Introduction

Laser radiation exhibits several properties which are not provided by conventional light sources such as incandescent or fluorescent lamps:

- Narrow frequency bandwidth (as low as 10^{-3} Hz) and high temporal coherence

- Low beam divergence (on the order of milliradians) and high spatial coherence

- High intensity in focal spots (up to 10^{15} W/cm^2)

Due to these beam characteristics the laser has found application in technologies where material has to be removed or transformed within defined, often small, areas (e.g. laser surgery, material processing) or where a narrow spectral frequency band is required (e.g. remote sensing and spectroscopy). In addition, the unique properties of laser diodes (small size, high efficiency, high modulation rates) have been the enabling force in the burgeoning fields of optical communications and optical processing.

Without the optical resonator, the radiation emitted by the laser medium (solid state material, gas or liquid dye) could hardly be used for any application. Since the active atoms generate photons mainly through the process of spontaneous emission, the radiation (superluminescence) does not differ from the light emitted by thermal light sources as far as spatial coherence and focusability are concerned. This is due to the fact that the photons are emitted independently over a range of light frequencies that are characteristic for the laser material. By feeding back the emitted photons into the active medium, this lack of synchronization between emitting atoms can be overcome to generate the exciting properties of laser beams. One could say that the feedback forces the output to stabilize on one preferred channel.

In all laser systems this feedback is accomplished by the optical resonator. Beam properties such as spatial intensity structure, output power, and minimum focal spot size attainable with a focusing lens, are mainly determined by the optical resonator. This crucial role played by the resonator in laser systems is the reason that even after forty years of research in this field, scientists are still working on optical resonators to further understand and improve laser beam characteristics.

The principle of feedback to stabilize the output of a physical system to a desired level is not only limited to optics but has been applied to different engineering areas for eighty years. The best known example is the Meissner circuit, invented in 1913 by the physicist Alexander Meissner, which generates undamped electromagnetic oscillations (Fig.1).

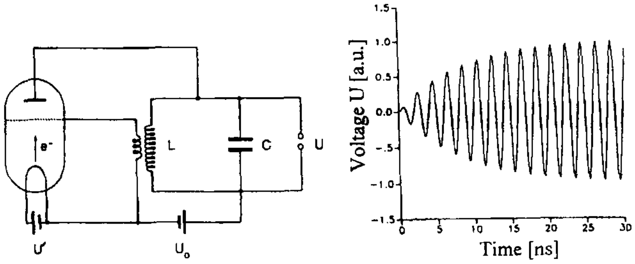

Fig.1 Meissner Circuit for generating undamped electrical oscillation as an example for the feedback principle. The right hand graph shows the normalized output voltage *U* as a function of time for the case that the initial gain factor overcompensates the loss in the LC circuit.

The attenuation losses of a LC-oscillation circuit are compensated by feeding back the right amount of electrons by means of a triode. Without feedback, the output voltage *U* would oscillate sinusoidally with an exponential amplitude decay. The feedback is accomplished by coupling the voltage at the inductance *L* to the acceleration grid of the triode. As soon as the grid voltage turns positive, electrons are forced to enter the LC-circuit to compensate for resistance losses. This process maintains a temporally constant amplitude of the oscillating output voltage *U*. By using this circuit, electric oscillations with frequencies of up to 10^9 Hz can be generated. For higher frequencies the time of flight of the electrons in the triode becomes a problem.

In steady state the gain factor G ($G>1$) by which the voltage is increased per period by the triode exactly compensates the attenuation losses by the loss factor V ($V<1$). The steady state oscillation condition therefore reads:

$$G\,V = 1$$

The gain factor G is a function of the acceleration voltage at the triode grid and thus defined by the characteristics of the triode and the coupling between the inductors. In general, the gain factor increases as the voltage increases but it reaches a saturation value at higher voltages. Only if the gain factor G is greater than the loss factor V, which means $GV>1$, the oscillation will build up as shown in Fig.1. As the output voltage increases, the gain factor saturates. The steady state output voltage amplitude is reached as soon as the condition $GV=1$ is realized. Since the gain factor G can be expressed as a function of the output voltage U (the grid acceleration voltage and the output voltage are proportional!) the steady state output voltage U_S is given by:

$$G(U_S) = \frac{1}{V}$$

This equation can be used to calculate the steady state output voltage amplitude since the expressions for both the gain and loss as a function of output voltage U can easily be derived.

The laser can be considered as the extension of the feedback principle into the frequency range of 10^{15} Hz, the frequency range of UV, visible, and infrared light. There is a strong similarity between the laser and the Meissner circuit since the principle of operation is basically the same. The triode is now represented by the active medium and the oscillation of electrons in the LC circuit is now replaced by the oscillation of light between the resonator mirrors (Fig.2). The light first generated by spontaneous emission in the active medium bounces back and forth inside the resonator. The light intensity is decreased by a loss factor of R_l (mirror reflectivity) when it bounces off the output coupling mirror 1 and increased by a gain factor G when it travels through the active medium. Similar to the triode the gain factor is a function of intensity and it decreases as the intensity increases. The laser is forced to attain a steady state intensity I_O inside the resonator at which loss and gain per round trip are balanced. Since the light travels twice through the active medium before experiencing a loss, the steady state condition reads:

$$G(I_O)\, G(I_O)\, R_1 = 1 \quad \Rightarrow \quad G(I_O) = \frac{1}{\sqrt{R_1}}$$

The gain factor G is generated by an external pumping process by which energy is transferred into the active medium to generate a population inversion between atomic or molecular energy levels. In solid state lasers, this is accomplished by using high power flashlamps or diodes; gas lasers are generally pumped by means of a high voltage gas discharge, and a diode laser by an electric current.

The laser starts oscillating as soon as the gain factor is greater than $1/(V\sqrt{R_1})$ with all other losses caused by diffraction, scattering, and absorption being included in the loss factor per transit V. The light intensity experiences an amplification with every round trip inside the resonator. With increasing intensity, however, the gain factor G starts to decrease and the intensity will reach the steady state solution I_O for which the above shown steady state condition holds. A higher intensity can only be achieved if the pumping power is increased by increasing the flashlamp power, the pump diode current, or the discharge voltage, respectively.

The reader should always keep in mind that our definition of an optical resonator includes the active medium. This means that we are basically dealing with a laser. However, the laser is only a part of the whole field since optical resonators can also be used without an active medium. Examples in this area are interferometers, multi-pass optical delay lines, and optical coatings. The calculation of the properties of an optical resonator thus far seems to be as straightforward as a design of a Meissner circuit. With our current knowledge we can determine the steady state intensity I_O inside the resonator which means that we also know the output power emitted by the laser. Unfortunately, this model is far too simplified to be useful for the description of optical resonators.

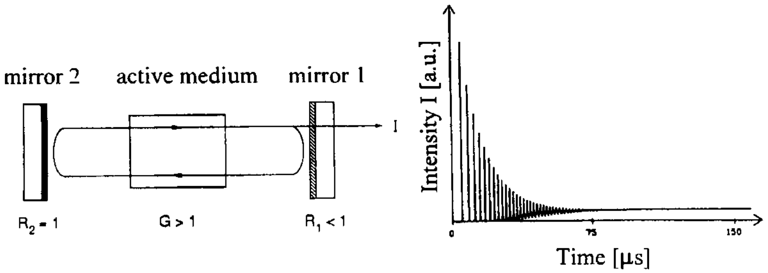

Fig.2 Principle of an optical resonator. In order to attain laser oscillation, the gain factor *G* of the active medium must be greater than the loss factor caused by the output coupling at mirror 1. After an initial intensity oscillation, a steady state intensity is reached for which the gain exactly compensates the output coupling losses (right hand graph). The laser then emits light in continuous wave (cw) operation.

In contrast to the Meissner circuit where we only have to deal with one parameter, the voltage amplitude, electromagnetic radiation exhibits more parameters than only the amplitude of the electric field or the intensity. Steady state solutions also exist for the spatial field structure, the polarization, and the phase distribution of the electric field. Furthermore, the shape of the resonator mirrors, the size of internal apertures as well as the properties of polarizing optics inside the resonator have a considerable influence on the beam properties. The nonlinear interaction of the oscillating electric field with the atoms in the active medium makes the treatment even more challenging.

This complexity of optical resonators makes it necessary to first discuss the physical properties of light and to derive methods for their calculation. The equations for the calculation of beam propagation, transverse beam structure, polarization, coherence and their application to optical resonators are derived in Part I; "The Electromagnetic Field". The mathematical content of this chapter was kept as low as possible by translating exact mathematical derivations into a more intuitive physical language. All equations printed in this book can be derived by using the mathematical tools presented in Part I. Parts II-V, which deal with optical resonators and their design, contain only short descriptions of how to obtain the result rather than going through the whole derivation.

Part II starts with the most basic optical resonator, the Fabry-Perot-Interferometer, and will help readers new to the field to become familiar with the properties of optical resonators. After discussing linear resonators without the influence of the active medium (Part III), the properties of the laser medium and its effect on the laser characteristics will be presented in Part IV. Part V is a collection of resonator concepts that exhibit specific advantages, like low misalignment sensitivity, narrow bandwidth, or excellent beam quality. These resonators have either a limited applicability or represent new concepts which may become more important in the near future. Readers interested in designing and optimizing laser resonators may find the measurement techniques presented in Part VI helpful. A detailed reference list will help readers to find more information on their chosen subject.

Part I

The Electromagnetic Field