Chapter 11

SYNCHRONIZED PRODUCTION-DISTRIBUTION PLANNING IN THE PULP AND PAPER INDUSTRY

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Abstract This chapter examines the short term production, transportation and inventory planning problems encountered in the fine-paper industry. After positioning the problems in the context of a general supply chain planning system for the pulp and paper industry, a comprehensive synchronized production-distribution model is gradually developed. First, a model for the dynamic lot-sizing of intermediate products on a single paper machine with a predetermined production cycle is proposed. The model also plans the production and inventory of finished products. Then, we consider the lot-sizing of intermediate products on multiple parallel paper machines with a predetermined production sequence. Finally, simultaneous production and distribution planning for a single mill multiple distribution centers network is studied by considering different transportation modes between the mill and its Distribution Centers (DCs).

1. Introduction

The pulp and paper industry is one of the most important industries of Canada in terms of contribution to its balance of trade. In 2001, it represented 3% of Canada's Gross Domestic Product (FPAC, 2002). The expertise of the Canadian pulp and paper industry is well renowned. Over the years, the industry has been confronted with different market pressures. For example, currently, global production capacity is abundant due to major consolidations in the sector. Companies are working closely to integrate the different business units of their supply chain due to this consolidation. They are reengineering their supply chain, which means they are trying to define the optimal network structure and planning approach in order to maximize profit.

1.1 The pulp and paper supply chain

Total shipments within the industry supply chain in 2002 included pulp (10.5 million tons), newsprint (8.5 million tons), printing and writing paper (6.3 million tons) as well as other paper and paperboard (5.2 million tons). These products are produced and distributed in complex supply chains composed of harvesting, transformation, production, conversion and distribution units, as shown in Figure 11.1. The main components of the pulp and paper supply chains are their supply network, their manufacturing network, their distribution network and the product-markets targeted. Different companies in the world are structured in different ways. Some are vertically integrated: they possess and control all the facilities involved in this value creation chain, from woodlands to markets. Others are not integrated and they rely on outsourcing to fulfill part of their commitments to their customers. For example, some companies buy pulp on the market, produce the paper and convert it through a network of external converters, before distributing the final products. All these possibilities are illustrated in Figure 11.1. The links between the external network and the internal network define these outsourcing alternatives.

An important problem is therefore to determine the supply chain structure and capacity, to decide how and where intermediate and finished products should be manufactured and how they should be distributed. These decisions relate to the company's business model as well as to its strategic supply chain design. In the pulp and paper industry, these decisions are tightly linked to the availability of fiber and the supply of raw materials. For example, Canadian paper is made from 55% chips and sawmill residues, 20% recovered paper and 25% round wood. The quality of the paper produced depends directly on the quality of the fiber used. Therefore, designing the supply chain imposes a thorough analysis of the supply network. Also, the industry is very capital intensive. Even the modification of a single paper machine is a long-term investment project. A planning horizon of at least five years must be considered to evaluate such projects. The final output of this strategic decision process defines the supply chain network structure, that is its internal and external business units (woodlands, mills warehouses, etc.), their location, their capacity, their technology as well as the transporta-

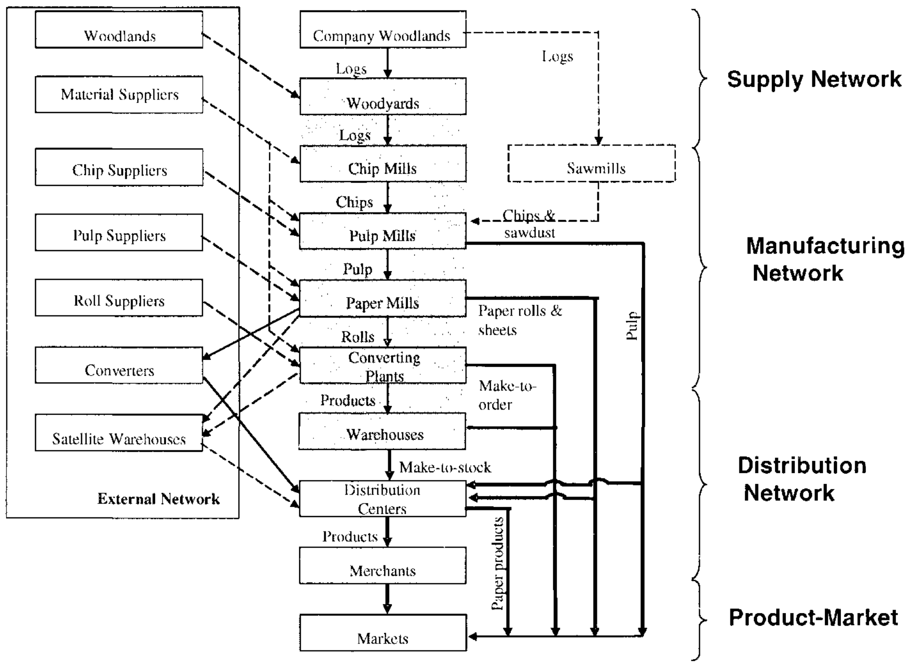

Figure 11.1. The pulp and paper supply chain

tion modes to favor. Large scale mixed-integer programming models can often be formulated to support this complex design process. In order to take the uncertainty of the future business environment into account, these models must be used in conjunction with a scenario planning approach. The application of such a modeling approach to capital budgeting problems at Fletcher Challenge Canada and Australasia are documented respectively in Everett, Philpott and Cook (2000) and in Everett, Aoude and Philpott (2001).

Once the structure of the supply chain is decided, managers need to plan supply, production and distribution over a rolling horizon. Usually this process is conducted in two phases: tactical planning and operational planning. Tactical planning deals with resource allocation problems and it defines some of the rules-of-the-game to be used at the operational planning level. The tactical plans elaborated usually cover a one year horizon divided into enough planning periods to properly reflect seasonal effects. The rules-of-the-game relate to supply, production, distribution and transportation policies such as: customer service levels, safety stocks, the assignment of customers to warehouses or to mills, the selection of external converters, the size of parent rolls to manufacture, the assignment of paper grades to paper machines and the determination of their production sequence, sourcing decisions for the mills, etc. These tactical decisions frame the operational planning decisions by identifying operational targets and constraints. They are made to convey an integrated view of the supply chain without having to plan all activities for all business units within a central planning engine. Again, mathematical programming models can often be used to support tactical planning decisions. Philpott and Everett (2001) present the development of such a model for Fletcher Challenge Paper Australasia.

At the operational planning level, managers are really tackling material, resource and activity synchronization problems. They have to prepare short-term supply, production and distribution plans. Usually at this planning level, information is no longer aggregated and the planning horizon considered covers a few months divided into daily planning periods. The plans obtained are usually sufficiently detailed to be converted into real-time execution instructions without great difficulty. The procurement, lot-sizing, scheduling and shipping plans made at this level are based on trade-offs between set-up costs, production and trim loss costs, inventory holding costs and transportation economy of scales, and they take into consideration production and delivery lead times, capacity, etc. The objective pursued at the operational planning level is usually to minimize operating costs while meeting targeted service levels and resource availability constraints. Mathematical programming models can often be used to support operational planning decisions. Everett and Philpott (2002) describe a mixed integer programming model for scheduling mechanical pulp production with uncertain electricity prices. Bredström et al. (2003) present an operational planning model for a network of pulp mills. Keskinocak et al. (2002) propose a production scheduling system for make-to-order paper companies. An integrated diagram of the system of strategic, tactical and operational planning decisions required to manage the pulp and paper supply chain is provided in Figure 11.2.

In an integrated pulp and paper plant, the production process can be decomposed in four main stages. The first stage (the chip mill) transforms logs into chips. The second stage (the pulp mill) transforms chips and chemicals into pulp. The third stage (the paper mill) transforms pulp into paper rolls. The paper mill is usually composed of a set of parallel paper machines. Finally, the last stage (convention mill) converts paper rolls into the smaller rolls or sheets which are demanded by external customers. Figure 11.3 illustrates the material flow within an integrated pulp and paper mill. As can be noted, some production stages can be partially or completely bypassed through external provi-

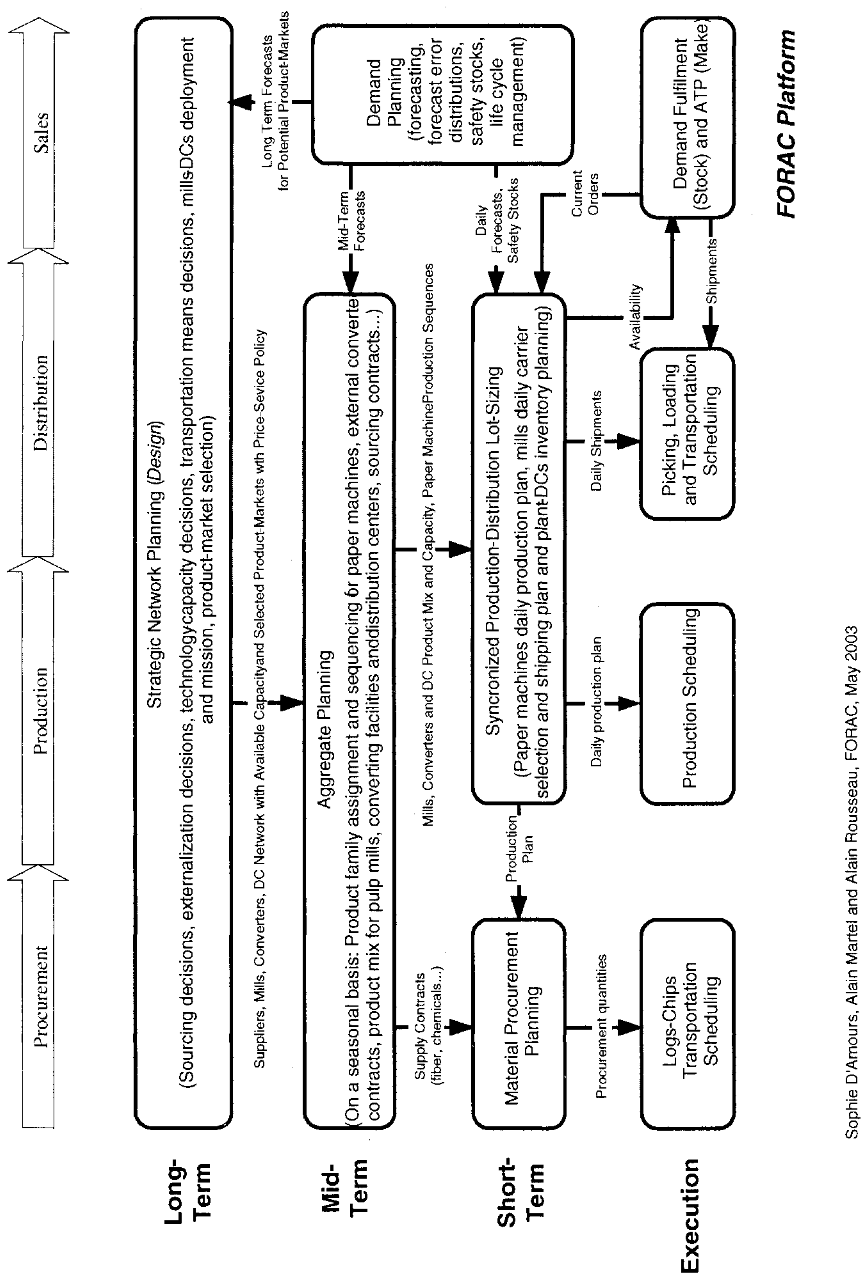

Figure 11.2. Supply chain decision processes in the pulp and paper industry

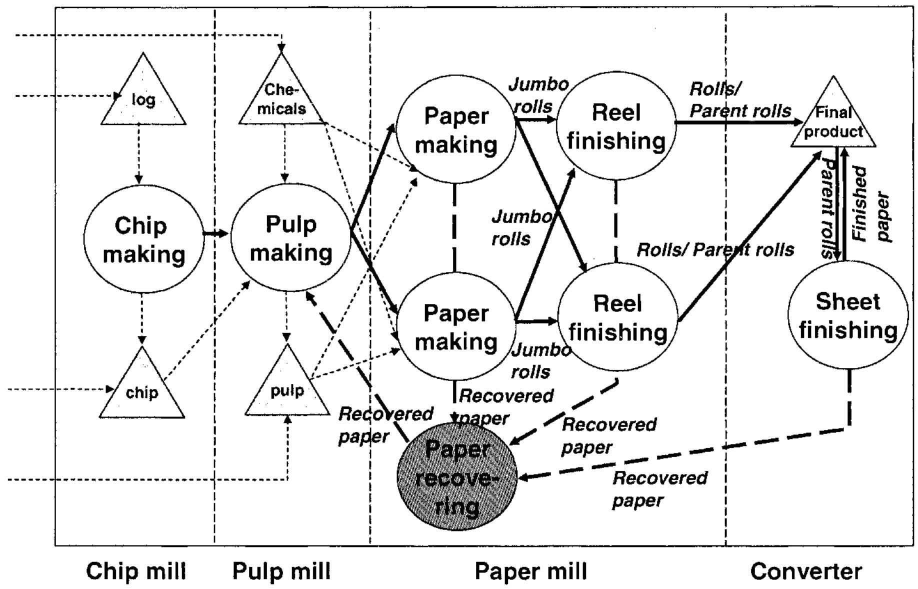

Figure 11.3. Processes and material flows in an integrated pulp and paper mill

sioning of intermediate products (chips and/or pulp). Also, although, some paper is lost during the paper making, reel finishing and sheet finishing operations, it is recovered and fed back into the pulp production process.

The planning challenge is to synchronize the material flow as it moves through the different production stages, to meet customer demand and to minimize operations costs. The paper machine is often the bottleneck of this production system and this is why production plans are usually defined in terms of this bottleneck.

The problem we focus on in this chapter is the synchronized production-distribution planning problem for a single mill and the set of distribution centers it replenishes for a make to stock and to order paper company. It is addressed gradually, starting by current industrial practices where production and distribution are planned independently and moving toward the integration of production and distribution decisions. Under the first paradigm two business contexts have attracted our attention. The first one refers to production planning on a single machine constrained by a production cycle, within which all different products are produced in a pre-defined sequence. It is referred to as the *Single-Machine Lot-Sizing Model.* The second context considered relates to production planning on several parallel machines each constrained by a pre-defined production sequence. It is referred to as the

Multiple-Machine Lot-Sizing Model. Finally, distribution considerations are introduced in the last part of this paper and the problem is set in its complete form as the *Synchronized Production-Distribution Planning Model.* Harvesting decisions and pulp making planning decisions are not taken into consideration in this chapter as they are in Bredström et al. (2001). Their work, however, fits within the planning paradigm presented in this chapter.

1.2 Production and distribution planning problems

In what follows, we concentrate our attention on the short term production and distribution planning problems encountered in the fine-paper industry. The specific context considered is illustrated in Figure 11.4 (see Tables 11.1 and 11.2 for notations). In this industry, some products are made-to-stock, others are made-to-order and others are shipped to external converting plants. Although the demand for products is partly planned and partly random, we assume, as is customary in ERP and APS systems, that it is deterministic and time-varying (dynamic). This demand is based on orders received and on forecasts, and we assume that the safety stocks required as protection against the randomness of demand are determined exogenously, prior to the solution of our problem. In order to provide a competitive service level, the make-to-stock products must be stored in distribution centers (DCs) which are close to the market. Part of the company demand is therefore fulfilled from these DCs. Make-to-order demand, converter demand and local make-to-stock demand is however fulfilled directly from the mills.

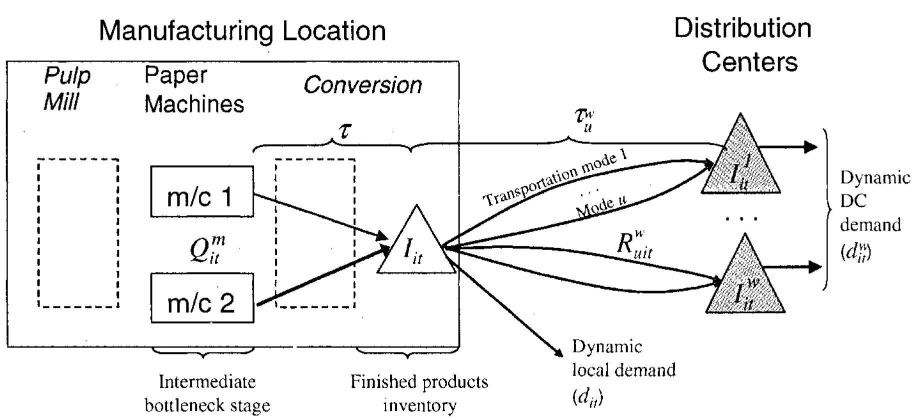

Figure 11.4. Paper industry production and distribution context

As indicated before, production at the mills involves multiple stages, with one of them, the paper machines, creating a bottleneck. Paper machines can run 24 hours a day during the whole year, but they can also be stopped (or slowed down) from time to time to adapt to low market demand or for maintenance purposes. In the bottleneck stage, a small number of *intermediate products* (IP) are manufactured by parallel paper machines, each machine producing a predetermined set of intermediate products with a *fixed* production sequence. A changeover time is required to change products on a paper machine, which means that capacity is lost when there is a production switch. In the succeeding stages, the intermediate products are transformed into a large number of finished products (FP). However, in the paper industry, any given finished product is made from a single intermediate product (divergent Bill-of-Material). The conversion operations can be done within a predetermined planned lead-time. We assume that no inventory of intermediate products is kept, but the finished products can be stocked at the plant before they are shipped.

Several transportation modes (mainly truck, train and intermodal), can be used to ship products from the plants to the warehouses. The transit time for a given origin-destination depends on the transportation mode used. For each mode, there are economies of scale in transportation costs, depending on the total loads shipped during a time period, independently of the type of finished products in the shipments.

Planning is done on a rolling horizon basis, with daily time buckets. Within this context, three different problems are examined in the next sections of the chapter:

(1) Single machine lot-sizing of intermediate and finished products with a predetermined IP production cycle.

(2) Single-mill multiple-machine lot-sizing of intermediate and finished products with a predetermined IP production sequence.

(3) Synchronized production-distribution planning for a single-mill multiple-DC subnetwork, with a predetermined IP production sequence.

The general notation used in the chapter is introduced in Tables 11.1 and 11.2. Additional notation specific to the three problems studied is defined in their respective sections.

1.3 Literature review

The three problems studied in the chapter relate to the multi-item capacitated dynamic lot-sizing literature. A recent survey of the lot-sizing literature covering these problems is found in Rizk and Martel (2001). Under the assumptions that there is a single production stage, that set-up costs and times are sequence independent and that capacity is constrained by a single resource, three formulations of the problem have been studied extensively: the *Capacitated Lot-Sizing Problem* (CLSP), the *Continuous Setup Lot-Sizing Problem* (CSLP) and the *Discrete Lot-Sizing and Scheduling Problem* (DLSP). The CLSP involves

Table 11.1. Indices, parameters and sets

T	Number of planning periods in the planning horizon.
t	A planning period ($t = 1, \ldots, T$).
IP	Set of intermediate products ($\{1, \ldots, N\}$).
FP	Set of finished products ($\{n+1, \ldots, N\}$).
i, i'	Product type indexes ($i, i' \in \mathrm{IP} \cup \mathrm{FP}$).
m	A paper machine for the production of IP ($m = 1, \ldots, M$).
M_i	The set of machines m that manufacture product i ($i \in \mathrm{IP}$).
IP_m	The set of intermediate products manufactured by machine $m(\mathrm{IP}_m \subset \mathrm{IP})$.
f_m	Number of intermediate products manufactured on machine m (i.e., $\lvert\mathrm{IP}_m\rvert$).
W	Set of distribution centers, $w \in W$.
U^w	Set of transportation modes available to ship products to DC w ($u \in U^w$).
τ	Planned production lead-time.
τ_u^w	Supply lead-time of DC w when transportation mode u is used.
d_{it}^w	Effective external demand for product i at DC w during period t.
d_{it}	Effective demand at the mill for product i during period t.
C_t^m	Production capacity of machine m in period t (in time units).
k_{it}^m	Changeover time required at the beginning of period t to produce $i \in \mathrm{IP}_m$ on machine m.
K_{it}^m	Product i changeover cost on machine m in period t ($i \in \mathrm{IP}_m$).
r_i	Transportation resource absorption rate for product i (in tons).
h_{it}^w	Inventory holding cost of product i at DC w in period t.
h_{it}	Inventory holding cost of product i at the mill in period t.
$g_{ii'}$	Number of product i units required to produce one unit of product i'.
a_{it}^m	Machine m capacity consumption rate of product $i \in \mathrm{IP}_m$ in period t.
SC_i	Set of finished products manufactured with intermediate product i ($\mathrm{SC}_i = \{i' \mid g_{ii'} > 0\}$).

Table 11.2. Decision variables

R_{it}	Quantity of finished product $i \in \mathrm{FP}$ added to the mill inventory for the beginning of period t.
Q_{it}^m	Quantity of intermediate product $i \in \mathrm{IP}$ produced with machine m during period t.
I_{it}	Inventory level of finished product $i \in \mathrm{FP}$ on hand in the mill at the end of period t.
I_{it}^w	Inventory level of finished product $i \in \mathrm{FP}$ on hand at DC w at the end of period t.
R_{uit}^w	Quantity of item i shipped by transportation mode u from the mill to DC w at the beginning of period t.

the elaboration of a production schedule for multiple items on a single machine over a planning horizon, in order to minimize total set-up, production and inventory costs. The main differences between the CLSP and the CSLP are that in the latter, at most one product is produced in a period and a changeover cost is incurred only in the periods where the production of a new item starts. In the CLSP, several products can be produced in each period and, for a given product, a set-up is necessary in each period that production takes place. For this reason, CLSP is considered as a large time bucket model and CSLP as a *small* time bucket model. DLSP is similar to CSLP in that it also assumes at most one item to be produced per period. The difference is that in DLSP, the quantity produced in each period is either zero or the full production capacity.

The first problem studied in this chapter can be considered as an extension of the CLSP to the case where the items manufactured include both intermediate products and finished products made from the IP products. When a predetermined fixed production cycle is used, production planning on a single paper machine reduces to such a problem. The length of the IP production cycle to use can be determined by first solving an *Economic Lot-Sizing and Scheduling Problem* (Elmaghraby, 1978; Boctor, 1985). Florian et al. (1980) and Bitran and Yanasse (1982) showed that CLSP is NP-hard even when there is a single product and Trigeiro et al. (1989) proved that when set-up times are considered, even finding a feasible solution is NP-hard. Exact *mixed integer programming* solution procedures to solve different versions of the problem were proposed by Barany et al. (1984), Gelders et al. (1986), Eppen and Martin (1987), Leung et al. (1989) and Diaby et al. (1992). Heuristic methods based on mathematical programming were proposed by Thizy and Wassenhove (1985), Trigeiro et al. (1989), Lasdon and Terjung (1971) and Solomon et al. (1993). Specialized heuristics were also proposed by Eisenhut (1975), Lambrecht and Vanderveken (1979), Dixon and Silver (1981), Dogramaci et al. (1981), Gunther (1987), and Maes and Van Wassenhove (1988).

When set-up costs are sequence dependent, the sequencing and lot-sizing problems must be considered simultaneously and the problem is more complex. This problem is known as *lot sizing and scheduling with sequence dependent set-up* and it has been studied by only a few authors (Haase, 1996; Haase and Kimms, 1996). Particular cases of the problem were also examined by Dilts and Ramsing (1989) and by Dobson (1992).

The second problem studied in this chapter can be considered as an extension of the CSLP to the case of several parallel machines with a predetermined production sequence, and with a two level (IP and FP) product structure. The multi-item CSLP has been studied by Karmarkar and Scharge (1985) who presented a Branch and Bound procedure based on Lagrangean relaxation to solve it. An extension to the basic CSLP that considers parallel machines was studied by De Matta and Guignard (1989) who proposed a heuristic solution method based on Lagrangean relaxation. The DLSP, which is also related to our second problem, has been studied mainly by Solomon (1991).

The third problem studied in this chapter is an extension of the second one involving the simultaneous planning of the production and distribution of several products. Coordinating flows in a one-origin multi-destination network has attracted the attention of some researchers (see Sarmiento and Nagi (1999), for a partial review). Most of the work done involves a distributor and its retailers and it considers a single product. Anily (1994), Gallego and Simchi-Levi (1990), Anily and Federgruen (1990, 1993), and Herer and Roundy (1997) tackle this problem in the case of a single product and deterministic static demand. In these papers, transportation costs are made up of a cost per mile plus a fixed charge for hiring a truck. The objective is to determine replenishment policies that specify the delivery quantities and the vehicle routes so as to minimize long-run average inventory and transportation costs. Viswanthan and Mathur (1997) generalized Anily and Federgruen (1990) with the multi-item version of the problem. Diaby and Martel (1993) and Chan et al. (2002) consider the single-item deterministic dynamic demand case with a general piece-wise linear transportation cost. Martel et al. (2002) consider the multi-item dynamic demand case with a general piece-wise linear transportation cost but they do not include production decisions in their model. To the best of our knowledge, the only models including production-distribution decisions for multi-item dynamic demands are Chandra and Fisher (1994), Haq et al. (1991) and Ishii et al. (1988).

2. Single-machine lot-sizing problem

2.1 Problem definition and assumptions

In order to reduce the complexity of the complete production-distribution problem defined in Figure 11.4, the current practice in most paper mills is to plan production for each paper machine separately. Furthermore, as indicated earlier, in order to simplify the planning problem and the implementation of the plans produced, the set of IP products to be manufactured on a given paper machine, the sequence in which the products must be manufactured and the length of the production cycles (in planning periods) to be used are predetermined (at the tactical planning level). The fixed sequence context also implies that the intermediate products are all manufactured in each cycle. However, as illustrated in Figure 11.3, the paper rolls (jumbo) coming out of the paper machines are not inventoried: they are transformed immediately into finished products. The finished products however are stored in the mill warehouse and it is from this stock that products are shipped, every planning period, to distribution centers or customers. In order to prepare adequate production plans, the relationships between the IP lot-sizes and the FP inventories and demands must be considered explicitly. Our aim in this section is to present a model to determine the lot-size of the IP to manufacture on a single paper machine which minimizes total relevant costs for all the production cycles in the planning horizon considered.

In order to relate the model proposed to the general problem, the timing conventions used must be clarified. Figure 11.5 illustrates the relationships between *planning periods, production cycles, production lead-times* and the *planning horizon.* As can be seen, a production cycle p, is defined by a set T_p of planning periods and there are P production cycles in the planning horizon. For the finished products, the planning horizon is offset by the production lead-time. This planned lead-time is assumed to be the same for all finished products and it is expressed in planning periods. It includes the total elapsed time from the beginning of the period in which an IP production order is released until the finished products are available to be shipped from the mill warehouse. In

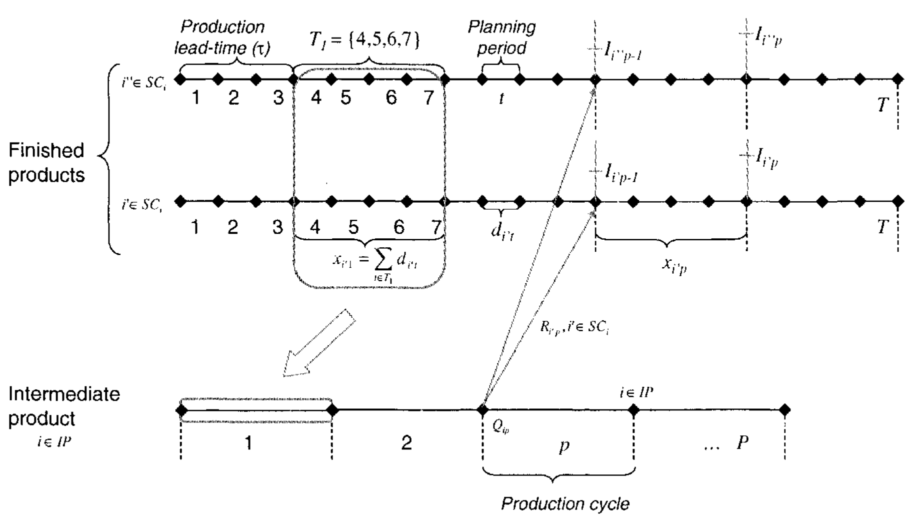

Figure 11.5. Planning horizon for IP and FP products

other words, it is assumed that the finished products made from the intermediate products produced in a production cycle will be available in inventory τ planning periods after the beginning of the cycle, independently of the position of the IP product in the predetermined machine production sequence. Clearly, this is a gross approximation and it is reasonable only when the production cycles are relatively short. This assumption, however, provides a rational for aggregating planning period effective demands into production cycles effective demands.

In what follows, we assume that planning is based on the finished products *effective demands*. Following Hax and Candea (1984), the effective demand d_{it} of a finished product $i \in \mathrm{FP}$ in planning period $t > \tau$ is defined as the demand for the period which cannot be covered by the projected inventory on hand $I_{i\tau}$ at the end of period τ, taking the desired safety stock level SS_i for the product into account. More precisely,

$$d_{it} = \begin{cases} \max\{0, \sum_{t'=\tau+1}^{t} \underline{d}_{it'} - I_{i\tau} + SS_i\}, & \text{if } d_{i,t-1} = 0, \\ \underline{d}_{it}, & \text{otherwise} \end{cases},$$

$$t = \tau + 1, \ldots, T \ (d_{i\tau} = 0),$$

where $\underline{d}_{it}$ is the demand for finished product i at the mill in planning period t. We also assume that, for each cycle p in the planning horizon, the cumulative capacity available is greater than or equal to the cumulative effective demand. When this condition is not satisfied, there is no feasible solution. We also assume that there is a lower bound on the production lot-size for each product made in a cycle. Finally, we assume that the unit production costs for an intermediate product are the same in every production cycle.

2.2 Single-machine lot-sizing model

Since this is a single-machine problem, the index m is dropped in what follows from the notation defined in Tables 11.1 and 11.2. The additional notations in Table 11.3, are also required to formulate our fixed cycle lot-sizing model.

In order to formulate the model, we first need to define the aggregate effective demand for the production cycles. As illustrated in Figure 11.5, the cycles' effective demands are given by:

$$x_{ip} = \sum_{t \in T_p} d_{it}, \quad i \in \mathrm{FP}, \ p = 1, \ldots, P.$$

Note next that one of the implications of using predetermined fixed production cycles is that every IP is manufactured during each cycle.

Table 11.3. Additional notation

P	Number of production cycles in the planning horizon.
p	A production cycle.
T_p	Set of planning periods in production cycle p.
$\tilde{C}_p$	Production capacity available in cycle p, net of set-up times (in time units).
$\underline{Q}_i$	Minimum lot-size for product $i \in$ IP (minimum hours/a_i).
x_{ip}	Product $i \in$ FP effective demand for production cycle p.

This implies that the total set-up costs over the planning horizon are constant and that they do not have to be taken into account explicitly.

In order to economize set-up times for the entire planning horizon, while maintaining the fixed sequence, as illustrated in Figure 11.6, we can impose that the last item scheduled at the end of a given cycle is scheduled at the beginning of the next cycle. The example in Figure 11.6 assumes that product 1 was the last product manufactured in cycle 0. Given this, the net capacity available in each cycle, $\tilde{C}_p$, $p = 1, \dots, P$, can be calculated *a priori*. For example, for cycle 2 in Figure 11.6, the net capacity available is $\tilde{C}_p = \sum_{t \in T_2} C_t - k_1 - k_2$, where k_i is the changeover time for product i. More generally, the capacity available can be calculated with the expression:

$$\tilde{C}_p = \sum_{t \in T_p} C_t - \sum_{i \neq \text{first}(p)} k_i,$$

where first(p) is the index of the product scheduled for production at the beginning of cycle p.

Also, since we have a deterministic demand and since the variable production costs do not change from cycle to cycle, the total production

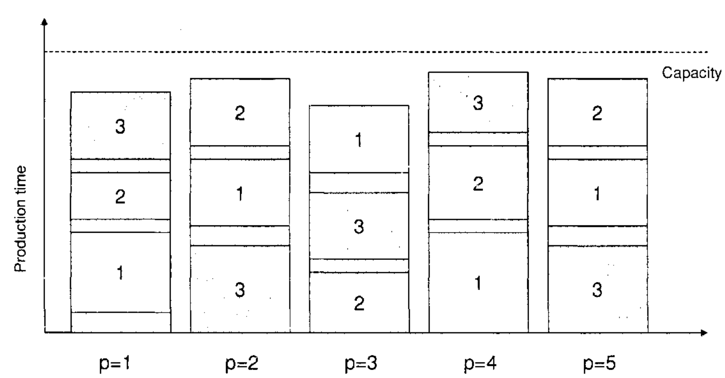

Figure 11.6. Example of a fixed cycle production plan for three products

cost is a constant and it does not have to be taken into account explicitly. Consequently, the only relevant costs under our assumptions are the intermediate products inventory holding costs. The lot-sizing problem to solve in order to minimize these costs is the following:

$$\text{Min} \sum_{p=1}^{P} \sum_{i \in \text{IP}} h_{ip} I_{ip} \tag{11.1}$$

subject to:

$$Q_{ip} - \sum_{i' \in \text{SC}_i} g_{ii'} R_{i'p} = 0 \quad i \in P \tag{11.2}$$

$$R_{ip} + I_{i(p-1)} - I_{ip} = x_{ip} \quad i \in \text{FP};\ p = 1, \ldots, P\ (I_{i0} = 0) \tag{11.3}$$

$$\sum_{i \in \text{IP}} a_{ip} Q_{ip} \le \tilde{C}_p \quad p = 1, \ldots, P \tag{11.4}$$

$$Q_{ip} \ge \underline{Q}_{ip} \quad i \in \text{IP}; p = 1, \ldots, P \tag{11.5}$$

$$I_{ip} \ge 0 \quad i \in \text{FP} \tag{11.6}$$

$$Q_{ip} \ge 0 \quad i \in \text{IP};\ p = 1, \ldots, P \tag{11.7}$$

$$R_{ip} \ge 0 \quad i \in \text{FP};\ p = 1, \ldots, P \tag{11.8}$$

The constraints include product bills of material (11.2), inventory accounting equations for finished products (11.3), and production output capacity of the machine (11.4), taking into account set-up times incurred for each cycle. Constraints imposing a minimum production quantity for each cycle (11.5) were also included. These constraints guarantee that each product can be manufactured in each cycle according to the predetermined sequence. Backorders are not allowed (11.6). Finally, nonnegativity constraints for production variables are also included (11.7) and (11.8). The experimental evaluation of the impact of this model and its various parameters are discussed is Bouchriha, D'Amours, and Ouhimmou (2003).

Although it is common practice in the fine paper industry to prepare fixed cycle length production plans for the paper machines and to use all the capacity available (i.e., to replace the inequality by an equality in constraint (11.4)), it is clear that the planning approach, developed in this section, is not really satisfactory. When the market demand for paper is low, as it currently is, the approach may lead to the production of products which are not required or to high inventory levels which could be avoided. It is therefore clear that this approach is suboptimal. Moreover, depending on the cycle length used and the demand variability, the approach could even lead to unfeasible solutions. For all these reasons, in the following sections, the fixed cycle length assumption is relaxed but the assumption that the production sequence is predetermined is maintained.

3. Multiple-machines lot-sizing problem

3.1 Problem definition and assumptions

In this section, we consider the simultaneous planning of the lot-sizes of intermediate products on all the paper machines in a mill, as well as the production and inventory planning of its finished products. We assume that the paper machines are capacity constrained but that the conversion stages are not capacity constrained. This is realistic, since it is always possible to subcontract part of the finishing operations if additional capacity is required. Although it is important in the industry to preserve the predetermined production sequence on the paper machines (launching production according to increasing paper thickness minimizes paper waste and set-up times), the use of fixed length production cycles is not imposed by any technological constraints. In this section we therefore relax the assumption of a fixed length production cycle. However, we assume that at most one production changeover is allowed per paper machine per planning period. This is reasonable provided that the planning periods used are relatively short (a day or a shift). We also assume that it is not necessary to use the total capacity available in a given period. Although in practice this is rarely the case, it is possible to reduce the production paste in order to produce less during a planning period without stopping the machine. The approach proposed in this section and the next is based on Rizk, Martel, and D'Amours (2003).

Let $g_{ii'}$ be the number of units of IP i required to produce one unit of FP i', taking any waste incurred in the transformation process into account. Since each FP is made from a single IP product, the set of FP can be partitioned according to the IP it is made of. In addition, it is assumed that a standard production sequence of IP must be maintained for each machine $m = 1, \ldots, M$, and that at most one product type can be produced in a given time period. Let e_m denote the index of the IP in the eth position in machine m production sequence, so that $e_m = 1_m, \ldots, f_m$, where f_m represents the product in the final position in machine m production sequence. Thus, when $e < f$ product $(e+1)_m$ can be produced on machine m, only after product e_m has finished its production batch (see Figure 11.7). The production resource consumption for intermediate products is assumed to be concave, that is, a fixed resource capacity consumption is incurred whenever production switches from one IP to another (changeover time), and linear resource consump-

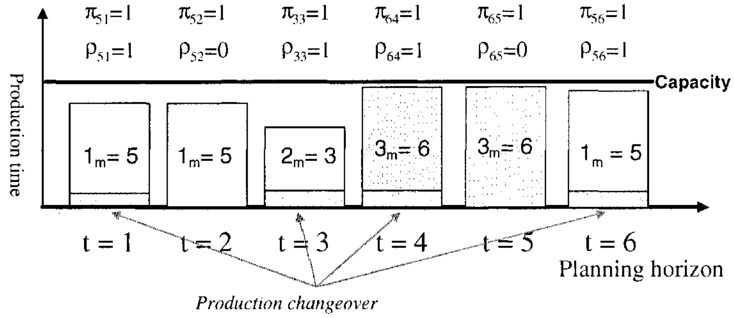

Figure 11.7. Example of a production plan for machine m

tion is incurred during the production of a batch of IP. Inventory holding costs are assumed to be linear.

3.2 Multiple-machines lot-sizing model

Using the notation in Tables 11.1, 11.2 and 11.4, the production planning problem of the manufacturing plant can be represented by the following optimization model:

$$\text{Min } \sum_{t=1}^{T}\left[\sum_{m=1}^{M}\sum_{i\in \mathrm{IP}_m} K_{it}^m \rho_{it}^m\right] + \sum_{t=\tau+1}^{T+\tau}\left[\sum_{i=n+1}^{N} h_{it} I_{it}\right] \tag{11.9}$$

subject to

$$\sum_{\eta\in M_i} Q_{it}^m - \sum_{i'\in \mathrm{SC}_i} g_{ii'} R_{i't} = 0 \qquad i=1,\ldots,n; t=1,\ldots,T \tag{11.10}$$

$$R_{it} + I_{i(t+\tau-1)} - I_{i(t+\tau)} = d_{i(t+\tau)} \qquad i=n+1,\ldots,N; t=1,\ldots,T; I_{i\tau}=0 \tag{11.11}$$

$$k_{it}^m \rho_{it}^m + a_{it}^m Q_{it}^m - C_t^m \pi_{it}^m \le 0 \qquad m=1,\ldots,M; i\in \mathrm{IP}_m; t=1,\ldots,T \tag{11.12}$$

Table 11.4. Additional notation

e_m	The eth item in the production sequence of machine m, $e_m = 1_m, \ldots, f_m$ $(f \le n)$
ρ_{it}^m	Binary variable equal to 1 if a new production batch of product i is started on machine m at the beginning of period t and to 0 otherwise
π_{it}^m	Binary variable equal to 1 if product i is made on machine m in period t and to 0 otherwise

$$\pi_{e_m t}^m - \sum_{u=1}^{t} \rho_{e_m u}^m + \sum_{u=1}^{t} \rho_{(e+1)_m u}^m = 0 \qquad m = 1, \dots, M; e_m = 1_m, \dots, (f-1)_m; \; t = 1, \dots, T \tag{11.13}$$

$$\pi_{f_m t}^m - \sum_{u=1}^{t} \rho_{f_m u}^m + \sum_{u=1}^{t} \rho_{1_m u}^m = 1 \qquad m = 1, \dots, M; t = 1, \dots, T \tag{11.14}$$

$$\sum_{i \in \mathrm{IP}_m} \pi_{it}^m \leq 1 \qquad m = 1, \dots, M; t = 1, \dots, T \tag{11.15}$$

$$\rho_{it}^m \leq \pi_{it}^m, \pi_{it}^m \in \{0,1\}, \rho^m \in \{0,1\}, Q_{it}^m \geq 0 \qquad m = 1, \dots, M; u \in \mathrm{IP}_m; t = 1, \dots, T \tag{11.16}$$

$$I_{it} \geq 0 \qquad i = n+1, \dots, N; t = \tau + 1 \dots, \tau + T \tag{11.17}$$

$$R_{it} \geq 0 \qquad i = n+1, \dots, N; t = 1, \dots T \tag{11.18}$$

In model 2, (11.10) and (11.11) are the flow conservation constraints of IP and FP products at the manufacturing location. Constraints (11.12) ensure that production capacity is respected. Constraints (11.13) and (11.14) make sure that the production sequence is respected for each machine. For a given machine m, when $e < f$, constraint (11.13) enforces the number of product $(e+1)_m$ changeovers to be less than or equal to the number of product em changeovers for any given period of time. Hence, it forces product $(e+1)_m$ production to start only after the production batch of product e_m is completed. Constraints (11.14) do the same job for product f_m which has the particularity of being last in the machine m production sequence. Thus, after its production batch, machine m has to switch production to product 1_m and start another sequence. Constraints (11.15) makes sure that at most one product is manufactured per period of time for each machine. Finally, constraints (11.16) restrict the changeovers on a machine to the periods in which there is some production.

This is a mixed-integer programming model of moderate size and it can be solved efficiently with commercial solvers such as Cplex. Rizk, Martel and D'Amours (2003) showed, however, that its solution time can be decreased significantly by the addition of appropriate valid inequalities (cuts).

4. Synchronized production-distribution planning problem

4.1 Problem definition and assumptions

In this section, we consider the flow coordination problem of multiple products in a single plant multi-warehouse network. In this network, one or multiple transportation modes are used to replenish different distribution centers with finished goods. The different transportation modes may have different transportation lead times from the plant to its clients and their cost structure can be represented by a general piece-wise linear function $z(S)$ to reflect economies of scale. These transportation economies of scale may have a major impact on inventory planning and replenishment strategies for both the plant and its clients. Transit inventory costs may have an impact on which transportation mode to use between the plant and a destination. Transit inventory costs can be embedded in each transportation mode cost structure as shown in Figure 11.8. Figure 11.8 also shows that, when different transportation modes have the same lead time to a given destination, their cost structures can be amalgamated in a single piece-wise linear function. Major cost savings can be achieved by integrating inventory control and transportation planning.

4.2 Synchronized production-distribution planning model

The type of general piece-wise linear function used to model transportation costs can be represented as a series of linear functions, as shown in Figure 11.9. Let S_j, $j = 0, \ldots, \gamma$, $S_0 = 0$ denote the break points of

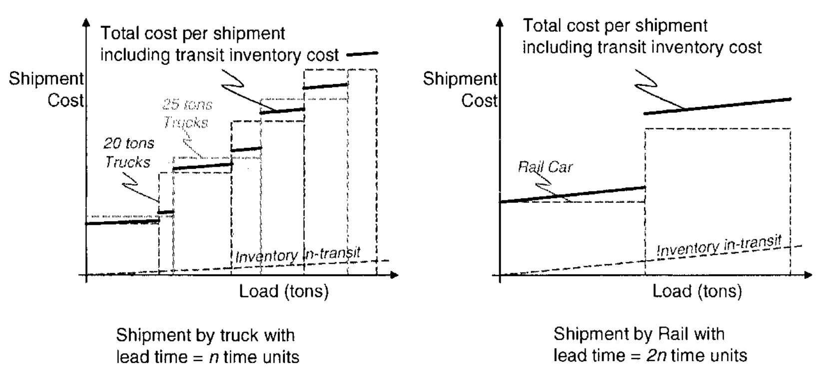

Figure 11.8. Cost structures for two different transportation modes

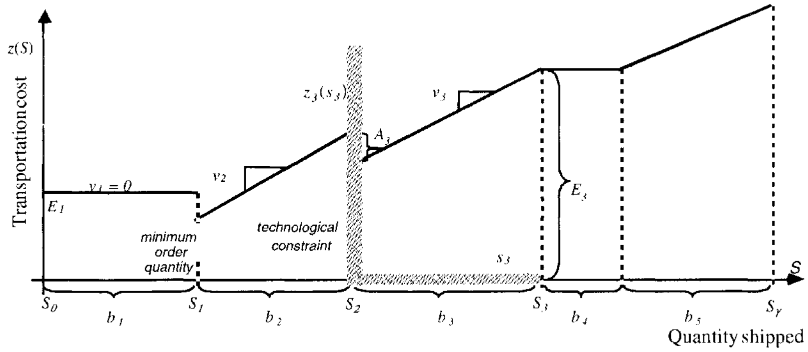

Figure 11.9. General transportation cost function

the piece-wise linear function and let $b_j = S_j - S_{j-1}$, $j = 1\ldots,\gamma$ denote the length of the jth interval on the S-axis defined by the break points $(S_0,\ldots,S_\gamma)$. Finally, for interval j, let ν_j be the slope of its straight line (variable cost), A_j be the discontinuity gap at the beginning of the interval and E_j be the value of the function at the end of the interval, i.e. $E_j = z(S_j)$. Then, it is seen that for $S_{j-1} < S < S_j$, we have $z(S) = (E_{j-1} + A_j) + \nu_j s_j, s_j = (S - S_{j-1})$.

For an amount S to be shipped in a given period of time, let j be the interval for which $S_{j-1} < S < S_j$, $j \geq 1$, $S_0 = 0$. S can then be expressed as $S = \lambda_j S_j$ where $\lambda_j = S/S_j$ for $j \geq 1$. Based on the above, S can be written in general as $S = \sum_{j=0}^{\gamma} \lambda_j S_j$ where $(S_{j-1}/S_j) < \lambda_j \leq 1$ if $S_{j-1} < S \leq S_j$ and $\lambda_j = 0$ otherwise, for $j = 1,\ldots,\gamma$.The last two conditions can be represented by a binary variable α_j where

$$\alpha_j = \begin{cases} 1, & \text{if } S_{j-1} < S \leq S_j; \\ 0, & \text{otherwise,} \end{cases} \quad j = 1,\ldots,\gamma \text{ and } \alpha_0 = \begin{cases} 1, & \text{if } S = 0; \\ 0, & \text{otherwise.} \end{cases}$$

Using the above observation, S can be expressed in an LP model by the following set of constraints:

$$S = \sum_{j=0}^{\gamma} \lambda_j S_j \tag{11.19}$$

$$\frac{S_{(j-1)}}{S_j}\alpha_j \leq \lambda_j \leq \alpha_j, \quad j = 1,\ldots,\gamma \tag{11.20}$$

$$\sum_{j=0}^{\gamma} \alpha_j = 1 \tag{11.21}$$

$$\alpha_j \in \{0,1\}, \qquad j = 0,\ldots,\gamma \tag{11.22}$$

From the definition of E_j and the above set of constraints, it is seen that $z(S)$ can be expressed as a linear function of variables α_j and λ_j, $j = 1, \ldots, \gamma$:

$$z(S) = \sum_{j=1}^{\gamma}[(E_j - v_j S_j)\alpha_j + (v_j S_j)\lambda_j] \tag{11.23}$$

In addition, we can observe from constraints (11.21) and (11.22) that α_j, $j = 0, \ldots, \gamma$ form a *Special Ordered Set of type 1* (SOS1) as defined by Beale and Tomlin (1970). Declaring α_j, $j = 0, \ldots, \gamma$ as SOS1, the process of Branch and Bound can be further improved (see Beale and Tomlin, 1970). In addition, by defining α_j, $j = 0, \ldots, \gamma$ as SOS1 along with constraints (11.21), constraints (11.22) are not needed.

For a given destination $w \in W$, let $\beta^w = \text{Min}_{u \in U^w}(\tau_u^w)$. β^w is the shortest transportation lead time to destination w. Let's assume that in period 1, a quantity of product $i \in$ FP is manufactured at the plant and at the end of period 1 we decide to ship an amount of product i to destination w. Because of the production and transportation lead times, the quantity of product i shipped cannot get to destination w earlier than time period $\tau + \beta^w + 1$. Thus, destination w replenishment planning can only start at period $\tau + \beta^w + 1$. Figure 11.10 illustrates different transportation mode shipments (R_{uit}^w) to satisfy the demand for product i at destination w. In this example, the planning horizon includes five ($T = 5$) planning periods. There are three transportation modes available to ship finished products from the plant to destination w ($U^w = \{1, 2, 3\}$). The transportation modes lead time from the plant

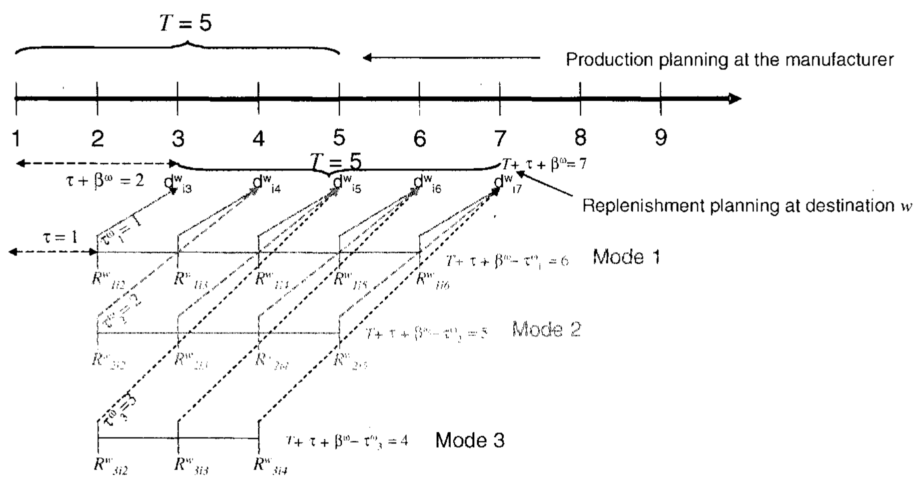

Figure 11.10. Example of multiple transportation mode shipments

to destination w are $\tau_1^w = 1$, $\tau_2^w = 2$, and $\tau_3^w = 3$. Production planning in the plant starts at period 1 and ends at period $T = 5$. On the other hand, because of production and transportation lead times, as stated above, replenishment planning for destination w starts at period $\tau + \beta^w + 1$ and ends at period $T + \tau + \beta^w$. Note that for a given transportation mode $u \in U^w$, only shipments that are made before time period $T + \tau + \beta^w - \tau_u^w$ can get to destination w within its replenishment planning horizon ($[\tau + \beta^w + 1, T + \tau + \beta^w]$). In practice, to get around this difficulty, planning must be done on a rolling horizon basis and the number of periods in the planning horizon must be sufficiently long to have a significant horizon for all the transportation modes, i.e, $T \gg \tau_u^w$, $\forall w \in W$, $u \in U^w$.

Using the notation in Tables 11.1, 11.2, 11.4 and 11.5, the flow coordination problem in a single manufacturer multi-destination network with multiple transportation modes can be formulated as follows:

Table 11.5. Additional notation

j	jth interval of the piece wise linear cost function of a transportation mode u to destination w in period t, $j = 0, \ldots, \gamma_{ut}^w$
S_{utj}^w	The maximum volume (in tons) that can be shipped by transportation mode u to destination w to incur the fixed plus linear cost associated to interval j in period t.
A_{utj}^w	Fixed cost associated to the jth interval of transportation mode u piece-wise linear cost function to destination w in period t.
E_{utj}^w	Cost of shipping the volume S_{utj}^w to destination w by transportation mode u in period t.
v_{utj}^w	Variable cost associated to the jth interval of transportation mode u to destination w piece-wise linear cost function in period t.
τ_u^w	Transportation lead-time to destination w by transportation mode u in period t.
α_{utj}^w	Binary variable associated with the jth interval of mode u to destination w transportation cost function in period t.
λ_{utj}^w	Multiplier associated to interval j of the quantity shipped by transportation mode u from the plant to location w for period t.
d_{it}^w	Effective external demand at destination w for item i during period t.
r_i	Transportation resource absorption rate for item i (in cwt, cube...).
I_{it}^w	Inventory level of finished item i in destination w at the end of period t.
R_{uit}^w	Quantity of finished item i shipped by transportation mode u from the plant to destination w in period t.

Min

$$\sum_{t=1}^{T}\Bigg[\sum_{m\in M}\Bigg[\sum_{i\in \mathrm{IP}_m} K_{it}^m\rho_{it}^m\Bigg]+\sum_{i\in \mathrm{FP}} h_{i(t+\tau)}I_{i(t+\tau)}\Bigg]+\sum_{w\in W}\Bigg[\sum_{t=\tau+\beta^w+1}^{T+\tau+\beta^w}\Bigg[\sum_{i\in \mathrm{FP}} h_{it}^w I_{it}^w\Bigg]\Bigg]$$
$$+\sum_{w\in W}\Bigg[\sum_{u\in U^w}\Bigg[\sum_{t=\tau+1}^{T+\tau-\eta_u^w}\Bigg[\sum_{j=1}^{\gamma_{ut}^w}(E_{utj}^w - v_{utj}^w S_{utj}^w)\alpha_{utj}^w + (v_{utj}^w S_{utj}^w)\lambda_{utj}^w\Bigg]\Bigg]\Bigg] \tag{11.24}$$

subject to

$$\sum_{m\in M_i} Q_{it}^m - \sum_{i'\in \mathrm{SC}_i} g_{ii'}R_{i't} = 0, \quad i\in \mathrm{IP}; 1\le t\le T \tag{11.25}$$

$$R_{it} + I_{i(t+\tau-1)} - I_{i(t+\tau)} - \sum_{w\in W}\Bigg[\sum_{u\in U^w} R_{ui(t+\tau)}^w\Bigg] = d_{i(t+\tau)},$$
$$i\in \mathrm{FP}; 1\le t\le T; R_{uit}^w = 0, \forall t\ge T+\tau-\eta_u^w \tag{11.26}$$

$$K_{it}^m\rho_{it}^m + a_{it}Q_{it}^m - C_t^m\pi_{it}^m \le 0, \quad m\in M; i\in \mathrm{IP}_m; 1\le t\le T \tag{11.27}$$

$$\pi_{e_m t}^m - \sum_{u=1}^{t}\rho_{e_m u}^m + \sum_{u=1}^{t}\rho_{(e+1)_m u}^m \le 0, \quad m\in M;$$
$$1_m\le i_m\le (f-1)_m; 1\le t\le T \tag{11.28}$$

$$\pi_{f_m t}^m - \sum_{u=1}^{t}\rho_{f_m u}^m + \sum_{u=1}^{t}\rho_{1_m u}^m = 1, \quad m\in M; 1\le t\le T \tag{11.29}$$

$$\sum_{i\in \mathrm{IP}_m}\pi_{it}^m \le 1, \quad m\in M; 1\le t\le T \tag{11.30}$$

$$\sum_{u\in U^w} R_{ui(t-\tau_u^w)}^w + I_{i(t-1)}^w - I_{it}^w = d_{it}^w, \quad w\in W; i\in \mathrm{FP};$$
$$\tau+\beta^w+1\le t\le T+\tau+\beta^w; R_{uit}^w = 0, \forall t<\tau+1 \tag{11.31}$$

$$\sum_{i\in \mathrm{FP}} r_i R_{uit}^w - \sum_{j=1}^{\gamma_{ut}^w}\lambda_{utj}^w S_{utj}^w = 0,$$
$$w\in W; u\in U^w; \tau+1\le t\le T+\tau-\eta_u^w \tag{11.32}$$

$$(S_{ut(j-1)}^w/S_{utj}^w)\alpha_{utj}^w \le \lambda_{utj}^w \le \alpha_{utj}^w,$$
$$w\in W; u\in U^w; \tau+1\le t\le T+\tau-\eta_u^w; 1\le j\le \gamma_{ut}^w \tag{11.33}$$

$$\sum_{j=0}^{\gamma_{ut}^w}\alpha_{utj}^w = 1, \quad w\in W; u\in U^w; \tau+1\le t\le T+\tau-\eta_u^w \tag{11.34}$$

$$\rho_{it}^m \le \pi_{it}^m, \pi_{it}^m\in\{0,1\}, \rho^m\in\{0,1\}, Q_{it}^m\ge 0$$
$$m\in M; i\in \mathrm{IP}_m; 1\le t\le T \tag{11.35}$$

$$I_{it}^w \ge 0, \quad w\in W; i\in \mathrm{FP}; \tau+\beta^w+1\le t\le T+\tau+\beta^w \tag{11.36}$$

$$R_{it} \geq 0, I_{i(t+\tau)} \geq 0, \quad i \in \text{FP}; 1 \leq t \leq T \tag{11.37}$$

$$R_{uit}^w \geq 0, \quad w \in W; u \in U^w; i \in \text{FP}; \tau + 1 \leq t \leq T + \tau - \eta_u^w \tag{11.38}$$

$$0 \leq \alpha_{utj}^w \leq 1, 0 \leq \lambda_{utj}^w \leq 1,$$
$$w \in W; u \in U^w; \tau + 1 \leq t \leq T + \tau - \eta_u^w; 1 \leq j \leq \gamma_{ut}^w \tag{11.39}$$

$$(\alpha_{ut0}^w, \ldots, \alpha_{utj}^w) \in \text{SOS1},$$
$$w \in W; u \in U^w; \tau + 1 \leq t \leq T + \tau - \eta_u^w. \tag{11.40}$$

This is a large scale mixed-integer programing model and only small cases can be solved efficiently with commercial solvers such as Cplex. For the case when there is a single distribution center, Rizk, Martel, and D'Amours (2003) proposed valid inequalities which can be added to the model to speed up the calculations. Work on the development of an efficient heuristic method to solve the problem is also currently under way.

5. Conclusion

This chapter presents a review of the supply chain decision processes needed in the pulp and paper industry, from strategic supply chain design to operational planning, but with a particular emphasis on production and distribution planning for a paper mill logistic network. Gradually more relevant and comprehensive planning models are sequentially introduced starting from current industry practice and ending with a sophisticated synchronized production-distribution planning model.

The implementation of these models raises some interesting questions. From a practical point of view, solving the distribution and the production planning problem in sequence may seem interesting, since it reduces the problem size and complexity. Although the size of the problem may increase with the number of intermediary products and planning periods, large linear problems of this sort are easily solved with today's commercial solvers. Under this planning approach, the multi-machine lot-sizing problem provides a better solution than the single-machine lot sizing model where a production cycle constraint is imposed. However, for some demand contexts, experimental work has shown that the potential gains may be small in regard to the planning simplicity induced by the latter approach. Moreover, the imposition of a production cycle time is often useful to synchronize sales and operations, especially when order-promising is conducted on the web.

The last model proposed integrates both production and distribution planning processes. It takes advantage of transportation economies of scale and permits a better selection of transportation modes. However, in order to solve the model within practical time limits, specialized solution methods taking the structure of the problem into account must be developed. An approach which has shown interesting potential, is the addition of valid inequalities (cuts) to the original model. Initial experimentation has shown that the use of appropriate cuts can reduce computation times by an order of magnitude for this class of problem. The application of various decomposition approaches to the solution of the problem is also under study.

It is important to remember that the synchronized production-distribution model assumes that converting facilities are in-house (transportation between roll production and converting facilities is not considered) and over-capacitated in comparison with the bottleneck which was assumed in this chapter to be the paper making machines. Therefore, rolls can be converted within a known delay. Obviously, before applying the model this assumption should be assessed with regard to the company's situation.

Finally, the models presented in this chapter were designed to plan production and distribution over a two-week to a month rolling planning horizon. Since such a short horizon may limit visibility over seasonal parameters, tactical planning models should be used to supply key information to the production-distribution planning model. More specifically they should define end-of-horizon inventory targets for each product produced. Not doing so may results in very bad planning decisions over time, especially in the context of cyclic or highly variable demand. Including such end-of-horizon inventory targets in the model proposed presents no difficulty.

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