Chapter 14

Resonators with Several Active Media

14.1 General Aspects

In high power lasers, the maximum output power scales proportionally with the length of the active medium. In general, there is an upper bound for the output power that can be attained with a medium of a certain length. For solid state lasers this limit is caused by the pump induced thermal heat P_H which has to stay below the fracture limit to avoid damage to the active material. For a laser rod the maximum output power P_{out} per length is given by:

$$\frac{P_{out,\max}}{\ell} = \frac{\eta_{extr}}{\chi}\frac{P_{H,\max}}{\ell} \tag{14.1}$$

where η_{extr} is the extraction efficiency, χ is the thermal load parameter (ratio of heat to power transferred to inversion, ~0.33 for 808nm diode-pumped Nd:YAG lasers emitting at 1064nm), and $P_{H.\max}$ is the maximally allowed thermal heat (see Table 14.1 for typical values). In gas lasers the limitation of the output power per tube length is also a result of the heat deposition in the gas. For sealed-off CO_2 lasers in which the gas is not constantly removed, the maximum output power is on the order of 100W/m for tube lasers. Better heat removal can be achieved by flowing the gas through the electrical discharge region. In fast flow CO_2 lasers, maximum output powers per length on the order of 2,000W/m can be realized. Unfortunately, using a longer active medium to increase the output power is often not feasible due to space requirements, limitations on the length (solid state laser rods, for instance, are usually not grown longer than 200mm), or technical difficulties in providing efficient heat removal. It is for these reasons that solid state lasers with output powers greater than 600W and CO_2 lasers in the multi-kW power range utilize several active media inside the resonator (Fig. 14.1). The use of additional external amplifiers has also found application, particularly in solid state lasers.

Table 14.1 Maximally allowed heat and maximum output power per unit length of different solid state laser rods (flashlamp pumping)..

Material	Nd:YAG	Nd:glass (LG76)	Nd:Cr:GSGG	Alexandrite
$P_{H,\max}/\ell$ [kW/m]	14	0.7	12.5	64
$P_{out,\max}/\ell$ [kW/m]	4.0	0.2	3.5	13

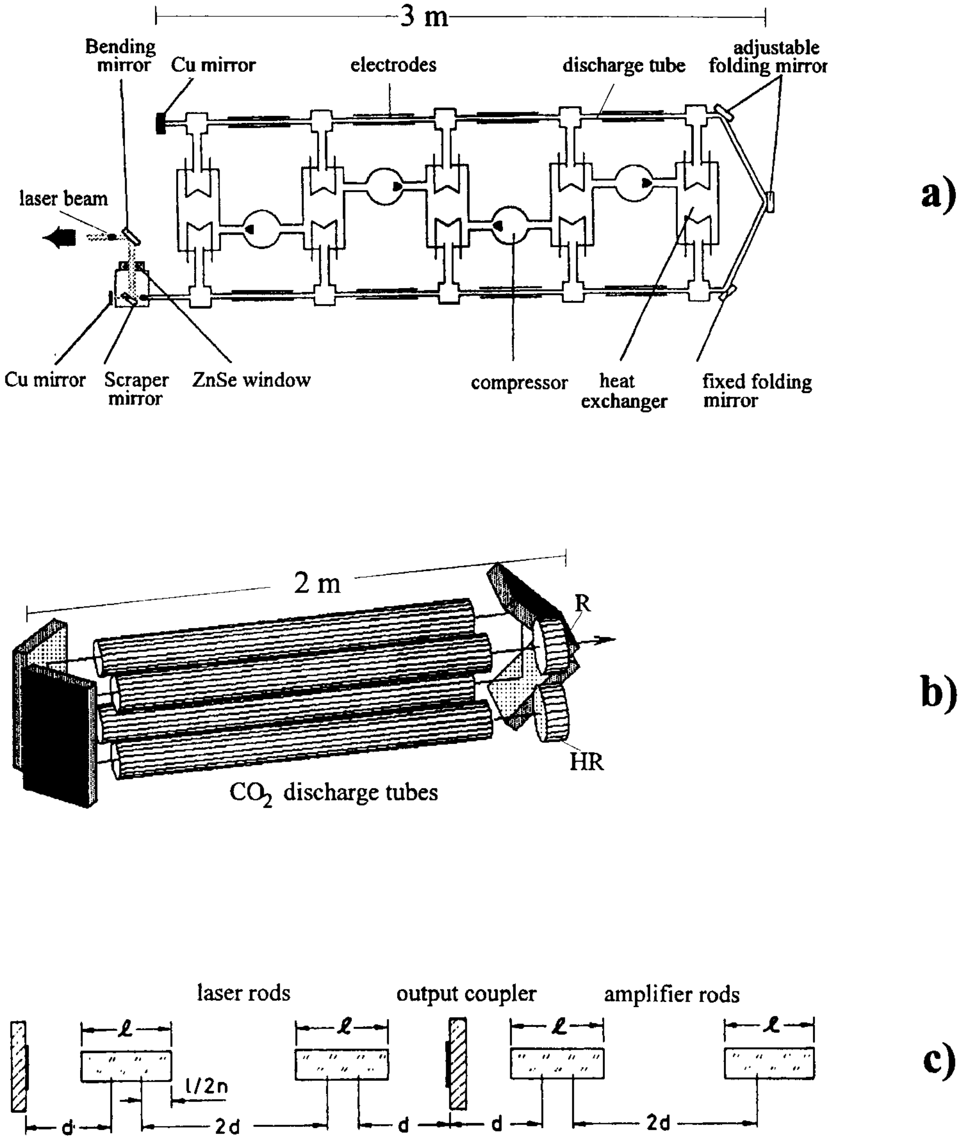

Fig. 14.1 Lasers with several active media. a) 20kW CO_2 laser [4.131] (© SPIE 1994), b) 6kW CO_2 laser [4.132] (© SPIE 1994), c) symmetric flat-flat resonator for a 2kW Nd:YAG rod laser with two amplifiers (*ℓ=15cm, d=25cm*).

14.2 Output Power and Efficiency

14.2.1 Oscillator Arrangement

Let us assume that the laser material is homogeneously broadened. We saw in Chapter 10 that the maximum extraction efficiency $\eta_{extr,max}$ and the corresponding optimum output coupling reflectance R_{opt}, to a good approximation, are given by:

$$\eta_{extr,max} = \frac{\alpha_0\ell}{g_0\ell}\left[\sqrt{\frac{g_0\ell}{\alpha_0\ell}} - 1\right]^2 \qquad (14.2)$$

$$\ln R_{opt} = -2\alpha_0\ell\left[\sqrt{\frac{g_0\ell}{\alpha_0\ell}} - 1\right] \qquad (14.3)$$

where g_0 is the small-signal gain coefficient, α_0 is the loss coefficient, and ℓ is the length of the active medium. If N equally pumped active media are used, the small-signal gain $g_0\ell$ and the loss $\alpha_0\ell$ are N times higher. According to (14.2), the maximum extraction efficiency is the same and the optimum output coupling reflectance reads:

$$\ln R_{opt}^{(N)} = -2N\,\alpha_0\ell\left[\sqrt{\frac{g_0\ell}{\alpha_0\ell}} - 1\right] \qquad (14.4)$$

Thus we get an N times higher output power if the reflectance of the output coupling mirror is the N-th power of the optimum reflectance for a single medium. An accurate calculation of the output power, however, shows that the extraction efficiency decreases slightly for a high number of active media. Considering the fact that usually no more than ten media are used this roll-off is negligible.

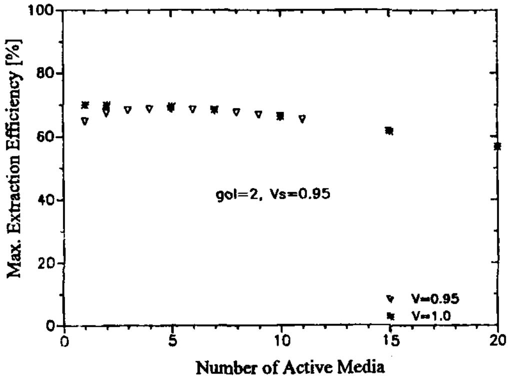

Fig. 14.2 Maximum extraction efficiency versus the number of active media. Each medium exhibits a small-signal gain of $g_0\ell=2$ and a loss of $\alpha_0\ell=-\ln V_S=0.053$. V is the diffraction loss factor per round trip.

13.2.2 Oscillator-Amplifier Arrangement

If some of the N active media are placed behind the output coupler to amplify the laser beam, the output power behind the amplifiers can approach the maximum output power of the oscillator arrangement with N media. However, the maximum output power can only be achieved if the intensity incident on the first amplifier is greater than the saturation intensity I_S of the active laser material. For Nd:YAG ($I_S = 2{,}000\mathrm{W/cm^2}$), this means that we need an output power from the oscillator of at least 1kW (in cw-operation) to get comparable extraction efficiencies in the amplifier rods. In each amplifier medium the amplification of the intensity I along the optical axis (z-axis) is determined by:

$$\frac{dI}{dz} = \left[\frac{g_0}{1 + I/I_S} - \alpha_0\right] I \tag{14.5}$$

If I_0 denotes the intensity incident on the amplifier and $P_{out,0}$ is the corresponding power, the power $P_{out,l}$ after amplification is given by:

$$P_{out,1} = G\, P_{out,0} \tag{14.6}$$

where the gain factor G is the solution of the equation:

$$\ln G - \frac{g_0}{\alpha_0} \ln\left[\frac{g_0 - \alpha_0(1 + GI_0/I_S)}{g_0 - \alpha_0(1 + I_0/I_S)}\right] = (g_0 - \alpha_0)\ell \tag{14.7}$$

The extraction efficiency is related to the gain factor via:

$$\eta_{extr} = \frac{(G-1)\, P_{out,0}}{A\, I_S\, g_0 \ell} \tag{14.8}$$

where A is the cross sectional area of the laser beam. For lossy media, the gain factor G as a function of the incident intensity exhibits a maximum and for high intensities the gain factor approaches unity, which means that the amplifier medium does not contribute any power to the beam (zero extraction efficiency) (Fig.14.3). The maximum power that can be attained with amplification can be found by setting the bracket in (14.5) equal to zero:

$$P_{out,\max} = A\, I_S\, [g_0/\alpha_0 - 1] \tag{14.9}$$

For Nd:YAG rods with 10mm diameter and 150mm length and each rod being pumped continuously with an electrical pump power to the flashlamps of 10kW ($g_0\ell = 0.4$, $\alpha_0\ell = 0.07$), Eq. (14.8) yields a maximum output power of 7.4kW. Lower losses will result in higher output power; high quality rods can provide more than 10kW.

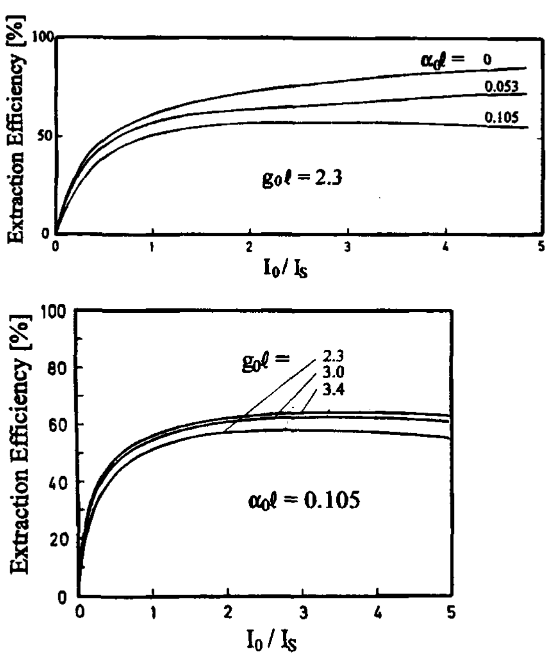

Fig. 14.3 Extraction efficiency of a laser amplifier as a function of the ratio of the incident intensity I_0 to the saturation intensity I_S. a) for a small-signal gain of $g_0\ell=2.3$ and different losses $\alpha_0\ell$ b) for a loss of $\alpha_0\ell=0.105$ and different small-signal gains.

14.3 Multirod Solid State Lasers

14.3.1 The Equivalent g-Diagram

In solid state lasers the active media cannot be placed arbitrarily inside the resonator since the thermal lensing may drive the resonator into unstable regions. Similarly to single rod resonators discussed in the preceding chapter, an equivalent, empty, resonator that exhibits the same beam radii at the mirrors can be defined. If M denotes the ray transfer matrix for the transit from mirror 1 to mirror 2 (remember that each mirror with radius of curvature ρ is replaced by a pair of lenses with focal length $f=\rho$ and the propagation starts and ends at the planes between the lenses of each pair), the equivalent g-parameters and the equivalent length are given by:

$$g_1^* = A \; ; \quad g_2^* = D \; ; \quad L^* = B \tag{14.10}$$

with: $$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

In general, the equivalent resonator does not drive through the g-diagram on a straight line (Fig. 14.4). If N lenses are inside the resonator, the resonator moves along a curve that exhibits N-1 inflection points and may cross unstable zones up to $2N$ times [4.128,4.129]. In order to prevent the resonator from penetrating into unstable zones, a symmetric resonator set-up with two flat mirrors has to be chosen. The active media are positioned at an equal distance Δ from one another which corresponds to a distance $2d$ between the principal planes of adjacent rods (Fig. 14.5). The equivalent resonator moves along a straight line through the origin of the g-diagram and reverses its direction N-1 times at the stability limits. For N laser rods the refractive power per rod D_{SL} at which the equivalent resonator hits a stability limit is given by:

$$D_{SL} = \frac{2}{d}\,\sin^2\left[\frac{m\,\pi}{2\,N}\right] \quad ; \qquad m = 0,1,2,...,N \tag{14.11}$$

Thus the resonator remains stable for a refractive power per rod of $D < 2/d$.

Example : *d=30 cm, N=5*:

m	1	2	3	4	5
D_{SL} [m^{-1}]	0.64	2.30	4.36	6.03	6.67

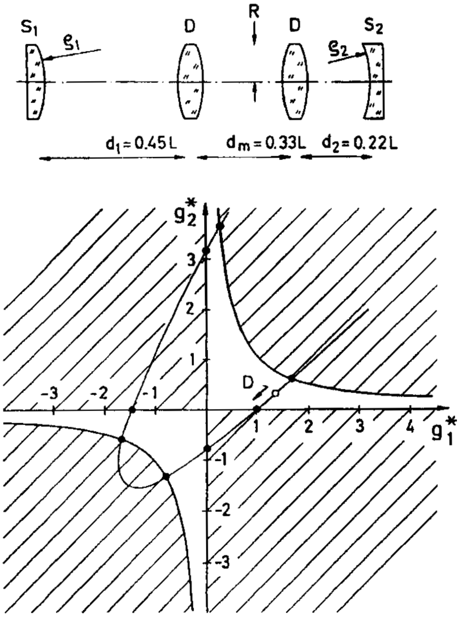

Fig. 14.4 The equivalent g-diagram for a resonator wit h two rods as a function of the refractive power D per rod. Both rods exhibit the same refractive power [4.128] (© IEEE 1988).

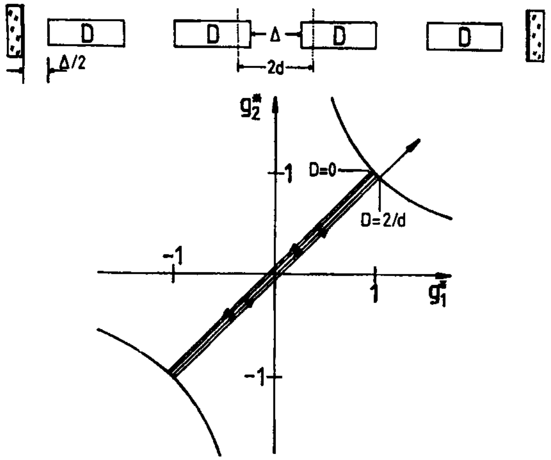

Fig. 14.5 The symmetric flat-flat multirod resonator and its path in the equivalent g-diagram. The distance d is related to the rod spacing Δ via: $d = \Delta/2 + \ell/(2n)$ where ℓ is the rod length.

14.3.2 Beam Quality and Output Power

As was shown in the last chapter, stable resonators with one thermal lens exhibit a fundamental relationship between the output power range ΔP_{out} and the maximum beam parameter product $(w\theta)_{max}$:

$$\frac{(w\theta)_{max}}{\Delta P_{out}} = \frac{k}{4\pi}\,\frac{\alpha}{\eta_{slope}} \tag{14.12}$$

where α is the thermal lensing coefficient, η_{slope} is the slope efficiency, and the integer k is equal to 1 if the confocal point $g_1^*=g_2^*=0$ is passed, and $k=2$ otherwise. A similar relationship holds for multirod systems. At any refractive power the symmetric flat-flat resonator exhibits planes with mirror symmetry in the middle between two rods. The beam waists are always located at this symmetry plane and the beam caustic is the periodic repetition of the beam caustic in a single rod resonator (Fig. 14.6). Thus, the beam parameter product does not depend on the number N of active elements placed inside or outside the resonator if all media are spaced at an equal distance. Figure 14.7 presents measured beam parameter products of a cw Nd:YAG laser for one and six intracavity rods. Since the output power P_{out} is now N times higher and the lens resonator remains always in the stable zone ($\Delta P_{out}=P_{out}$), expression (14.12) reads for symmetric flat-flat multirod resonators:

$$\frac{(w\theta)_{max}}{P_{out}} = \frac{k}{4\pi}\,\frac{\alpha}{\eta_{slope}}\,\frac{1}{N} \tag{14.13}$$

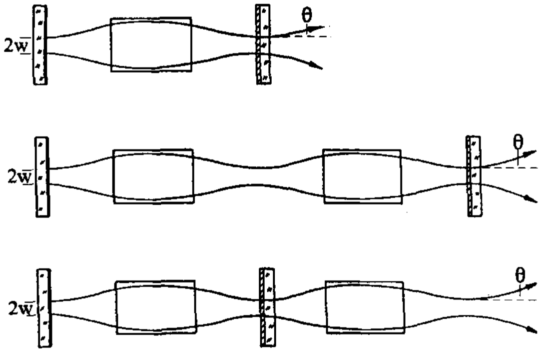

Fig. 14.6 If the rods are equally spaced in a flat-flat resonator the beam parameter product $w\theta$ does not depend on the number N of rods used, inside or outside the resonator.

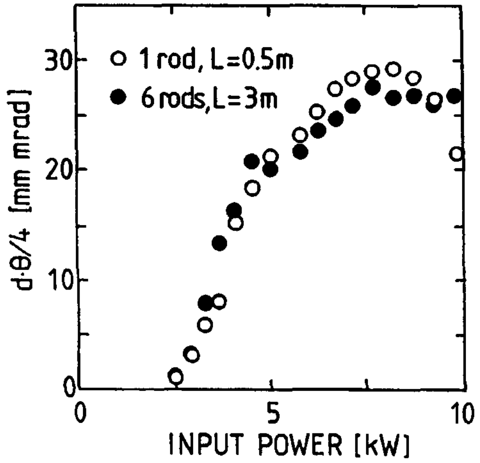

Fig. 14.7 Measured beam parameter product (d: beam waist diameter, θ: full angle of divergence, 86.5% power content) as a function of the electrical input power per rod for a cw flashlamp pumped Nd:YAG laser with one and six rods (diameter: 8mm, length: 150mm). The rods are positioned in a symmetric flat-flat resonator with a distance Δ=20cm between the rod endfaces. The output power is 300W and 1,800W with one and six rods, respectively. L is the geometrical resonator length. The refractive power of each rod is 0.52m^{-1} per kW of input power.

At present multirod solid Nd:YAG lasers providing average output powers of up to 5 kW are commercially available that use 6-8 rods. Figure 14.8 gives an overview of the maximum beam parameter products and the maximum output power of flashlamp-pumped Nd:YAG lasers in comparison with corresponding data of typical high power CO_2 lasers. Similar to single rod systems (Fig. 14.15), the beam quality is worse than predicted by (14.13). For an optimized dual-rod 1kW laser, for instance, we would expect a beam parameter product of about 16 mm mrad (using 0.4 mm/kW for α/η_{slope}, see Table 13.6). One explanation for this discrepancy is the fact that most commercial lasers do not make use of the whole stable refractive power range $\Delta D=2/d$. In general, the resonators are chosen shorter to avoid the decrease in efficiency due to spherical aberration. Furthermore, there are also space requirements to be considered. In order to cover the stable zone with the six rod laser of Fig. 14.7, the resonator length has to be increased from 3m to 4.5m. This is a length that is considered too long, even though the maximum beam parameter product would decrease by a factor of 1.5.

Better beam qualities are generally observed in diode-pumped Nd:YAG lasers (Fig. 14.9) due to the fact that for the same generated inversion, the heat dissipation is about 30% lower compared to lamp pumped systems. This translates into a 30% improvement of the beam quality, which is rather low compared to the expectations people had for diode-pumped solid state lasers a decade ago. The poorer than expected thermal characteristics, together with the high cost and limited lifetime of the pump diodes (~10,000 hours), are still limiting factors for the introduction of DPSSLs into industrial applications that require kW of average output power. At present the vast majority of this market is served by flashlamp pumped YAG lasers. A photograph of the laser head of an industrial flashlamp pumped cw Nd:YAG laser used for cutting and welding is presented in Fig. 14.10. This laser was the first commercially available kW Nd:YAG system and provided a maximum output power of 1.2kW with four rods (length: 6 inch, diameter 5/16 inch) at a total electrical pump power of 40kW. Today, kW lasers use only two rods, and 5kW output power can be easily achieved with 8 rods, at total efficiencies of 4-5% (power out versus electrical power to flashlamps). Multirod Nd:YAG systems are also widely used in laboratories to generate high output powers in TEM_{00} mode operation. By restricting the stability range and compensating for thermally induced birefringence with quartz-rotators, output powers of up to 208W have been achieved with dual-rod systems [4.112,4.116,4.120,4.121,4.123]. Without restricting the stability range, TEM_{00} mode output powers of 48W at 1064nm have been achieved with diode-pumped $Nd:YVO_4$ [4.77,4.115].

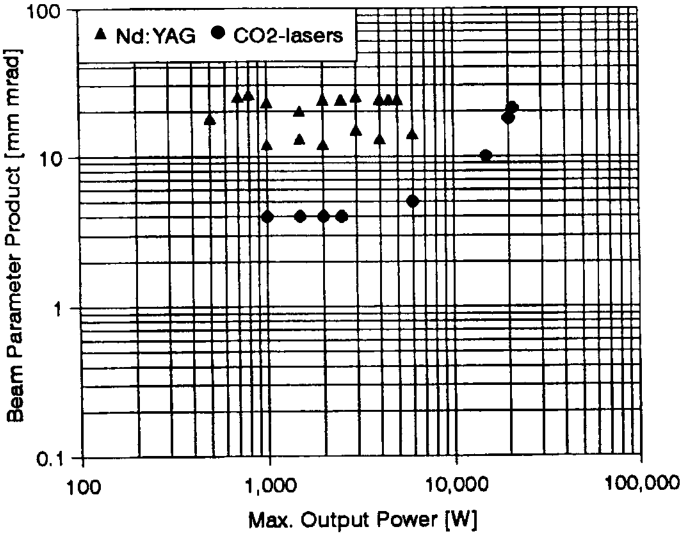

Fig. 14.8 Maximum beam parameter products versus maximum output powers for commercially available flashlamp-pumped Nd:YAG multirod lasers (λ=1.064µm) and high power CO_2 lasers (λ=10.6µm). The YAG data includes systems introduced to the market since 1988. Early systems used more rods to achieve a certain output power, resulting in abetter beam quality (row of points around 15mm mrad). The beam parameter product corresponding to M^2=1 are 0.34mm mrad for Nd;YAG lasers and 3.4 mm mrad for CO_2-lasers.

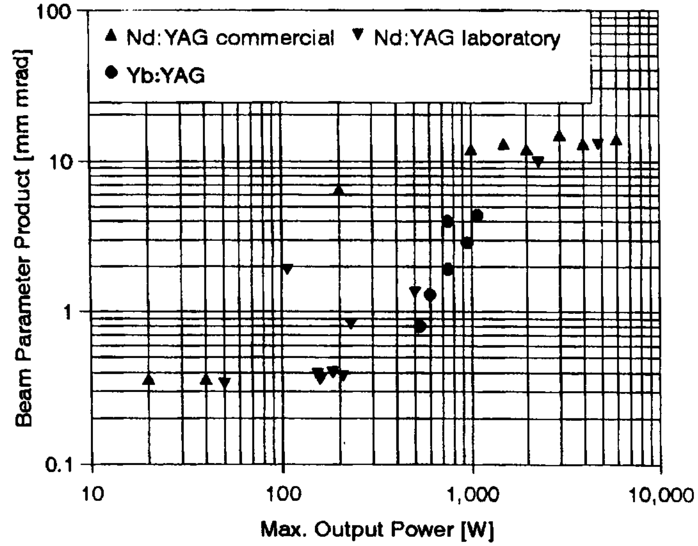

Fig. 14.9 Beam Parameter Product as a function of output power for diode-pumped solid state lasers.

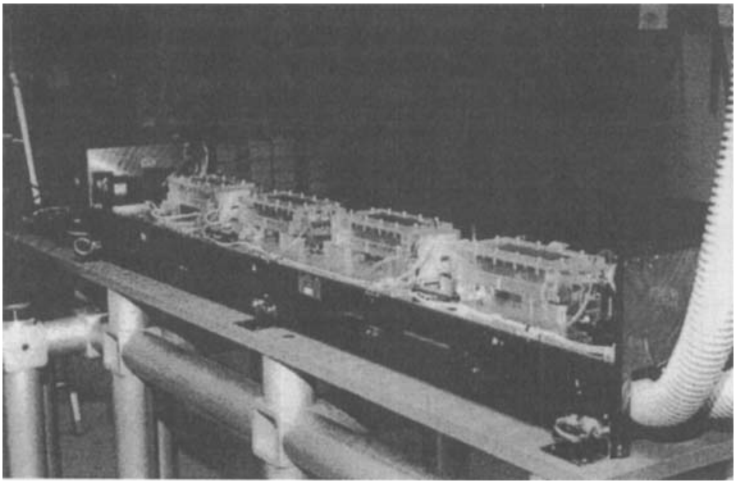

Fig. 14.10 The first industrial kW Nd:YAG laser with four rods in a symmetric flat-flat resonator (NEC 1988). The distance Δ between adjacent rod endfaces is 20cm. The overall length of the laser head is 1.6m (courtesy of Laser- und Medizin-Technologie Berlin gGmbH, Berlin, Germany) [S.14].

14.3.3 Multirod Resonators with Variable Reflectivity Mirrors

It is possible to improve the beam quality of stable solid state multirod resonators by using a variable reflectivity mirror (VRM) as an output coupler. Similar to a hard aperture, the VRM controls the number of transverse modes, but the output power which is absorbed by a conventional aperture is now coupled out of the resonator. As the lens resonator moves through the stability diagram, the multimode beam radius at the output coupler is decreased from a value close to the rod radius at zero pump power, to a radius on the order of 200μm shortly before the resonator goes unstable. Ideally, the resonator should provide the same maximum output power as compared to a conventional output coupling mirror. This can be accomplished by choosing the VRM profile radius about the size of the beam radius at the highest pump power. At lower pump powers, the VRM decreases the number of transverse modes, resulting in a considerable decrease of the maximum beam parameter product. Due to gain saturation, which results in a spreading of the beam profile in the active medium, the decrease of the output power is generally kept low. An experimental example for a dual rod Nd:YAG laser with stable resonator and VRM is presented in Fig. 14.11.

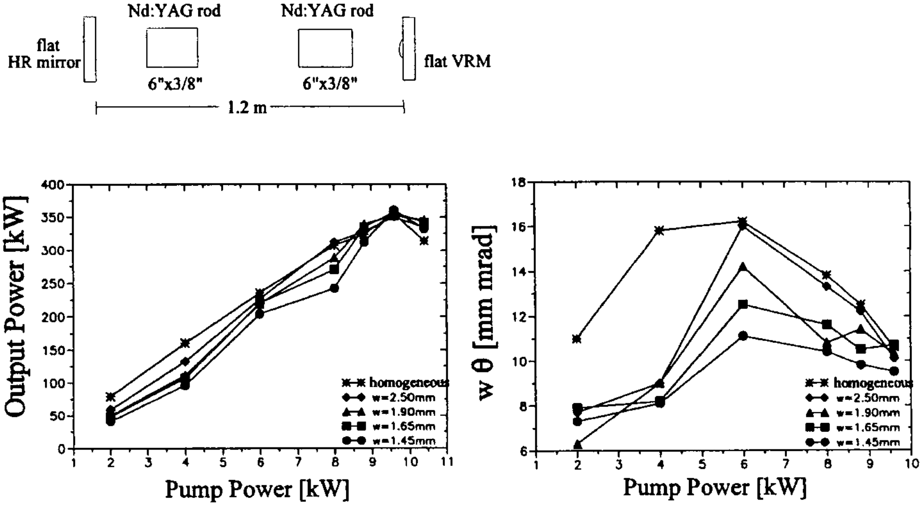

Fig. 14.11 Measured output power and measured beam parameter product (w: beam waist radius, θ: half angle of divergence, 86.5% power content) as a function of the electric pump power per rod for a flashlamp pumped, pulsed, dual rod Nd:YAG laser with different VRM output couplers. The curve parameter is the reflectivity profile radius *w* of the VRM. The center reflectivity of the VRM is 46% and the super-Gaussian index of the VRM is *n=12*. In both graphs, the upper curve corresponds to a conventional output coupling mirror with reflectance *R=46%*. The refractive power of each rod is 0.38m^{-1} per kW of pump power [S.15].