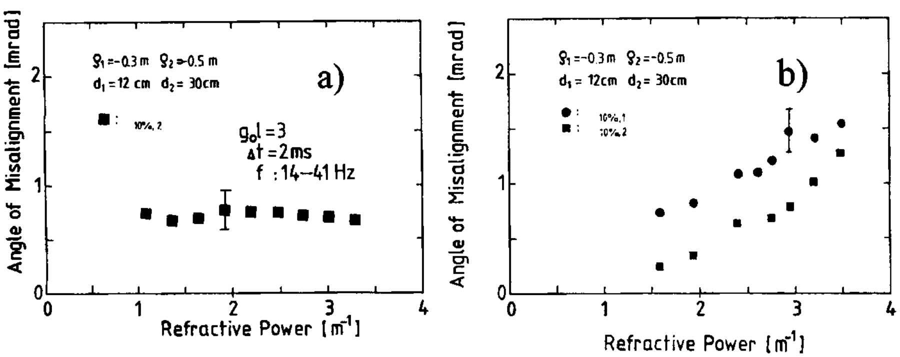

Fig. 15.19 Measured angles of misalignment $\beta_{10\%,i}$ at which the output power has decreased by 10% for a pulsed Nd:YAG laser with a positive branch unstable resonator (rod radius: 5mm). Mirror i is misaligned, mirror 1 is the output coupler with a radius of a=1.75mm. a) for a constant small-signal gain of 3.0 and the repetition rate varied between 14 and 41Hz. b) for a constant repetition rate of 30Hz and the small-signal gain varied between 2.0 and 4.6.

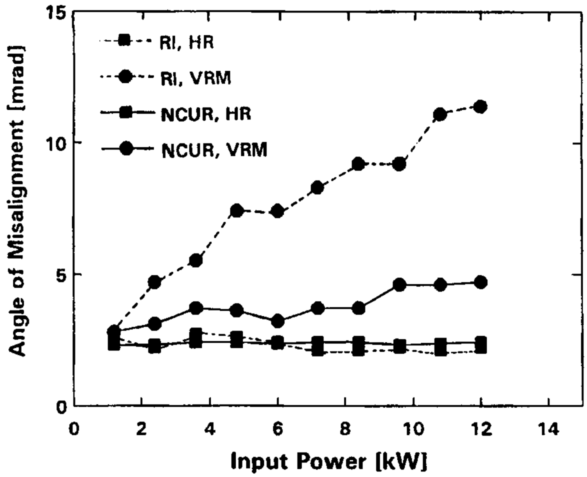

Fig. 15.20 Measured angle of misalignment at which the output power has decreased by 10% of a pulsed Nd:YAG laser (*b=4.75mm, ℓ=150mm*) with a rod-imaging unstable resonator (RI) and a near concentric unstable resonator (NCUR) as a function of the electrical pump power. The output coupler is a variable reflectivity mirror (VRM) with 70% center reflectivity. Please see Fig. 13.49 for resonator geometries and output power. Data marked VRM refer to the tilt of the output coupler, data marked HR refer to the tilt of the high-reflecting mirror [S.9].

According to (15.33), the angles $\alpha_{10\%,i}$ become infinite if the distances d_i are equal to the mirror curvatures ρ_i. Unfortunately, our geometrical model holds only for small angles of misalignment and the infinite solutions of (15.33) thus make no physical sense. However, an infinite or very large 10%-angle indicates that the misalignment sensitivity of the resonator must be very low. Two unstable resonator schemes exhibit large 10%-angles; the rod-imaging unstable resonator with $d_2=\rho_2$, and the near concentric unstable resonator (NCUR) whose distances d_i are slightly larger than the radii of curvature of the mirrors. For both resonator schemes, Fig. 15.20 presents measured angles of misalignment at which the output power decreased by 10%. A comparison with the corresponding 10%-angles of stable lens resonators (Figs. 15.9 and 15.11) indicates that unstable resonators exhibit similar misalignment sensitivities of the output power as stable multimode resonators. By using optimized unstable resonator designs, like the rod-imaging resonator, it is even possible to attain a lower misalignment sensitivity.