Chapter 12

Q-switched Laser Resonators

12.1 General Aspects

In the previous chapters we have discussed the output power of laser resonators emitting in continuous wave operation. For non-military laser systems, the cw output powers are limited to about 10kW for solid state lasers and 20kW for gas lasers. Much higher peak powers can be achieved by utilizing the energy storage capability of the active medium and releasing the energy in short pulses. Peak powers up to several hundred MW with pulse duration on the order of 10ns can be realized using a technique collectively referred to as Q-switching. With this technique, the round trip loss of the resonator is increased to prevent the laser from reaching threshold during a defined pump time interval. The energy stored during this time is then partially released in form of a short pulse after the round trip loss has been decreased to its normal value. In other words, in order to store and release the energy, the quality factor Q of the resonator (see Chapter 4) is switched from a low value to a high value and back again. Figure 12.1 shows the typical time sequence of resonator loss, stored energy and output of a repetitively Q-switched laser.

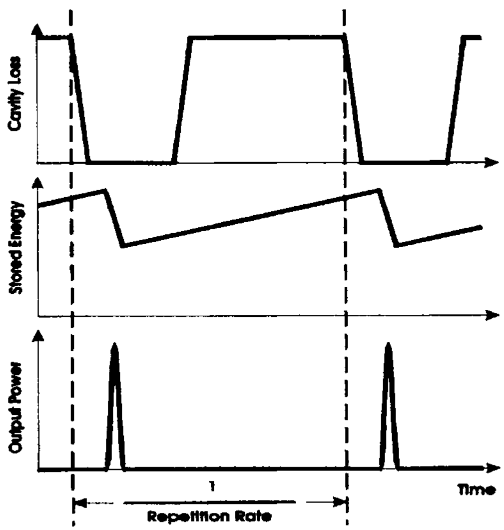


Fig.12.1 Qualitative time sequence of resonator loss, stored energy and output power of a cw pumped Q-switched laser at high repetition rate.

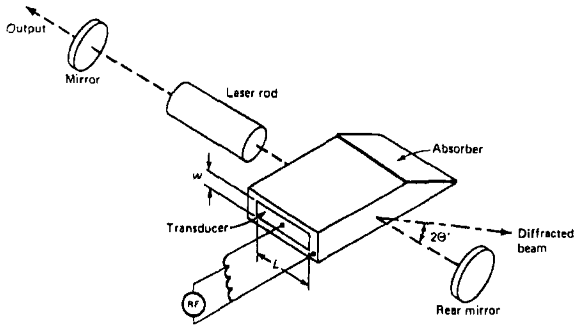


Fig. 12.2 Schematic of a Q-switched solid state laser using an acousto-optic modulator (AOM). The AOM generates a high resonator loss (low Q) by steering the optical axis when RF power is applied [4.2].

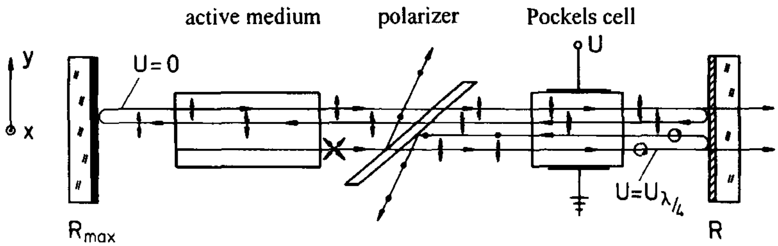


Fig. 12.3 Pockels cell resonator for Q-switch operation. Applying the quarter wave voltage U will rotate the polarization by 90° after a double pass, generating a high loss at the polarizer. Dropping the voltage to zero will lead to the emission of a Q-switch pulse.

A general detailed discussion of Q-switching is given in Siegman's book [4.1]. This description applies to modern practical realization of Q-switched lasers. Commercial laser systems use one of three techniques to temporarily increase the cavity loss: acousto-optic Q-switching using an acousto-optic modulator (Fig.12.2) [4.61,4.76,4.77], electro-optic Q-switching using a nonlinear crystal (Pockels-cell) [4.63,4.65,4.76,4.79], and passive Q-switching with a saturable absorber [4.72-75]. Mechanical Q-switching devices such as rotating mirrors [4.50,4.53,4.54] or piezo-driven etalons [4.56] have not been standardized for commercial applications.

Acousto-optic Q-switching is the method most commonly used because the AOM provides switching capabilities at very high repetition rates (up to tens of MHz, depending on the beam size) and can handle high average beam powers of up to kWs. By applying modulated RF power to a piezo-electric transducer, an ultrasonic wave is launched into the AOM material, typically made of fused silica or crystalline quartz. Due to the photoelastic effect, the acoustic wave generates an index grating that moves across the beam at the speed of sound. This phase grating leads to diffraction of the incoming light, generating a secondary beam at an angle of typically less than 1°. Diffraction efficiencies (deflected beam power/total power) can be as high as 85% for RF powers of about 10W per mm of transducer height w, and RF frequencies between 25 and 80MHz. By applying RF power, most of the intracavity power is deflected away from the optical axis for each transit through the AOM, resulting in an additional round trip loss of up to 98%, thereby surpressing laser emission. The fact that there is a residual zero-order transmission of a couple of percent makes it necessary to use several AOMs to prevent laser emission in high gain lasers (small-signal gain > 5). In addition, a certain resonator length (for example > 100 mm) is required to prevent overlap of the deflected and the transmitted beam at the resonator mirrors. Another disadvantage of the AOM is its relatively long turnoff time which is determined by the speed of sound of the AOM material, For typical acoustic speeds of 4-6 km/s it takes the index grating 150-250 ns per mm of beam diameter to clear the beam. In high gain lasers, this time may be too long compared to the build-up time of the Q-switch pulse, resulting in a decrease of laser efficiency due to the residual loss.

Much shorter turnoff times on the order of ns and higher intr acavity losses (>99%) can be realized with electro-optic modulators (Fig.12.3). With this method, voltage is applied to an electro-optic crystal (most commonly KDP, KD^*P, $LiNbO_3$, or BBO) which acts as a quarter wave plate with its principal axes rotated by 45° with respect to the polarizer. After a double pass, the polarization vector of the linearly polarized input beam is rotated by 90°, leading to a reflection at the polarizer. Ideally, this should generate a 100% loss inside the resonator, but due to crystal imperfections, stress induced birefringence and polarizer imperfections, a residual transmission of 0.5-1% is typically observed. The high contrast ratio combined with the fact that there is no lower limit on the resonator length, makes the Pockels cell superior to an AOM in Q-switched lasers. However, switching the required voltage of several kVs becomes electronically challenging for switching frequencies beyond 10kHz. Although switching frequencies of up to 50kHz have been demonstrated [4.79], electro-optic Q-switching is generally utilized in low-to-medium repetition rate lasers.

In passive Q-switching a saturable absorber is inserted into the resonator which increases its transmission with increasing intracavity intensity due to the saturation of a spectral transition. The original absorbers were organic dyes dissolved in a solution or incorporated into a thin plastic film. Nowadays, much higher durability and damage thresholds are being provided by crystals doped with absorbing ions, such as Cr^{4+}:YAG [4.72-4.75]. Since the laser radiation itself provides the switching mechanism, no expensive drive electronics are required, making passive Q-switching a low-cost alternative to AO and EO techniques. Unfortunately, the absolute change in transmission is limited to about 30%. A Cr^{4+}:YAG, crystal with an initial transmission of 50% will absorb 20% of the Q-switch pulse energy, resulting in thermal problems in higher power lasers. Another disadvantage is the lack of control of the repetition rate. The Q-switch pulse is generated as soon as the inversion density is high enough to overcome the initial absorber transmission. Therefore, the pulse energy cannot be adjusted and the repetition rate can only be varied by changing the pump power. In addition, passive Q-switching is subject to timing jitter and high rms-noise. Despite these disadvantages, compactness and cost-effectiveness makes passive Q-switching the method of choice for low cost, low power lasers providing pulse durations down to the sub-ns range.

Independent of the Q-switching technique used, the laser performance can be calculated using the rate equations presented in Chapter 9.

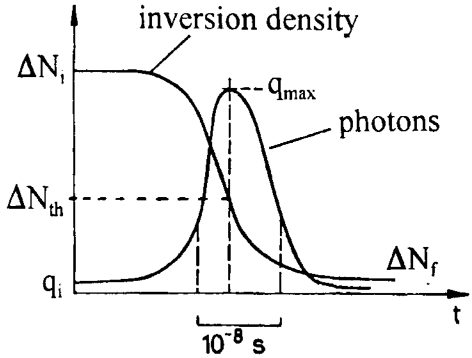


Fig. 12.4 Temporal evolution of inversion density and photon number after the loss induced by the Q-switch has decreased to zero. After a build-up time, the photon number will increase exponentially resulting in a depletion of the inversion. The photon number will decrease again after the inversion density has reached the threshold inversion density ΔN_{th}.

12.2 Rate Equations for Q-switching

12.2.1 Inversion Densities

The analysis of Q-switching performance starts with the rate equations for inversion density ΔN and photon number q inside the resonator, derived in Chapter 9 [4.46,4.49,4.51]:

$$\frac{d\Delta N}{dt} = W(N_0-\Delta N) - \frac{\Delta N}{\tau} - \frac{\sigma_0 c \Delta N q}{A\,L} \tag{12.1}$$

$$\frac{dq}{dt} = \frac{\sigma_0 c_0 \ell \Delta N}{L}(q+1) - \frac{q}{\tau_C} \tag{12.2}$$

with:	W	:	pump rate [s^{-1}]
	N_0	:	doping concentration
	σ_0	:	cross section of stimulated emission [cm^2]
	τ	:	upper laser level lifetime
	L	:	length of resonator
	ℓ	:	geometrical length of active medium
	A	:	cross-sectional area of laser beam
	c	:	speed of light in active medium
	τ_C	:	cavity lifetime = $L/[c_0(\alpha_0\ell-\ln\sqrt{R})]$

At the very beginning of the pulse emission process, the inversion density has been built up by the pumping process. This initial inversion density is determined by the time interval Δt during which the Q-switch suppressed laser emission. For an ideal four-level system we have (see Chapter 9):

interval Δt during which the Q-switch suppressed laser emission. For an ideal four-level system we have (see Chapter 9):

$$\Delta N_i = \frac{WN_0}{W+1/\tau}\left(1-\exp[-(W+1/\tau)\Delta t]\right) \tag{12.3}$$

The emission of the Q-switch pulse happens during a time interval that is much shorter than the upper level lifetime. Any change of the inversion density due to pumping or spontaneous emission can therefore be neglected (first and second term on right hand side of (12.1)). By introducing the intensity $I=qh\nu c_0/(AL)$, the Q-switched rate equations can then be written as:

$$\frac{d\Delta N}{dt} = -\frac{I}{\tau}\frac{\Delta N}{I_S} \tag{12.4}$$

$$\frac{dI}{dt} = \frac{\sigma_0 c_0 \ell(\Delta N-\Delta N_{th})}{L}I \tag{12.5}$$

where we used the threshold inversion density:

$$\Delta N_{th} = \frac{\alpha_0\ell-\ln(\sqrt{R})}{\sigma_0\ell} \tag{12.6}$$

Equation (12.5) indicates that the output pulse will have its maximum ($dI/dt=0$) when the inversion density is equal to the threshold inversion density, an important result that will help us later to determine the peak power. It is not possible to solve these equations analytically to obtain inversion and intensity as a function of time. However, since we know the relationship between intensity and inversion density at the beginning and the end of the pulse as well as at the intensity maximum, one possible approach is to express the intensity as a function of the inversion density. With $dI/dt = dI/d\Delta N \cdot d\Delta N/dt$, we get:

$$\frac{dI}{d\Delta N} = \frac{h\nu c_0\ell}{L}\left[\frac{\Delta N_{th}}{\Delta N}-1\right] \tag{12.7}$$

Integration of (12.7) from time 0 to time t yields:

$$I(t) = \frac{h\nu c_0\ell}{L}\left[\Delta N_{th}\ln\left(\frac{\Delta N(t)}{\Delta N_i}\right)+\Delta N_i-\Delta N(t)\right] \tag{12.8}$$

where we used $I(t=0)=0$ and $\Delta N(t=0)$ equals the initial inversion density ΔN_i. At the end of the output pulse, the inversion density has been depleted to the final inversion density

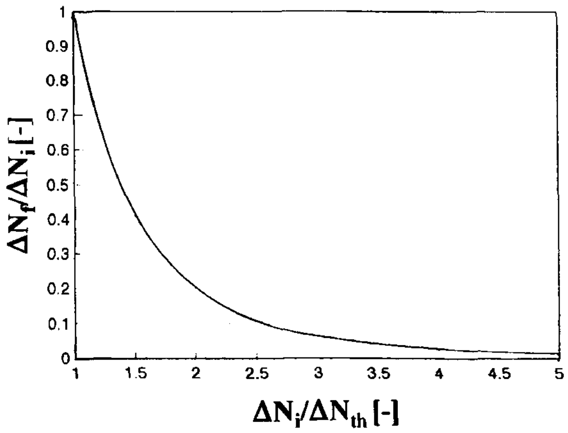


Fig. 12.5 Relationship between final and initial inversion densities according to (12.9).

$$\Delta N_i - \Delta N_f = \Delta N_{th} \ln\left(\frac{\Delta N_i}{\Delta N_f}\right) \tag{12.9}$$

This equation can be solved numerically by successive approximation. Figure 12.5 shows the final inversion density as a function of the initial inversion density. Since the threshold inversion density and the initial inversion density are known, Eq.(12.9) can be used to determine the residual inversion ΔN_f after the pulse has been emitted.

12.2.2 Energy, Pulse Duration and Peak Power

The output energy can be obtained by integrating (12.7) using the output coupling transmission |lnR|/2:

$$E_{out} = A\,\frac{h\nu}{\sigma_0}\,\frac{|\ln R|}{2}\,\frac{\Delta N_i - \Delta N_f}{\Delta N_{th}} = A\,\frac{h\nu}{\sigma_0}\,\frac{|\ln R|}{2}\,\ln\left(\frac{\Delta N_i}{\Delta N_f}\right) \tag{12.10}$$

The peak power is determined by (12.7) with $\Delta N(t) = \Delta N_{th}$:

$$P_{peak} = A\,\frac{h\nu c_0 \ell}{L}\,\frac{|\ln R|}{2}\,\Delta N_{th}\left[\frac{\Delta N_i}{\Delta N_{th}} - \ln\left(\frac{\Delta N_i}{\Delta N_{th}}\right) - 1\right] \tag{12.11}$$

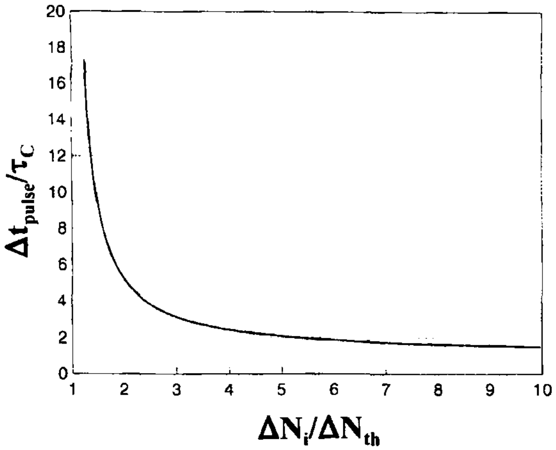


Fig.12.6 Ratio of pulse duration to cavity lifetime as a function of the initial inversion ratio, according to (12.12) and (12.9).

Since we do not know the temporal shape of the intensity, we have to define the pulse duration of the Q-switch pulse by the ratio of energy to peak power:

$$\Delta t_{pulse} = \tau_C \left[\frac{\Delta N_i}{\Delta N_{th}} - \frac{\Delta N_f}{\Delta N_{th}}\right] \Big/ \left[\frac{\Delta N_i}{\Delta N_{th}} - \ln\left(\frac{\Delta N_i}{\Delta N_{th}}\right) - 1\right] \tag{12.12}$$

where τ_C is the cavity lifetime (see (12.2). By combining (12.12) with (12.9), the ratio of pulse duration to cavity lifetime becomes dependent only on the initial inversion ratio $\Delta N/\Delta N_{th}$. Figure 12.6 shows this relationship graphically. The pulse duration is proportional to the resonator length L and roughly inversely proportional to the initial inversion ratio. In other words, for a given hold-off time Δt, the pulse duration will decrease proportionally to the inverse pump power.

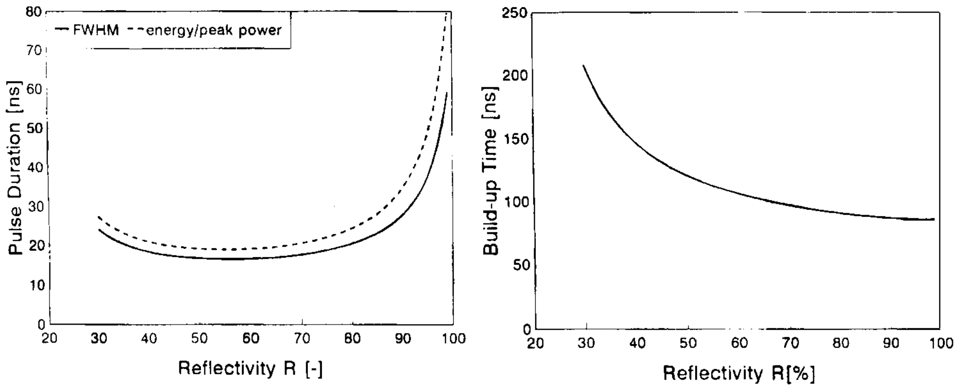
Let us now calculate an example to demonstrate how to apply (12.9) through (12.12) to predict the performance of a Q-switched laser.

Example:

A diode-pumped Nd:YAG (τ=230 µs, σ_0=$4.1\cdot10^{-19}$cm^2) with a rod diameter of 4mm, a rod length of 100mm, and 0.8% doping concentration (N_0=$1.1\text{x}10^{20}$ cm^{-3}) is pumped with an optical pump power of 1kW at 808nm. The resonator has a length of 0.5m with 50% output coupling (R=0.5) and a loss of 2% per transit ($\alpha_0\ell$=0.02). Assuming 95% pump light absorption, the pump rate can be calculated to be W=27.6 s^{-1}, using $W\cdot N_0\cdot V\cdot h\nu_{pump}=P_{pump,abs}$, where V is the volume of the rod. The Q-switch is holding off the cavity for 2ms. With (12.3) we get the initial inversion density ΔN_i= $6.31\cdot10^{-3}$ N_0. Equations (12.7)-(12.12) then yield:

threshold inversion density ΔN_{th}	=	$8.13 \cdot 10^{-4} N_0$
initial inversion ratio $\Delta N_i/\Delta N_{th}$	=	7.76
final inversion ratio $\Delta N_f/\Delta N_{th}$	=	$3.32\ 10^{-3}$
output energy E_{out}	=	156 mJ
cavity lifetime τ_C	=	4.55 ns
pulse duration Δt_{pulse}	=	7.49 ns
peak power P_{peak}	=	20.8 MW

Given a pump time of 2ms, this system can be operated at a maximum repetition rate of nearly 500 Hz, with an average output power of over 70 W.



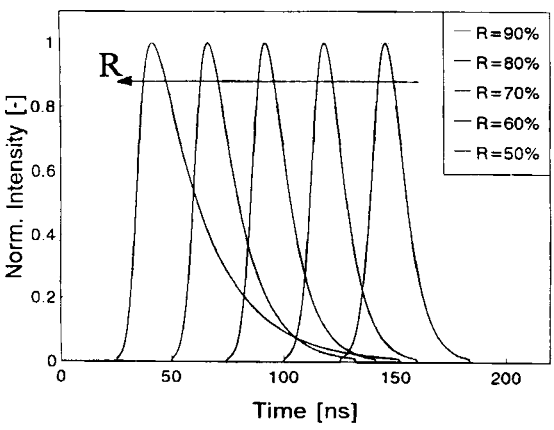


Fig. 12.7 Calculated FWHM pulse duration, build-up time and pulse shapes for a Nd:YVO_4 laser with $g_0\ell=\Delta N_i\sigma_0\ell=2.0$, $\alpha_0\ell=0.02$, and L= 500mm as a function of the reflectivity of the output coupler. Equations (12.1) and (12.2) were solved numerically. The dotted line shows the pulse duration as defined by (12.12). Note that in the lower graph, the pulses have been equally spaced to improve the presentation. The initial inversion ratio decreases from 79.3 for R=99% to 3.21 for R=30%.

Numerical solutions of the rate equations (12.1) and (12.2) are shown in Fig. 12.7 for a diode-pumped Vanadate laser with 2mm diameter pump spot and 104W absorbed pump power. With increasing output coupling $(1-R)$, the pulses become shorter until the decrease in cavity lifetime is overcompensated by the right hand factor of (12.12) containing the inversion densities. This example indicates that increasing the output coupling may not necessarily result in shorter pulse durations. The build-up time is defined as the time interval between switching the Q-switch and the maximum of the output pulse. No analytical expression for this build-up time exists, but similar to the pulse duration (12.12), the build-up time is proportional to the resonator length L and will increase with decreasing ratio $\Delta N_i/\Delta N_{th}$. As a rule of thumb, for Q-switched lasers with near optimum output coupling, the build-up time is about ten times the pulse duration.

In preparation for the next section, where we will calculate the optimum output coupler reflectivity, let us define the initial and the final small-signal gain:

$$g_0\ell = \Delta N_i \, \sigma_0 \, \ell \qquad\qquad (g_0\ell)_f = \Delta N_f \, \sigma_0 \, \ell$$

Equations (12.9) through (12.12) can then be expressed in terms of the small–signal gains, the reflectivity R and the loss per transit $\alpha_0\ell$. Equation (12.9) and (12.10) then become:

$$g_0\ell - [g_0\ell]_f = (\alpha_0\ell - \ln\sqrt{R}) \, \ln\left(\frac{g_0\ell}{[g_0\ell]_f}\right) \tag{12.13}$$

$$E_{out} = A \, \frac{h\nu}{\sigma_0} \, \frac{|\ln R|}{2} \, \ln\left(\frac{g_0\ell}{[g_0\ell]_f}\right) \tag{12.14}$$

By using these two equations, we can now determine the optimum reflectivity R_{opt} for which the output energy exhibits a maximum.

12.3 Optimum Output Coupling

Similar to the output power of cw laser resonators presented in Chapter 10, maximum output energy and optimum mirror reflectivity are a function of the small-signal gain $g_0\ell$ and the loss per transit $\alpha_0\ell$ [4.71]. The Q-switch extraction efficiency is defined as ratio of the output energy to the stored energy present at the time when the Q-switch opens:

$$\eta_{extr} = \frac{E_{out}}{\Delta N_i \, h\nu \, A\ell} = \frac{|\ln R|}{2 g_0\ell} \, \ln\left(\frac{g_0\ell}{[g_0\ell]_f}\right) \tag{12.15}$$

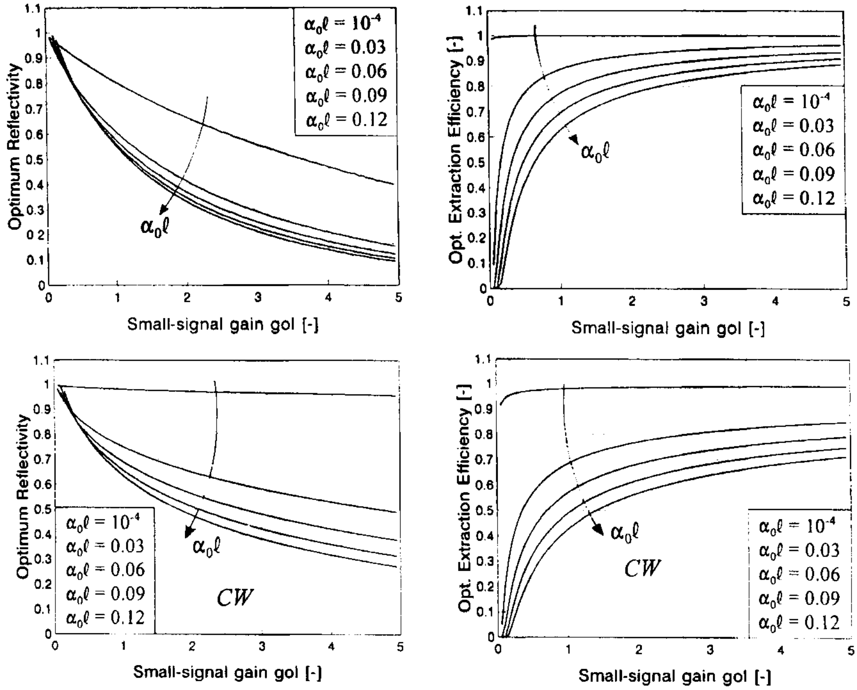


Fig. 12.8 Optimum reflectivity and maximum extraction efficiency for Q-switching (upper graphs) and cw-emission (lower graphs) as a function of the small-signal gain. Curve parameter is the loss per transit $\alpha_0\ell$. For Q-switching, the small-signal gain is determined by the initial inversion density: $g_0\ell=\Delta N_i\sigma_0\ell$.

Optimum reflectivity R_{opt} and maximum extraction efficiency $\eta_{extr,max}$ can be found analytically [4.71]:

$$\ln R_{opt} = -2\alpha_0\ell \left[\frac{x - 1 - \ln x}{\ln x}\right] \tag{12.16}$$

$$\eta_{extr,\max} = \frac{x - 1 - \ln x}{x} \tag{12.17}$$

with $x=g_0\ell/\alpha_0\ell$. These expressions are very similar to the corresponding equations for a cw laser presented in Chapter 10. Figure 12.8 shows optimum extraction efficiencies and optimum reflectivities as a function of small-signal gain and loss for both, Q-switching (Eqs. (12.16) and (12.17)) and cw emission (Eqs. (10.14) and (10.15)). These graphs indicate that for the same ratio x, Q-switched operation requires a higher output coupling. This relative insensitivity to the total cavity loss is also apparent in the extraction efficiency. Compared to a cw laser, Q-switching will provide higher extraction efficiencies for the same loss per transit $\alpha_0\ell$. At the optimum output coupling, the pulse duration (12.12) is given by [4.71]:

$$\Delta t_{pulse} = \frac{L}{c_0\alpha_0\ell}\left[\frac{\ln x}{x[1-y+y\ln y)]}\right] \qquad with\ \ y = \frac{x-1}{x\ln x} \tag{12.18}$$

Example:

Application of (12.16) through (12.18) to the diode-pumped Nd:YAG laser discussed on page 439 yields:

small-signal gain $g_0\ell$	=	2.85
ratio $x=g_0\ell/\alpha_0\ell$	=	142.5
optimum reflectivity R_{opt}	=	33.2%
maximum Extraction Efficiency	=	95.8%
pulse duration	=	6.1 ns

For the same laser, decreasing the pump power by a factor 2 will result in:

small-signal gain $g_0\ell$	=	1.425
ratio $x=g_0\ell/\alpha_0\ell$	=	71.25
optimum reflectivity R_{opt}	=	53.9%
maximum extraction efficiency	=	92.6%
pulse duration	=	11.6 ns

12.4 Repetitive Q-switching

In the previous sections, we have assumed that the inversion at the start of the pump process is equal to zero (see Eq. (12.3)). This implies that after a Q-switch pulse, a certain time must elapse in order to let the final inversion density go to zero via spontaneous emission prior to starting the next pump and Q-switch cycle. If, however, the repetition rate f gets close to or is greater than the inverse of the upper laser level lifetime τ, the left over inversion at the start of the pump cycle has to be considered. This condition is referred to as repetitive Q-switching [4.55,4.57,4.59,4.60]. For a cw-pumped laser, the initial inversion density ΔN_f depends on the final inversion density of the previous pulse (we consider an ideal four-level system), adding another term to Eq. (12.3):

$$\Delta N_i = \frac{WN_0}{W+1/\tau}\left(1-\exp[-(W+1/\tau)/f]\right) - \Delta N_f \exp[-(W+1/\tau)/f] \tag{12.19}$$

With the assumption $W\tau \ll 1$, this equation can be rewritten in terms of the small-signal gains:

$$g_0\ell = [g_0\ell]_{cw}\left(1-\exp[-1/(\tau f)]\right) - [g_0\ell]_f \exp[-1/(\tau f)] \tag{12.20}$$

where $[g_0\ell]_{cw}$ is the small-signal gain of the cw-laser. This equation has to be combined with (12.13) and (12.14) to find a numerical solution through successive approximation. The average output power can then be calculated using P_{out}=f E_{out}. Figs. 12.9 through 12.11 show calculated output power, pulse duration and build-up time for the same diode-pumped solid state laser resonator (L=500mm, 2mm pump spot diameter, 100W pump power, $\alpha_0\ell$=0.02), but for four different commonly used laser materials. As the curves indicate, the output characteristics depend mainly on the upper level lifetime τ and the cross-section for stimulated emission σ_0. The relatively short lifetime of Nd:YVO_4 requires high repetition rates to achieve output powers close to the cw power, but its high emission cross-section results in short pulse duration. Nd:YLF and Yb:YAG, on the other hand, lend themselves well to low repetition rate operation due to their energy storage capacities. Yb:YAG, however, exhibits a low emission cross-section, which leads to relatively long pulses and limits the extraction efficiency due to the low small-signal gain. Note that for all four lasers, the build-up time at the optimum output coupling is about ten times the pulse duration, as stated previously. With increasing repetition rate, the initial small-signal gain becomes lower (see (12.19), resulting in a shift of the optimum output coupling to higher reflectivities.

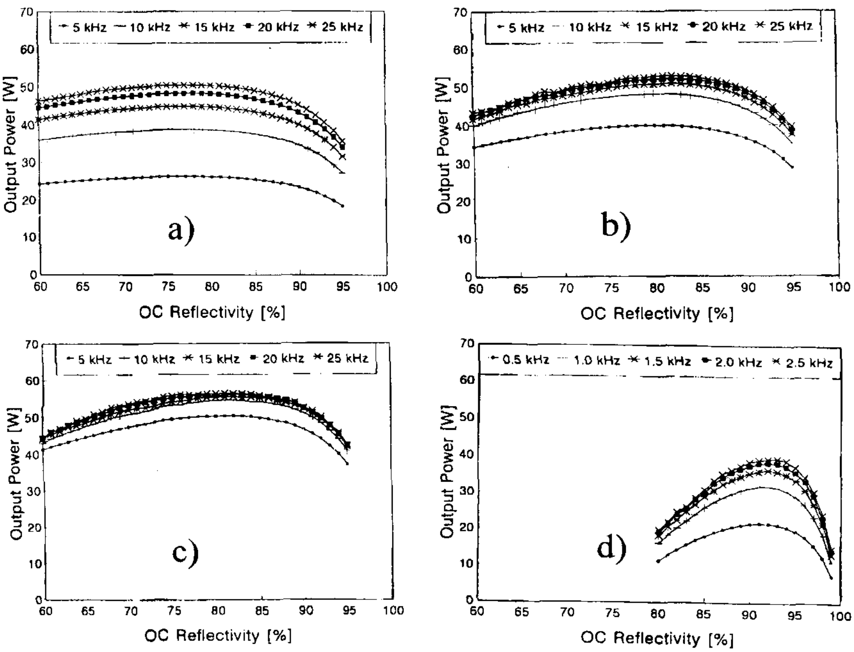


Fig. 12.9 Calculated average output power as a function of the output coupler reflectivity for four different diode-pumped solid state materials and the same resonator (L=500mm, 2mm pump spot diameter, 100W pump power, 2% loss per transit). a) Nd:YVO_4, λ_p=808nm, σ_0=15 x $10^{-19}cm^2$, τ=100μs , b) Nd:YAG, λ_p=808nm, σ_0=4.1x$10^{-19}cm^2$, τ=230μs c) Nd:YLF, λ_p=806nm, σ_0=1.8 x $10^{-19}cm^2$, τ=480μs d) Yb:YAG, λ_p=940nm, σ_0=0.21 x $10^{-19}cm^2$, τ=950μs.

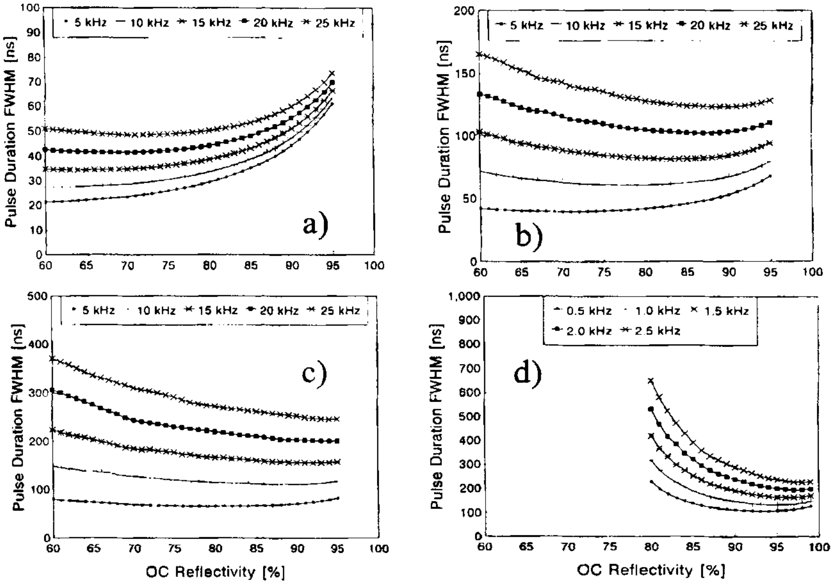


Fig. 12.10 Calculated pulse durations for the resonators of Fig. 12.9 (L=500mm, 2mm pump spot diameter, 100W pump power, 2% loss per transit). a) Nd:YVO_4, λ_p=808nm, b) Nd:YAG, λ_p=808nm, c) Nd:YLF, λ_p=806nm, d) Yb:YAG, λ_p=940nm.

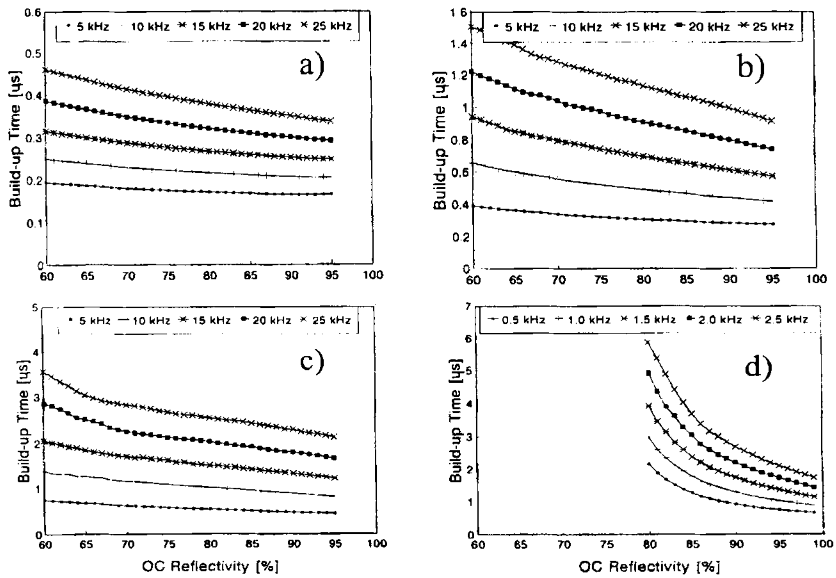
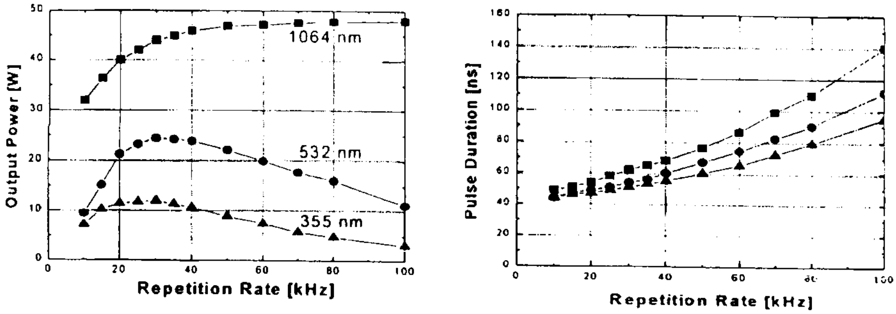


Fig. 12.11 Calculated pulse build-up time for the resonators of Figs. 12.9 and 12.10 (L=500mm, 2mm pump spot diameter, 100W pump power, 2% loss per transit). a) Nd:YVO_4, λ_p=808nm, b) Nd:YAG, λ_p=808nm, c) Nd:YLF, λ_p=806nm, d) Yb:YAG, λ_p=940nm.

Experimental examples of repetitively Q-switched Vanadate lasers are presented in Figs. 12.12 and 12.13. Both systems are designed to provide a diffraction limited output beam. The choice of laser material is usually determined by the application requirements on pulse energy, pulse duration and repetition rate. For repetition rates below 10kHz, Nd:YAG or Nd:YLF are the materials of choice, whereas Nd:YVO_4 finds widespread applicaton at high repetition rates (20-400 kHz).



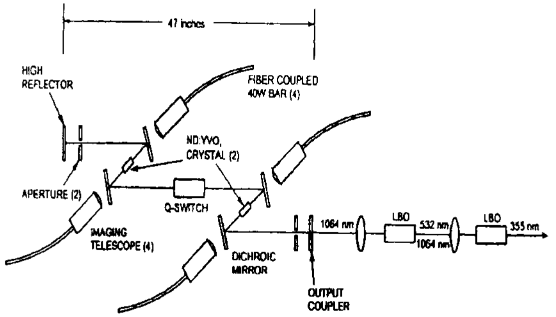


Fig. 12.12 Measured output power and pulse durations of a diode-end-pumped Nd:YVO_4 laser with AOM Q-switch in TEM_{00} mode operation at 1064nm and with external frequency doubling and tripling. The layout for frequency tripling is shown.. Total optical pump power at 808nm incident on the crystals is 104W [4.77] (© OSA 2001).

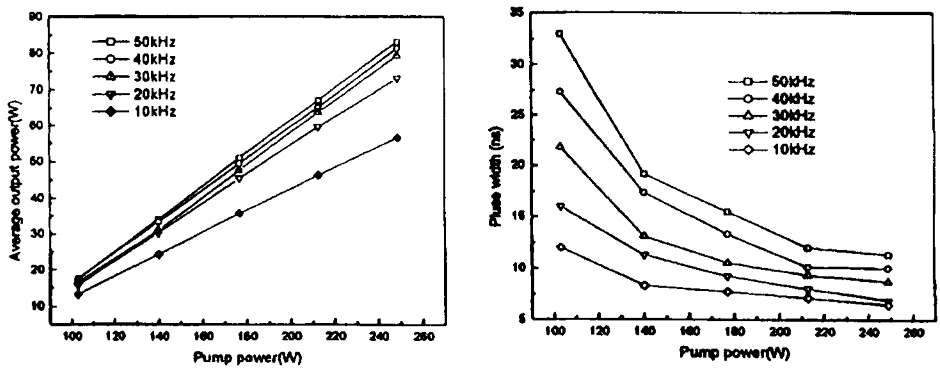


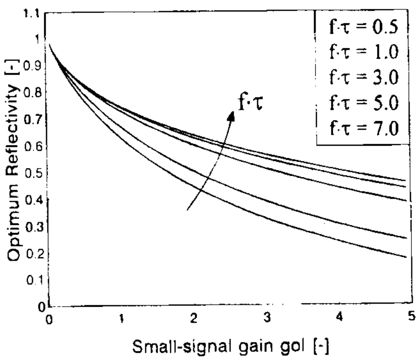
Fig. 12.13 Output power and pulse durations for a diode-pumped Nd:YVO_4 slab laser with an electro-optic BBO Pockels cell. The slab with dimensions 1x12x10mm is end pumped over an area of 0.5mmx12mm. A 67.5mm long flat-unstable hybrid resonator is used to obtain $M^2<1.5$ [4.79] (© OSA 2003).

Optimum Output Coupling

The optimum output coupling now depends on three rather than two parameters: the cw small signal gain $[g_0\ell]_{cw}$, the loss per transit $\alpha_0\ell$, and the product of lifetime and repetition rate τf. This makes it a little difficult to present the results graphically. However, since the loss per transit is usually within a small range of a couple of percent, a good compromise is to present the optimum values as a function of gain and repetition rate. To find the optimum, the three equations (12.13), (12.14), and (12.20) have to be solved numerically via successive approximation. In the limit $\tau f \to 0$, the final inversion density of the previous pulse has been depleted before the next Q-switch cycle starts, which means that the optimum reflectivities for Q-switching shown in Fig. 12.8 apply. For high repetition rates with $\tau f \gg 1$, we expect the optimum reflectivities to be close to those of a cw-laser. In any case, the previously discussed expressions (12.16) and (12.17) are still valid, provided that the initial inversion density is calculated with (12.20) rather than (12.3). Since we are considering a cw-pumped laser, it is reasonable to define the extraction efficiency as the ratio of the average output power to the inversion power:

$$\eta_{extr} = \frac{P_{out}}{A\ I_s\ [g_0\ell]_{cw}} = \frac{E_{out}\, f}{A\ I_s\ [g_0\ell]_{cw}} = \frac{|\ln R|}{2[g_0\ell]_{cw}} \ln\left(\frac{g_0\ell}{[g_0\ell]_f}\right) \tau f \qquad (12.21)$$

Figure 12.14 presents optimum output coupling and maximum extraction efficiencies as a function of the cw small-signal gain $[g_0\ell]_{cw}$ for a loss of $\alpha_0\ell=0.03$. As expected, both parameters approach the cw-values with increasing repetition rates. Typically, for $\tau f>5$, the average output power is within 5% of the cw power. Unfortunately, the repetition rate of repetitively Q-switched lasers can not be increased indefinitely, because the time it takes the inversion to reach the threshold inversion will eventually exceed the inverse frequency $1/f$. The pulse repetition rate will then automatically switch to half the Q-switch repetition rate.



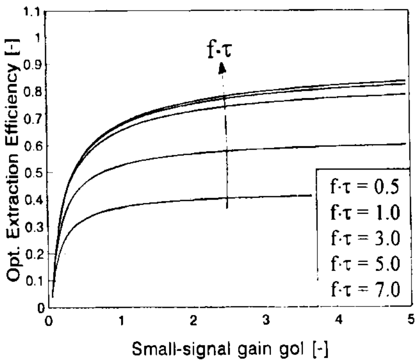


Fig. 12.14 Optimum output coupling reflectivities and maximum extraction efficiencies as a function of the cw small-signal gain $[g_0\ell]_{cw}$ for a loss per transit of $\alpha_0\ell=0.03$. Curve parameter is the product of upper level lifetime and repetition rate.

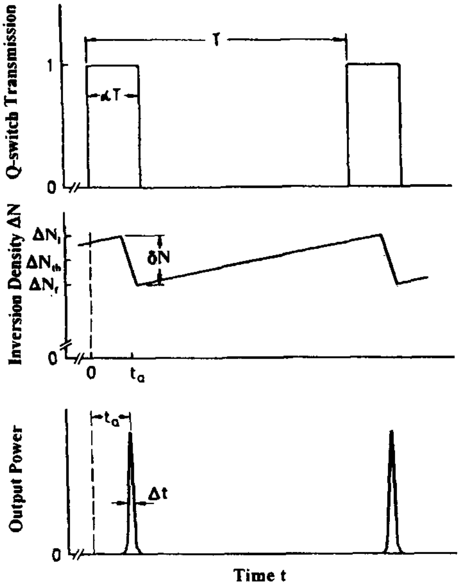


Fig. 12.15 Q-switch transmission, inversion density build-up and pulse emission for repetitive Q-switching with $\tau f \gg 1$. The Q-switch generates no loss during the time αT. The peak power is observed after the build-up time t_a.

At high repetition rates with $\tau f \gg 1$, the inversion increases linearly between pulses and the initial and final inversion densities stay close to the threshold inversion density (Fig. 12.15):

$$\Delta N_i - \Delta N_{th} \approx \Delta N_f - \Delta N_{th} \approx \delta N/2 \tag{12.22}$$

Insertion into (12.10), (12.11) and (12.12) and expansion of the logarithm to the second order results in:

$$E_{out} = \frac{A\,h\nu}{\sigma_0}\,\frac{|lnR|}{2}\,\frac{\delta N}{2\Delta N_{th}} \tag{12.23}$$

$$P_{peak} = A\,\frac{h\nu c_0 \ell}{L}\,\frac{|lnR|}{2}\,\Delta N_{th}\left[\frac{\delta N}{2\Delta N_{th}}\right]^2 \tag{12.24}$$

$$\Delta t_{pulse} = \frac{L}{c_0}\,\frac{2}{\delta N\,\sigma_0 \ell} \tag{12.25}$$

A surprising result is that the product of the pulse energy and the pulse duration is a constant of the laser resonator and does not depend on the pump power:

$$E_{out} \cdot \Delta t_{pulse} = \frac{AL}{c_0} \frac{h\nu}{\sigma_0} \frac{|\ln R|}{|\ln R| + 2\alpha_0 \ell} \tag{12.26}$$

If we introduce the cw-output power P_{cw} of the system:

$$P_{cw} = A\, I_S \frac{1-R}{2\sqrt{R}} \left(\frac{[g_0 \ell]_{cw}}{|\ln(\sqrt{R} V_S)|} - 1 \right)$$

the equations read [4.78]:

$$E_{out} \cong 2\, \tau\, m\, P_{cw} \tag{12.27}$$

$$P_{peak} = \frac{c\tau}{2L} \left| \frac{\ln^2(\sqrt{R} V_S)}{\ln\sqrt{R}} \right| \frac{m^2}{A\, I_S} P_{cw}^2 \tag{12.28}$$

$$\Delta t_{pulse} = A\, I_S \frac{2L}{c} \frac{2}{m\, P_{cw}} \tag{12.29}$$

with the modulation function:

$$m = \frac{1 - \exp[-1/(\tau f)]}{1 + \exp[-1/(\tau f)]} \tag{12.30}$$

Let us now find an expression for the maximum Q-switch repetition rate at which still one pulse is emitted per Q-switch period. It is safe to assume, that this is only possible if the inversion can build up beyond the threshold inversion within the time period $1/f$. By linearizing Eq. (12.19) with $W\tau \ll 1$ and using (12.22), we can find an expression for the modulation depth δN of the inversion density:

$$\frac{\delta N}{2\Delta N_{th}} = \frac{\tau W N_0}{\Delta N_{th}} - 2\tau f + 1 \tag{12.31}$$

This equation can be rewritten in terms of the pump power:

$$\frac{\delta N}{2\Delta N_{th}} = \frac{\eta_{excit} P_{pump}}{I_S\, A\, (\alpha_0 \ell - \ln\sqrt{R})} - 2\tau f + 1 \tag{12.32}$$

with the excitation efficiency η_{excit} defined by (9.19)-(9.21). The initial inversion ΔN_i can only be greater than the threshold inversion if the right-hand side of (12.32) is greater than zero. The condition for the allowed Q-switch repetition rates then reads:

$$f < \frac{1}{2\tau}\left[\frac{\eta_{excit}\,P_{pump}}{I_S\,A\,(\alpha_0\ell - \ln\sqrt{R})} + 1\right] \tag{12.33}$$

Example:
Diode pumped Nd:YVO_4 laser with emission wavelength λ_L=1064nm, diameter of pumped area: 2mm, pump power at λp=808nm: 100W with 95% absorption, R=0.9, $\alpha_0\ell$=0.02. With I_S=1.257 kW/cm^2 and τ=100 μs, Eq. (12.33) provides the condition:

f < 125.7 kHz

Increasing the repetition rate of the Q-switch beyond the limit given by (12.33) will result in a reduction of the pulse frequency by a factor of 2. It now takes two Q-switch cycles to build up the inversion. Further increase of the repetition rate will then result in further reductions of the pulse repetition rates. An experimental example of an cw flashlamp-pumped Nd:YAG with AOM is shown in Fig. 21.16. It should be mentioned that the time during which the Q-switch generates no loss, which is referred to as the gate-open time, has a strong influence on the laser performance. If the gate-open time is chosen too large, several pulses are emitted, or the laser may even emit in cw-operation after a sequence of spikes. On the other hand, if the gate-open time is too short, it takes two Q-switch cycles before the pulse can be emitted. Typical gate-open times for solid state lasers range from one to ten microseconds, depending on gain, lifetime and repetition rate.

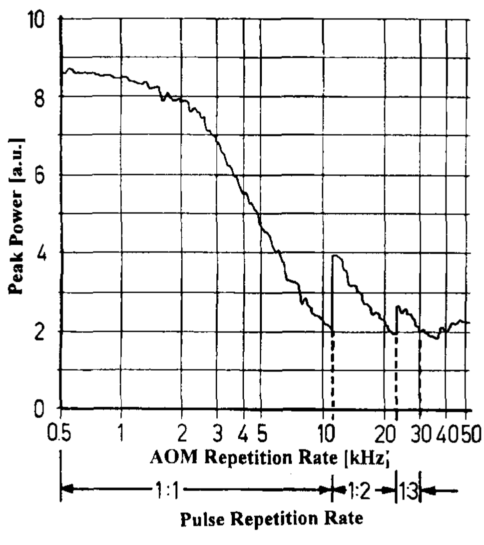


Fig. 12.16 Measured pulse peak power as a function of the AOM repetition rate for a cw flash-lamp pumped Nd:YAG laser in TEM_{00} mode operation with acousto-optic Q-switching. The pulse repetition rate is reduced by a factor 2 and 3 as the AOM repetition is increased and never exceeds 10.5kHz.