Chapter 4

OPERATIONAL RESEARCH METHODS FOR EFFICIENT WAREHOUSING

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Abstract The design and operation of a warehouse entail many challenging decision problems. We begin by providing definitions as well as qualitative descriptions of two actual warehouses. This will then set the stage for an overview of representative operational research models and solution methods for efficient warehousing. Problems which will be exposed can be classified into three major categories: throughput capacity models, storage capacity models, and warehouse design models. We conclude by identifying future research opportunities.

1. Introduction

Be they associated with grocery distribution, manufacturing or health care, warehouses are ubiquitous and come in almost all shapes and sizes. So it is certainly of considerable practical interest to identify methods for improving their design and operation, and these span the entire spectrum of analytical models (optimization and queuing) and simulation models. Problems which are surveyed here can be classified into three major categories: throughput capacity models, storage capacity models, and warehouse design models. Note that warehouse location models and container terminals (which serve as temporary buffers for inbound and outbound containers) are respectively examined in the chapters entitled "Facility Location in Supply Chain Design" and "Models and Methods for Operations in Port Container Terminals," in this book.

Throughput capacity models are comprised of order picking policies, akin to vehicle routing problems and which can be further subdivided between picking and batching policies, as well as storage assignment policies and dynamic control policies. Storage assignment policies attempt to match incoming product with available storage locations. Objective functions assumed in the study of these policies include the minimization of material handling cost (or equivalently, the maximization of throughput), as well as the minimization of material handling costs plus inventory holding and reordering costs.

Storage capacity models either find the optimal warehouse size or else maximize space utilization. Meantime, questions such as rack orientation, space allocation and overall building configuration are the purview of warehouse design models. Previous surveys on the use of operational research methods in warehouses were conducted by Ashayeri and Gelders (1985), who concluded that the most practical approach to studying the complexities of a warehousing system is to combine analytical and simulation models, and by Cormier and Gunn (1992), who pointed out that, while warehouses are usually part of a larger supply chain, studying the tradeoffs between all the latter's constituents poses both significant modelling and organizational challenges. The most recent such surveys are by Van den Berg and Zijm (1999) and Rouwenhorst et al. (2000).

Figure 4.1 is an attempt to categorize the various warehousing decision models and proposes a second classification based on a strategic, tactical and operational decision framework. Note that strategic decisions have a significant impact on long-term profitability and do not recur frequently, hence justifying the use of sophisticated analytical and simulation models. On the other hand, operational decisions tend to recur on a daily basis, or even more frequently for that matter, so that the main concern is in having algorithms which yield consistently good solutions quickly.

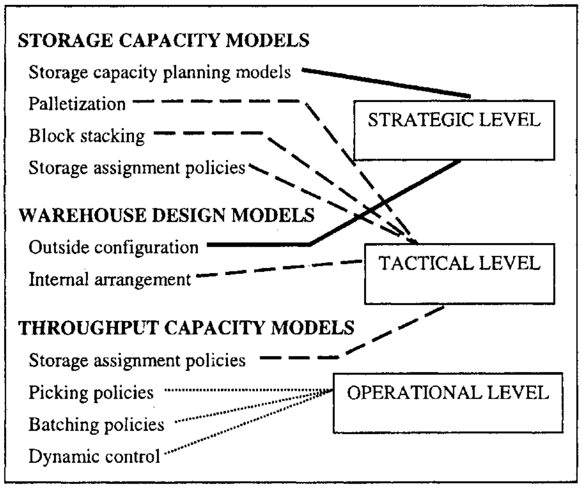

Figure 4.1. A taxonomy of warehousing decision models.

This therefore points the way to the development of efficient heuristics, given that most problems in this category are combinatorial. As for tactical models, they lie in-between strategic and operational models in importance and characterizing an ideal algorithm for them depends on specific circumstances, particularly execution frequency. Some readers might be surprised to find that storage assignment policies appear both under storage capacity models and throughput capacity models. Generally speaking, all storage capacity models exercise some influence over throughput capacity; for instance, think of how far you have to walk to get your groceries in a large grocery store as opposed to a small one.

The remainder of this chapter is organized as follows. In the next section, some terms which frequently appear in the warehousing literature are defined. In order to help the reader better understand the application context, we then describe, in Section 3, actual warehousing operations. This is followed in Section 4 by a presentation of performance evaluation models whose use transcends the three major decision model categories, namely, throughput capacity models, storage capacity models and warehouse design models. These are thereafter reviewed in Sections 5, 6 and 7, respectively. Finally, Section 8 recapitulates this chapter and identifies research gaps.

2. Definitions

Let us begin by defining some terms which are typically encountered in the warehousing literature. An *order* consists of a set of items destined to some customer and which must be retrieved from the warehouse. The *reorder quantity* is the amount of stock received by the warehouse at one time. A *rack* is a set of adjacent storage locations, while an *aisle* is the space in front of the rack where the order picking vehicle travels. The order picking vehicle can take several forms, for instance, a forklift truck, a hand cart, or, in the case of an automated storage and retrieval system (AS/RS), a S/R machine or crane.

Warehouses typically comprise a *reserve storage area*, where product is usually stored on pallets, as well as a *picking area*, where it is more common to place items on shelves or some other form of storage device. As open case stock in the picking area is depleted, new product is transferred from reserve storage to the picking area. Each area serves a specific purpose: in the reserve storage area the main concern is achieving high storage density whereas in the picking area the objective is to maximize picking efficiency.

Once the allocation of stock between the reserve and picking areas has been established, the items must be assigned to specific storage locations. In a *dedicated storage policy*, a set of locations is reserved for each product for the duration of the planning horizon. Furthermore, since the same priority is given to all units of the same product, these units are assigned to contiguous locations. A *shared storage policy*, on the other hand, allows units of different products to successively occupy the same locations. A common example of a shared storage policy is *random storage*, in which products are assigned to storage locations randomly. A popular hybrid approach is the *class-based dedicated storage policy*, which entails assigning products to a class of storage locations based on their class of turnover, while within any given class products are stored randomly.

A *cross-docking warehouse* is one in which incoming items are moved directly from receiving to shipping, thereby avoiding intermediate storage and retrieval. The *pickup and delivery (P/D) point* is the transfer point in and out of the warehouse (this term is most often associated with AS/RS's, and is analogous to a dock). *Single*, *dual* and *multi-command systems* refer to the number of locations which the order picking vehicle can visit between consecutive trips to the P/D point. The term *interleaving* signifies dual command as well, which consists of one leg from the P/D point to the first rack location where a pallet is placed, a second leg from the first location to a second location where a pallet is retrieved, and a final leg back to the P/D point where the retrieved pallet is deposited.

Travel time and distance in a two dimensional warehouse (that is, one in which the order picking vehicle's travel time depends on two axes, as in the case of a rack where neither horizontal nor vertical travel time dominates the other) may either be determined by the rectilinear norm or the Chebyshev norm. Travel time measurement according to the *rectilinear norm* implies that travel between any pair of locations occurs along only one Cartesian axis at a time, while in the case of the *Chebyshev norm* travel occurs in both directions simultaneously, albeit usually at different speeds. Let t_H and t_V denote the travel times from the P/D point to the farthest horizontal and vertical rack locations, respectively, and define the *rack shape factor* as $b = \min\{t_H, t_V\} / \max\{t_H, t_V\}$. The resultant *normalized rack* is called *square-in-time* if $b = 1$, meaning that travel time between the P/D point and the farthest horizontal location is identical to that to the farthest vertical location.

3. Examples of warehousing systems

Two actual warehousing operations are briefly described next and their distinct features underlined. The purpose here is twofold; first, to give the reader some insights into the organizational and technical issues often characterizing warehousing environments, and second, to motivate the modelling approaches reviewed in the remainder of this chapter.

3.1 A warehouse for grocery products

In this section, we outline order picking procedures in a large grocery warehouse. In addition to having a top and a bottom cross-aisle, the warehouse has a central cross-aisle. The order picking vehicle can carry two pallets at once, and since each order exceeds its capacity, the computerized warehouse management system (WMS) partitions each order into several tours. A dedicated storage policy is enforced inside the shelves, while the top of each shelf constitutes the reserve storage area in which a random storage policy is employed. The order picking activities are directed by the WMS under the following assumptions:

(1) Each tour is restricted to a single order.
(2) The central cross-aisle is not used.
(3) Aisles are always traversed in the same direction.
(4) All tours begin with the left-most aisle where items are to be picked, if the direction of travel in that aisle is from the docks to the back of the warehouse. Otherwise, the picker has to first travel empty to the back of the warehouse.

Tours hence follow a serpentine path, from the left side to the right side of the warehouse, with new tours started whenever the vehicle is full. Intuitively, relaxing the above assumptions can only lead to more efficient order picking. Some evidence supporting this conjecture was obtained through the adaptation of Clarke and Wright's (1964) heuristic, which, using data from an actual order, yielded savings (in terms of distance) of 13% compared to the WMS. Moreover, some results obtained by Racine (2000) on the subject of order batching in this situation are given in Section 5.1.

3.2 A warehouse for hardware products

This zoned warehouse, used for storing hardware products, is divided into three major storage areas: i) a storage area where a dedicated storage policy is used and non-conveyable products are kept, for instance, shower units and lawn mowers; ii) a reserve storage area consisting of a number of zones in which a random storage policy is in force and conveyable high volume products are stored; and iii) a forward pick area, operating under a dedicated storage policy and where many of the products from the reserve storage area are also kept for the purpose of open case picking. Note that, given that the random storage policy makes no distinction between zones, a product could be stored in any number of locations and zones.

A *wave* regroups some or all the orders that constitute a truckload. There is often a single wave per truckload, but sometimes two or three. Orders cannot be split between waves, although portions of orders on the same wave are generally picked in different zones. The purpose of waves is mostly to keep the work flowing smoothly at the sortation station, just prior to shipping. On the one hand, a wave cannot be too big since the area around the sortation station only allows for the accumulation of about six orders at a time. On the other hand, a wave cannot be too small as this reduces the probability of each zone having items to pick, resulting in a workload imbalance.

A worker determines the number of waves, and the computer performs the assignment of orders to waves. It should further be noted that each zone picks on a single wave at a time, and that the waves are taken in the same sequence in all zones, although all zones are not necessarily working on the same wave simultaneously. Picking is done mostly during the day and midnight shifts, with re-stocking (put-away) carried out during the evening shift, along with some limited picking. A *pick list* is a document specifying a *work assignment*, i.e., the set of items constituting a *batch* which is a subset of a *wave* to be picked on a single tour of an order picking vehicle. Moreover, each *line* on a pick list specifies the number of units of a certain product demanded on an *order*, the latter of which originates with a particular customer.

Note that an order cannot be split between work assignments in the same zone; conversely, it is usual for several orders to appear on the same work assignment. However, batches in different zones are independent of one another. Furthermore, in each zone, a work assignment is restricted to a certain number of orders and lines owing to the configuration of the order picking vehicles.

4. Performance evaluation models

Travel-time models can be useful for comparing both alternative operating scenarios and warehouse designs. Such a model, derived by Bozer and White (1984), includes several P/D point locations and dwell point strategies, the latter of which involve dynamically positioning the S/R machine when it becomes idle after completing a cycle. Other factors that can be incorporated in travel-time models include, for instance, the acceleration and deceleration of the S/R machine, maximum velocity restrictions, and various travel speeds; see Hwang and Lee (1990) as well as Chang et al. (1995).

Seidmann (1988) developed a travel-time model for the situation in which the number of items to be picked is a random variable, while Elsayed and Unal (1989) obtained an expression to estimate the travel-time as a function of the number of locations to be visited and the physical configuration of the warehouse. Expressions for upper and lower bounds on travel time are also developed. Hwang and Ko (1988) derived travel-time expressions for multi-aisle AS/RS's, assuming that the S/R machine is transferred between adjacent aisles by a "traverser." They also investigate the problem of partitioning the aisles into a number of classes so as to minimize the required number of S/R machines subject to the throughput constraint, each class having a dedicated S/R machine. Kim and Seidmann (1990) show that previously published models are special cases of their own throughput rate expressions.

It is noteworthy that Riaz Khan (1984) appears to be the only author who devotes a paper to the efficiency measurement of warehouse personnel. The proposed model estimates the time required to complete a picking cycle, considering lead time, travel time and non-efficient time. Foley and Frazelle (1991) assume the time required for the picker to retrieve items from containers to be either deterministic or exponentially distributed. Their purpose is to determine the maximum throughput at which a miniload AS/RS can process requests, as a function of such parameters as rack dimensions, S/R machine speed, and so on. They also derive closed-form expressions for the probability distribution function of dual command travel time, the utilization of the picker, and the utilization of the S/R machine. Meanwhile, the expected value of S/R machine travel time for multi-command order picking was derived as a function of the number of addresses and rack area by Guenov and Raeside (1992). Kouvelis and Papanicolaou (1995) present travel-time formulas for a two-class-based storage and retrieval system, for both single and dual command cycles, and obtain the optimal boundary between the two storage areas. A review of travel-time models is provided by Sarker and Babu (1995).

A framework for a dual command cycle travel-time model under class-based storage assignment is described by Pan and Wang (1996). De Koster (1994) presents a method which is based on Jackson network modelling and analysis (Jackson, 1957) for estimating the throughput performance of pick-to-belt order picking systems. Malmborg and Al-Tassan (2000) formulate travel-time models for single and dual command transactions in less than unit load order picking systems. These are applied to predict the operating performance of a reorder point stock management system with respect to item retrieval throughput capacity, physical storage space requirements, inventory service level and system responsiveness. Throughput models for unit-load cross-docking were developed by Gue and Kang (2001). For that purpose, they introduce a new type of queue, called a *staging queue*, which is characterized partly by the fact that, as the server pulls a pallet from the queue, the remaining pallets do not automatically move forward. Other travel-time models were proposed by Chew and Tang (1999) and by Koh et al. (2002), the latter of which considered the crane to be located at the centre of a round storage area.

In many complex situations it is necessary to resort to simulation models in order to quantify performance measures. Assuming a single-aisle dual command AS/RS, Azadivar (1986) constructed a simulation model in order to evaluate system response under various operating policies. An optimization problem is solved which maximizes throughput while respecting upper bounds on maximum queue length and average waiting time, as well as the acceptable risks with which the constraints can be violated. In addition, since warehouses are an integral part of global supply chains, Mason et al. (2003) develop a discrete event simulation model of a multi-product supply chain to assess the total cost reductions that can be achieved through the increased global visibility provided by integrating transportation and warehouse management systems.

5. Throughput capacity models

By some estimates, order picking costs account for about 55% of the recurring costs of operating a warehouse (Tompkins et al., 2003). It is therefore hardly surprising that researchers and companies alike have devoted so much effort toward improving the efficiency of order picking operations, be it at the operational level, in particular routing and batching policies, or at the tactical level, namely, storage assignment policies. A review of these policies will be followed by a brief discussion of dynamic control of warehouses.

5.1 Order picking policies

We hereafter review order picking policies, which are further divided between routing policies, analogous to the TSP, and batching policies.

5.1.1 Routing policies. Kanet and Ramirez (1986) propose a mixed zero-one nonlinear programming formulation for the problem of selecting from alternate picking locations on single command tours so as to minimize a combination of breakdown cost along with fixed and variable picking costs. The variable cost is a function of travel time while the fixed cost depends on such things as pallet loading and unloading times. Whenever the batch of stock retrieved exceeds the quantity required, the excess quantity must either be returned to storage or applied to another requisition, thereby causing the so-called breakdown cost to be incurred.

Graves et al. (1977), Han et al. (1987), Eynan and Rosenblatt (1993) as well as Lee and Schaefer (1996) all analyzed dual command storage and retrieval systems (Han et al.'s paper is discussed in Section 5.3). Considering the operating characteristics of a man-on-board storage and retrieval system, Hwang and Song (1993) present a heuristic procedure for the problem of sequencing a given set of retrieval requests. They also develop expected travel-time models based on a probabilistic analysis for single and dual commands assuming a random storage assignment policy.

The following papers all assume a multi-command system. Given a Chebyshev rack, Bozer (1985) derived an analytical expression for the expected tour length of the band heuristic, as well as the optimal number of bands as a function of the number of picks. The band heuristic divides the rack into a number of horizontal bands, with picking performed following a serpentine path defined by those bands.

Bozer et al. (1986, 1990) compared a number of tour construction heuristics, enhanced by some tour improvement routines such as 2-*opt* and 3-*opt* exchanges. The best were found to be the half-band insertion heuristic and the convex hull heuristic, the former running about 60% faster than the latter. Moreover, the decrease in tour length does not seem to warrant the additional implementation complexity and increased computation time of the k-*opt* exchanges. The convex hull heuristic consists of two phases. First, the convex hull of all the points to be visited is determined. If all the points are on the boundary, the tour is optimal; otherwise, the remaining points are inserted individually so as to minimize additional travel time. The half-band insertion heuristic starts by crossing out the middle half area of the rack. Points in the remaining top quarter area are joined sequentially, followed by the same procedure for points in the remaining bottom quarter area. Finally, the points in the crossed-out middle half are inserted using the same procedure as the convex hull heuristic.

Assuming a warehouse consisting of a single block of parallel aisles with crossovers only at the ends of the aisles, Ratliff and Rosenthal (1983) developed a procedure for finding an optimal picking sequence requiring a computational effort linear in the number of aisles. The dynamic programming algorithm proposed is based on graph theory, with the nodes corresponding to the depot along with the top and bottom of every aisle while the arcs connect nodes that can be visited consecutively. This method was extended by De Koster and Van der Poort (1998) to the case where the vehicle can pickup and deposit loads at the head of every aisle. Roodbergen and De Koster (2001a,b) as well as Vaughan and Petersen (1999), show the benefits of using cross-aisles. The latter show that such benefits are a function of the length of the main aisle. Furthermore, the number of cross-aisles that should be provided depends upon the total number of aisles, the number of picks per aisle as well as the ratio between the main aisle length and the cross-aisle width. Indeed, too many cross-aisles can effectively increase tour lengths.

Goetschalckx and Ratliff (1988) show that the optimal traversal picking tour is obtained by finding the shortest path in an acyclic graph. A traversal tour is one in which the vehicle enters at one end of the aisle and exits at the other, picking from both sides simultaneously. They also discuss a procedure for finding the optimal z-pick tour, where each slot is picked in a fixed sequence that remains the same for all orders. Meanwhile, under the assumption of a dedicated storage policy, Hall (1993) compares several routing strategies, namely, the traversal, midpoint, largest gap, and double traversal strategies, on the basis of number of picks and the warehouse's geometry (aisle width and length). With the double traversal strategy the picker enters each aisle from both ends, picking from only one side each time. The midpoint and largest gap strategies, both variants of the return strategy in which each aisle is entered and exited from both ends, differ in the criteria used to determine at which point the picker turns around. That is, in the former, the picker simply turns around at the middle of each aisle, while in the latter, the picker turns around at the point where the gap between successive items is greatest.

For his part, Petersen (1997), assuming a random storage policy, analyzed various routing strategies as well as the effect of the warehouse dimensions and the location of the pickup and delivery point, while the impact of routing strategies and storage assignment policies on warehousing efficiency is reported in Petersen (1999). In addition, Caron et al. (1998) consider jointly storage assignment policies and routing policies, while Hwang et al. (2001) developed travel-time models for traversal and return travel policies, which were then compared with respect to various ABC curves, number of picks, and length to width ratios of the warehouse. Under the assumption that each product may be picked from alternative locations, Daniels et al. (1998) formulate a model for simultaneously assigning inventory to an order and routing. Assuming a given order sequence, Van den Berg (1996) presents an efficient dynamic programming algorithm for the problem of sequencing picks in a set of orders on a single carousel. He then considers the problem in which the orders are not sequenced and simplifies this problem to a rural postman problem on a circle, solving it to optimality. For their part, Bartholdi and Gue (2000) concentrate on labour costs in a cross-docking terminal, while Apte and Viswanathan (2000) give an overview of cross-docking and discuss its various managerial aspects.

Whereas the previous studies focused mainly on distance minimization, Cormier (1987) describes an order picking problem in which the objective is to minimize the total weighted tardiness incurred when items are not delivered to the pickup and delivery point before their respective due-dates. Lee and Kim (1995) consider the problem of scheduling storage and retrieval orders under dual command operations in a unit-load automated storage and retrieval system. The objective is to minimize the weighted sum of earliness and tardiness penalties about a common due-date. Note that some of the methods developed for generic vehicle routing problems may also be applied to warehouses, see for instance the chapter in this book entitled "New Heuristics for the Vehicle Routing Problem."

5.1.2 Batching policies. Bozer (1985) identifies the following batching alternatives: single-order picking, batch picking, and zone picking. Under single-order picking, different orders are never combined on the same trip of the order picking vehicle. By contrast, batch picking relaxes this restriction. In zone picking, each vehicle, or picker, operates within specific geographical boundaries of the warehouse (as in the warehouse in Section 3.2). Pick-to-pack systems are a type of zone picking where items are placed directly in the shipping container and the container is transferred between zones. Batch picking can result in savings over single-order picking whenever items on different orders can be processed together and are located in close proximity in the warehouse. In addition, recall from Section 3.2 the concept of waves, whose purpose is to smooth the workflow by essentially assigning orders to time windows so that only orders within the same wave can be batched together. Meller (1997) proposes an algorithm to assign orders to lanes based on the arrival sequence of items to the sortation system. Significant throughput increases are achieved, with throughput based on the time to sort a complete order pick-wave. Moreover, Gue (2001) seeks to determine the optimal timing of pick-waves to minimize average order cycle time. He also contends that his proposed "percent making cut-off" metric, which establishes a cut-off time for orders to be guaranteed shipping on the next delivery cycle, is better for warehouses using cyclical transportation providers. Almost all of the literature on order batching assumes that several orders are to be combined on the same tour of the order picking vehicle and that orders cannot be split between tours.

Hwang et al. (1988) describe a clustering algorithm for batching in a Chebyshev rack. Elsayed and Unal (1989) compared four different order batching heuristics, under the assumptions that the number of orders is normally distributed and that the number of different items in an order and the quantity of each item are uniformly distributed. Their best algorithm entails first classifying each order as large or small relative to a preset fraction of vehicle capacity. Large orders are then combined in pairs for which savings over single-order picking are computed. The pair yielding the largest savings is kept and the process is repeated until all large orders have been assigned to batches. Small orders are thereafter considered in the same fashion, starting with the one having the largest quantity.

Gibson and Sharp (1992) compared three order batching heuristics for different experimental factors, among others, the distance measure, the order size and the storage assignment policy. They found that the method which outperforms the others consists of starting a new batch with an arbitrary seed order, and then augmenting the batch with other orders by minimizing a certain distance measure, starting new batches as necessary to ensure that vehicle capacity is not exceeded. The key to the efficiency of this algorithm is obviously the accuracy of the distance measure, which is obtained without solving any TSP's. A comparative study of order batching algorithms was carried out by Pan and Liu (1995). De Koster et al. (1999) conducted further comparisons between order batching methods in combination with the routing method (traversal and largest gap) and the warehouse type. They recommend choosing the routing method before the batching method. In narrow-aisle warehouses with pallet racks, the best batching algorithm was found to be one based on Clarke and Wright savings.

By proposing a new distance measure between orders, called minimum additional aisle (MAA), used in the procedure for selecting the order which is added to the seed order, Rosenwein (1996) obtained better results than did Gibson and Sharp, both in terms of distance (33% reduction in number of aisles traversed) and number of tours (12% reduction), but at the expense of more computer time. Gademann et al. (2001) developed an optimal branch and bound method for order batching to minimize the maximum picking time in all zones in a zoned warehouse. Meanwhile, Jewkes et al. (2004), under the assumption of out-and-back routing, consider the concurrent problems of product location, picker home base location, and allocation of products to each picker so that the expected order cycle time is minimized.

Ruben and Jacobs (1999) studied jointly batching and storage assignment policies, the performance measures being person-hours, distance, as well as the utilization of workers and the capacity utilization of the order picking vehicles. They conclude that the "First Fit-Envelope Based Batching" (FF-EBB) method is the most effective, which models the batching problem as a bin-packing problem and uses the gap between the first and last aisles traversed by each order as the distance measure between them. For their part, Elsayed et al. (1993) tackled routing and batching problems in the presence of both earliness and tardiness penalties, for which they propose a priority index methodology. Sequencing and batching of storage and retrieval requests to minimize total tardiness was considered by Elsayed and Lee (1996).

By contrast to the foregoing, the size of individual orders in some situations exceeds the capacity of the vehicle, so that the methods for order batching studied in the literature do not apply. Racine (2000) hence tackled the batching problem arising in Section 3.1. Due to the fact that the savings resulting from combining orders are attributable mainly to the reduction in stopping and item identification times, the coupling model proposed uses as a measure of performance the number of items in common between each pair of orders. The resulting solution was compared with the existing single-order picking method and found to yield reductions of 3% in the number of tours, 15% to 20% in the distance and 6% to 10% in picking time. Somewhat smaller savings were also achieved by using the cross-aisle and by solving the TSP optimally.

5.2 Storage assignment policies

In the dedicated storage assignment case, many studies have been published for the purpose of minimizing average workload, beginning with Heskett (1963), who proposed the cube-per-order index (COI) rule. The COI of an item is defined as the ratio of its total required space to its turnover, while Heskett's algorithm locates the items with the lowest COI closest to the pickup and delivery point. Assuming that the travel independence condition holds, implying that the cost of moving all items is constant and proportional to the distance travelled, several studies have since proved the optimality of this rule under various conditions, e.g., Harmatuck (1976), for single command systems, Malmborg and Krishnakumar (1987), for dual command systems, and Malette and Francis (1972), for a single command multi-dock facility in which all items have the same probability mass function for selection of a dock. Malmborg (1995) extended the COI methodology to the case of zoned warehouses.

Assuming racks of equal sizes and the same space utilization for all methods, Hausman et al. (1976) demonstrated that turnover-based dedicated storage is significantly better than random storage, while Rosenblatt and Eynan (1989) report that a four-class (twelve-class) system provides 90% (99%) of the benefits of full turnover-based storage assignment. However, Goetschalckx (1983) shows that methods such as random storage can in fact increase space utilization. Thonemann and Brandeau (1998) demonstrate that both turnover-based and class-based storage assignment policies in a stochastic environment reduce expected storage and retrieval time compared with the random storage policy. Malmborg (1996) developed a method which can estimate space and retrieval efficiency of random and dedicated storage policies and notes that average retrieval costs may decline with the number of storage slots utilized. Using simulation, Linn and Wysk (1987) concluded that random storage is best for low space utilization, while turnover-based dedicated storage is better at very high space utilization. In the paper by Montulet et al. (1998), mixed integer programming models are presented for the problem of minimizing, over a fixed horizon, the peak load in single command dedicated storage systems.

Wilson (1977), Hodgson and Lowe (1982) along with Malmborg and Deutsch (1988) consider the problem of establishing jointly a dedicated storage policy and an inventory policy. For instance, Wilson's algorithm works by first setting all reorder quantities equal to the economic order quantity (EOQ) and allocating stock by the COI rule. A gradient search procedure is then used to generate a new reorder quantity vector, the COI is reapplied, and so on, until the variation in reorder quantities between successive iterations is very small. Furthermore, Hodgson and Lowe extend the COI rule to the case where the travel independence condition does not hold.

Situations in which items are picked together on multi-command tours were studied by Jarvis and McDowell (1991) as well as Rosenwein (1994), the latter of whom applied clustering analysis. Van Oudheusden et al. (1988), and Van Oudheusden and Zhu (1992), tackled the case in which orders recur according to a known probability. Moon-Kyu (1992) developed a heuristic for the storage assignment problem based on group technology considering both order structure and frequency. Space requirements along with storage location assignment were modelled jointly by Kim (1993), while a class-based storage assignment policy for a carousel system was developed by Ha and Hwang (1994).

Shared storage policies are the subject of studies by Goetschalckx and Ratliff (1990) and Montulet et al. (1997), who show that they yield reductions in both space and travel time compared to dedicated storage. Given the potential of these policies, we outline a heuristic proposed by Montulet et al. (1997) which generally outperforms that by Goetschalckx and Ratliff (1990). In fact, comparisons between the former and the optimal solutions, obtained by means of an exact formulation solved by column generation, reveal that Montulet et al. (1997)'s heuristic systematically finds the optimal solution. It is assumed that all items are identical from the point of view of the storage system and that single command travel is employed. Let $G = (X, A)$ where $X = \{1, 2, \ldots, T\}$ is the set of nodes which correspond to the dates over the planning horizon. Each unit of product to be stored has associated with it an arc in A, called *item arc*, which connects the arrival date node to the departure date node. Arcs connecting each consecutive dates constitute the remaining members of A and have zero weight. Let the weight of each item arc equal $K + DS_{item}$, where K is a constant greater than T and DS_{item} is the duration of stay of a particular item. The algorithm's pseudocode is as follows:

Repeat while A contains item arcs:

- Obtain the longest path (without cycles) from 1 to T.
- Assign the items corresponding to the arcs on this path to the most accessible remaining available locations.
- Delete from A the item arcs along this path.

Finally, the allocation of items to an AS/RS was studied by Hackman and Rosenblatt (1990), the tradeoff to be optimized being the cost of replenishing the items assigned to the AS/RS from their other warehouse locations versus the savings per retrieval request if an item is stored in the AS/RS. A heuristic algorithm is developed based on the relationship between this problem and the knapsack problem.

5.3 Dynamic control of warehouses

This section presents methods for coping with operating environments that vary over time. In order to meet short-term throughput requirements of a fluctuating demand pattern, Jaikumar and Solomon (1990) examine the relocation of pallets which have a high expectancy of retrieval in a future time period closer to the pickup and delivery point. Likewise, Muralidharan et al. (1995) present a shuffling heuristic where random storage is employed for the initial storage assignment, but when the crane is idle, the more frequently accessed product is shifted nearer to the pickup and delivery point while the less frequently accessed product is shifted farther. Along the same lines, Sadik et al. (1996) describe a heuristic for the dynamic reconfiguration of the order picking system.

Linn and Wysk (1990) developed a prototype expert system in which the hierarchical control structure consists of strategic, tactical and process control levels. A multi-pass simulation technique is employed to adapt control policies to real-time system behaviour. Seidman (1988) also used such an artificial intelligence approach, while Knapp and Wang (1992), Lin and Wang (1995), and Hsieh et al. (1998) propose Petri nets for AS/RS operation modelling.

Han et al. (1987) propose the nearest neighbour heuristic for routing in a dual command warehouse, whereby each storage location is matched with the closest available retrieval location. Since the list of retrievals changes over time, a block of retrievals is selected, these retrievals are sequenced following which another block of retrievals is considered. The expected throughput of the nearest neighbour heuristic is shown to be within 8% of the upper bound on throughput for any block sequencing rule. Eben-Chaime (1992) alternatively proposes the dynamic application of the same rule with a resulting increase in performance. Kim et al. (2002) study an order picking problem where the pick location of goods can be selected in near real time, to which they apply an intelligent agent-based model. Egbelu (1991), Egbulu and Wu (1993) along with Hwang and Lim (1993) focused their attention on dwell-point strategies.

Bartholdi et al. (2001) introduce the concept of bucket brigades, which are a way of sharing work on a flow line that results in the spontaneous emergence of balance and consequent high throughput while requiring neither traditional assembly line balancing technology nor any central planning. They report a 35% increase in productivity at the national distribution center of a major chain retailer after the workers began picking orders by bucket brigade (see also Bartholdi and Hackman, 2003).

6. Storage capacity models

The following papers assume demand for space to be given, so that lot sizing is not incorporated in the modelling framework, which is not unreasonable in those instances where the purchasing department functions independently of the warehouse. In fact, using a discounted inventory cost approximation and a linear warehousing cost model, Cormier and Gunn (1996a) showed that such a sequential policy is near-optimal if the products are characterized by a very high purchasing cost relative to the marginal cost of the storage space (expensive jewellery is a good example of this). Goh et al. (2001) extended this framework to allow for a step function of the warehouse space to be acquired. They consider the cases of a single item, multi-items with separable costs together with multi-items with joint inventory replenishment costs.

White and Francis (1971), and Lowe et al. (1979) describe network flow formulations for some single-location, multi-period warehouse leasing problems, with demand specified by a probability mass function. The problem of determining the capacity of a single production facility along with the amount of warehouse space to lease in each of several regions is studied by Jucker et al. (1982).

Integrated models such as that of Cokelez and Burns (1989) are useful for coordinating various inter-related decisions. They present a mixed-integer linear programming model incorporating product mix, transportation, warehouse location and warehouse capacity. Also of considerable interest in a multi-item context, provided that a shared storage policy is employed, is the coordination of inventory cycles, with objective functions such as minimizing maximum storage, e.g., Zoller (1977), and minimizing the sum of ordering and holding costs, plus a cost determined by peak inventory levels (Hall, 1988; Rosenblatt and Rothblum, 1990; Anily, 1991).

Some situations warrant establishing capacity in view of achieving a target service level rather than minimizing costs. For instance, Rosenblatt and Roll (1988) used simulation in order to generate the cumulative distribution of the number of days requiring a certain capacity. Meanwhile, algorithms based on queueing theory proposed by Sung and Han (1992) yield the minimum number of storage spaces for which the blocking probability does not exceed a specified threshold.

If the items to be stored are fairly voluminous and inexpensive, e.g., sheets of polystyrene thermal insulation, then it is more appropriate to optimize inventory costs and space costs simultaneously. Assuming stationary demand, Herron and Hawley (1969) present analytical and graphical procedures for such a situation, while Levy (1974) restricts his attention to a single expansion under non-stationary demand. For the case of arbitrarily increasing product demand, Rao (1976) minimizes the sum of discounted production (comparable to product procurement), carrying, investment, and idle capacity costs, all of them concave. He proposes a discrete-time dynamic programming algorithm which is equivalent to finding the shortest path in an acyclic network.

Cormier and Gunn (1999) tackled a warehouse sizing problem in which product demands vary arbitrarily over a finite planning horizon and the expansion cost consists of fixed and variable components. The state variable and the stages in their proposed dynamic programming formulation correspond to the warehouse size and time periods, respectively. Moreover, under constant product demand, Cormier and Gunn (1996b) demonstrated through both analytical and numerical means that it can be worthwhile to lease space temporarily at the beginning of each inventory cycle. Qualitative research has also been done on the subject of outsourcing logistics services, including warehouses; see for instance Maltz (1992, 1994). The paper by Chen et al. (2001) considers multi-period warehousing contracts under random space demand characterized by a starting space commitment plus a certain number of times at which the commitment can be modified. Furthermore, Colson and Dorigo (2003) developed a database which allows the user to make a multiple criteria selection of a subset of public warehouses fitting as well as possible his or her needs and preferences.

Another subject related to storage capacity is that of the utilization of storage space. The unitization problem has been investigated by Steudal (1979), his objective being to partition a pallet into smaller identical rectangular areas so as to minimize the amount of unused pallet area. This is a special case of the two-dimensional cutting stock problem which allows non-guillotine cuts. Tsai et al. (1988) address the use of linear programming to determine an optimal solution to a similar problem, except that they allow a wide product mix of different box sizes to be loaded on the same pallet. Balasubramanian (1992) provides a survey of research relating to models and solution procedures used in pallet packing, while Dowsland and Herbert (1996) propose two new crossover operators used with genetic algorithms for the same purpose.

Carrying out a case study at a large distribution centre, Carlson and Yao (1996) develop decision rules that reveal that storage capacity could increase by at least 4% through optimum pallet stacking and a further 5–7% by standardizing the wooden pallets themselves. Additionally, Abdou and El-Masry (2000) devised a heuristic for random stacking, which entails loading boxes in undefined patterns incorporating load stability and box demand requirements. The container loading problem is formulated by Chen et al. (1996) as a zero-one mixed integer programming model, with consideration of multiple containers, multiple carton sizes, carton orientations, and the overlapping of cartons in a container. See also Scheithauer (1996), and Morabito and Morales (1998).

Block stacking is used for storing large quantities of palletized or boxed products on top of each other in stacks, without racks. Usually, forklift trucks are used to manipulate the pallets one at a time. A storage lane remains unavailable for arriving pallets until its current content has been totally depleted by demand, thereby creating the need to optimize storage lane depth. Marsh (1979) developed a simulation model in order to investigate the effect of alternate lane depths on space utilization, using statistical analysis to determine if they significantly influence performance measures such as primary storage area and lineal aisle frontage. Goetschalckx and Ratliff (1991) describe a dynamic programming algorithm for a single product and integer multiple lane depths, the states and stages of which correspond to the length and number of storage lanes, respectively. Heuristics are described for the case where lane depths are restricted to a finite set due to practical implementation considerations.

7. Warehouse design models

Warehouse design models attempt to optimize such things as the orientation of storage racks, the allocation of space among competing uses, the number of cranes and the overall configuration of the facility. Let us first consider papers that deal with the tactical design questions arising inside warehouses. Bozer (1985) develops performance models for in-the-aisle picking versus end-of-aisle picking, with the objective of minimizing cost subject to throughput and storage space constraints. He also analyzes the tradeoff, in terms of the increase in picking time versus the decrease in replenishment frequency (from reserve area to picking area) as the picking area is increased. With demand described as a Poisson process and an objective function comprising storage cost, runout cost (the cost of a stockout in the active pick zone), and picking cost, Bhaskaran and Malmborg (1990) as well tackled the question of relative sizing between the reserve storage area and the active pick area. Van den Berg et al. (1998) consider a problem where the objective is to determine which replenishments minimize the expected amount of labour during the picking period, the decision taking place prior to the picking period. For their part, Bozer and White (1990) analyzed end-of-aisle picking.

Bassan et al. (1980) compared two alternative shelf arrangements, considering material handling cost, annual cost per unit of storage area, and annual cost per unit length of external walls. The analysis yields the optimum number of storage spaces along a shelf, number of shelves, location of doors, as well as warehouse dimensions. Recognizing the reality that decision makers must often allocate scarce resources, Pliskin and Dori (1982) proposed a method for ranking alternative area assignments subject to the amount of space available. Likewise, Azadivar (1989) looks at allocating scarce floor space between a random access area and a rack storage area. Larson et al. (1997) outline a procedure for warehouse layout consisting of determination of aisle layout and storage zone dimensions, assignment of material to a storage medium, and allocation of floor space.

Now, turning our attention to overall warehouse design, an optimization model for the design of a dual command AS/RS was proposed by Ashayeri et al. (1985). The relative proportions of the various objective function terms and the convexity of the objective function in the number of aisles allow a one-dimensional sequential search over the number of aisles to yield the optimal solution. Park and Webster (1989a) compared alternative warehousing systems through exhaustive enumeration on the basis of the following factors: control procedure, handling equipment movement, storage assignment rule, input and output patterns for product flow, storage rack structure, and the economics of each storage system. In addition, Gray et al. (1992) propose a multi-stage hierarchical decision approach to solve a design model which encompasses warehouse layout, equipment and technology selection, item location, zoning, picker routing, pick list generation and order batching.

Park and Webster (1989b) extended the concept of square-in-time to that of cubic-in-time and subsequently presented an algorithm to design pallet rack storage systems based upon equipment characteristics. Malmborg (1994) proposes an analytical model which incorporates the tradeoffs between handling and storage requirements to support development of layout alternatives. The operating dynamics of factors such as production scheduling and part routing as well as handling and storage parameters are all captured and the author claims that his model provides a higher degree of modelling ease than either simulation or stochastic Petri nets. Yoon and Sharp (1996) characterized the general structure of order picking systems and proposed a design procedure consisting of an input stage, a selection stage and an evaluation stage. Bartholdi and Gue (2004) address the question of the best shape for a cross-docking warehouse and make recommendations based on the size (number of doors) of the facility.

Also quite common in evaluating warehouse designs is the use of computer simulation, given the complexity and stochastic nature of such systems. An hybrid method was developed by Rosenblatt et al. (1993), in which output values are passed back and forth between an analytical optimization model and a simulation model until target values of the performance measures are attained. Taboun and Bhole (1993) developed a simulation model of an AS/RS which they then use to study the effect of four different warehouse configurations, large and small holding pallets, and four sizes of stored items on system performance.

A simulation model developed by Randhawa and Shroff (1995) allows for the evaluation of the effect of layout arrangements and scheduling policies on the performance of unit-load automated storage and retrieval systems. Finally, Eben-Chaime and Pliskin (1997) investigate an integrative model of a warehouse, containing several S/R machines and considering performance measures such as response times, queue lengths, and utilization of the S/R machines. A simulation of the model demonstrates that economic gains are possible as a result of decreasing the number of S/R machines and reducing building space as a consequence of shorter queues.

8. Concluding remarks

Let us now recapitulate some of the observations made in this chapter and identify research gaps that are ripe for future study. While computer simulation can provide valuable insights in comparing alternative designs and operating scenarios, time and budget constraints often preclude its use, particularly in small and medium sized companies. A warehouse simulation package with some standard components would thus be most useful as a rapid modelling tool.

We learned from the case of the grocery warehouse in Section 3.1 that warehouse management systems (WMS) do not always optimize certain elements of the order picking process. Furthermore, as remarked by Barnes (1999), "very seldom does the base price of a WMS comprise the majority of the total system cost." The bottom line is that recurring operating costs should be taken into account when designing a warehouse or selecting a WMS. As well, research has revealed that order batching can reduce order picking costs, but this has to be traded off against the equipment and operating costs of sortation. It thus appears that further research on batching methods is justified.

As for the warehouse discussed in Section 3.2, not a lot of research has been done on zoned warehouses, especially establishing the loading sequence of delivery trucks, determining the number of waves, assigning orders to waves, determining which part of each order to pick in each zone, and assigning pickers to zones. Literature is also quite scant on the dynamic control of warehouses, and tackling this subject together with order picking with due-dates is all the more important in the presence of a just-in-time requirement.

Let us also stress that there is still a lot of room left for studying storage capacity problems, for instance, under the assumption of various underlying inventory policies. And, while cross-docking and order picking by bucket brigades would also benefit from further research, these newer concepts actually remind us of something more fundamental: Think out of the box!

We leave you with two additional references which we believe open up another realm of possibilities for warehousing research. Recognizing the fact that on-line resources are becoming more prevalent, the first is the *ERASMUS Logistica Warehouse Website*, http://www.fbk.eur.nl/OZ/LOGISTICA/, which is very rich in information and moreover contains links to all other important web sites dealing with warehousing research that we have found. The second is a paper by Brockmann (1999), who gives his opinion on which warehousing innovations seem to be the most promising for the future. These are listed below along with our suggestions on how operations research can help.

***Focusing on the customer*:** This dictates, among other things, developing relevant performance measures.

***Consolidation*:** Achieving optimal economies-of-scale is a natural byproduct of certain storage capacity planning models.

***Continuous flow of material and information*:** This provides further justification for the cross-docking concept.

***Information technology*:** Information technology will become more prevalent in warehousing, justified in part by embedded operations research methods that will reduce operating costs.

***Space compression*:** Proliferation of products results in greater space requirements as well as the need for judicious space allocation. Thus, we foresee a need for joint planning of storage capacity and storage assignment policies.

***Time compression*:** This means installing systems and methods that will allow time-based performance measures to be optimized.

***Distribution requirements planning*:** It is imperative that distribution plans adapt to changing customer orders, thereby justifying further research into the dynamic control of warehouses.

***Reverse logistics*:** Returned product has given rise to this field, for which allocating space and controlling labour costs is becoming more and more of an issue.

***Global supply chain optimization*:** This concept, which can only be envisaged through sharing of information among partners, is making it possible for suppliers and their customers to jointly optimize their operations.

***Third-party warehousing*:** Further research into optimal leasing arrangements is suggested.

***Automation*:** The increased availability of various levels of automation gives rise to an important decision problem: determining its optimal level. This requires analyzing the tradeoffs between, among other things, throughput, labour costs, equipment costs and flexibility. Further complicating this type of study is the fact that many of the pertinent factors are not readily quantifiable.

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