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REFLECTIONS

As we come to end of our travels through the pathways of research into the teaching and learning of probability, it is instructive to look back briefly at where we have come from and also to look forward to where others might journey in the future. In writing this book we faced a special challenge because ours is one of the first treatises on probability education since the study of it became part of the entire grade span of the school curriculum. It was for this reason that three of our sections focused specifically on the teaching and learning of probability in the elementary, middle, and high school years. The mainstreaming of probability in the school mathematics curriculum during the last 15 years was also a powerful reason for examining issues that relate to and influence the teacher and learner in the probability classroom.

In discussing the nature of probability in the classroom, two of our predecessors, Kapadia and Borovcnik (1992) used grist gleaned from giants in making the following observation:

> Concepts cannot be defined sharply from the onset, they are revised or even rejected during the process of emerging mathematics. Fischbein's interplay indicates that the teacher has to pay full attention to the intuitive level of subjects, in order to develop their intuitions. Bauersfeld's subjective domains of experience likewise suggest the need for a suitable context of experiments and for feedback on subjective notions. Freudenthal wonders how to develop applications rich enough to reveal the organizing potency of mathematical concepts and yet tractable enough to let learners really develop their mental objects and see how mathematics structures reality. (pp. 19-20).

This statement, although pertaining to mathematics in general, is especially germane for the teaching and learning of probability. Moreover, although we have addressed these issues, we acknowledge that there is still much distance to travel in helping children to build on their own probabilistic intuitions rather than their teachers', to monitor and assess their subjective notions of probability through exploration and experimentation, and to experience rich problems and tasks that enable them to develop varied and appropriate models of the realities of chance.

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We started by looking at the nature of chance and probability, and the sharp philosophical and mathematical divides that have resulted in probability becoming a multifaceted yet coherent body of knowledge. Historically and culturally it was noted that chance, randomness, and probability are significant realities in a world that impacts and is impacted by our children. This not only provides a rationale for the study of probability in schools, it raises issues of what they need to learn: a discussion we initiated under the rubric of probability literacy. In a complex technological world there are serious issues about expanding knowledge bases and overcrowded school curricula. Hence we have tried to provide a frame for discourse on probability literacy that resonates with what we have presented about research on the teaching and learning of probability in the last 50 years.

With respect to the *elementary school* we have documented an extensive body of research on children's reasoning about deterministic and chance phenomena, random mixtures and distributions, and combinatorics. Our chapters have also revealed how young children think about probability constructs such as sample space, experimental and theoretical probability, conditional probability and independence, and how they deal with combinatorics in problem-solving tasks. More precisely, we have noted the emergence of several cognitive frameworks that characterize students' probabilistic reasoning according to hierarchical levels and have implications for designing, implementing, and monitoring instruction.

In spite of the apparent robustness of the research on elementary school children's probabilistic reasoning, it is evident that there is a void in the research associated with the frequentist approach to probability; that is, research dealing with children's cognitions on experimental probability. In fact, there is almost no research on whether children can make connections between classical and frequentist orientations to probability even though teachers are encouraged to use these connections in the classroom. There is also a need for further research that traces children's individual and collective thinking in probability during instruction; such research needs to document effective classroom practices including those that use the technology and software that is becoming available for young children.

The research on *middle school* students' thinking in probability reflects the same kind of robustness that is apparent in the research on elementary school children. A broad panorama of research has been presented dealing with students' probabilistic language and their reasoning about random behavior, luck, fairness, probability measures, sampling and variation. Much of this research has the added value of being longitudinal and having the benefit of large samples. We also provided a microanalysis of middle school students' changing conceptions about randomness, distribution, and the law of large numbers, and one focusing on their evolving cognitive mechanisms when they were confronted with tasks involving sample space and probability in the context of compound experiments. Teaching experiment designs greatly enhanced these microanalyses and, in the case of the former, a microworld environment provided added technological implications for learning and instruction. Finally, we discussed middle school students' thinking in conditional probability and independence; two concepts that are relatively new to the middle school curriculum. This documentation, incorporating students' individual and collective thinking about conditional probability, was based on both clinical interviews and teaching experiments. Accordingly, it also has the potential to provide valuable theoretical and practical knowledge for curriculum developers and teachers.

Similar to the probability research on younger elementary children, the research on middle school students has limitations with respect to students' thinking about experimental probability and the connection between experimental and theoretical probability. Although it seems to be particularly apposite for this age group to begin to deal with relative frequencies, classical likelihood estimates and the law of large numbers, there is almost no clinical or instructional research addressing these issues. With respect to instructional research, there is a need for further research that traces students' individual and collective thinking during classroom instruction on probability. Such research has the potential to provide much needed theoretical knowledge on teaching and learning strategies, including strategies associated with the classical and frequentist approaches to probability. More specifically, there is a need for classroom research to evaluate teaching approaches that introduce "probability through data" (Gigerenzer, 1994; Hopfsenberger, Kranendonk, & Scheaffer, 1999; Shaughnessy, 2003) and examine metacognitve aspects of students' probabilistic thinking.

Although research into *high school* students' reasoning about probability started later than research into middle and elementary school students' probabilistic reasoning, the high school research has burgeoned in recent years. We have examined high school students' perception of randomness, their combinatorial reasoning, and their conceptions of conditional probability and independence, association, simulation, probability distribution, and inference. Much of this research resulted from clinical studies of students' thinking prior to instruction but we have also been able to accumulate studies that incorporated high school students' thinking during instruction. In response to a critical area there was a careful analysis of research that focused on the connections that students need to make between statistics and probability. The vexed questions associated with the teaching and learning of probability and statistical inference were reviewed from two perspectives: a theoretical approach (classical inference) and a simulation approach (informal inference).

Much of the research dealing with high school students' thinking about probability has focused on the misconceptions that they bring to the classroom. Although this has provided valuable background for teachers, there is a critical need for research that traces high school students' probabilistic reasoning and dispositions as they engage in instruction. We have documented evidence about students' misconceptions in areas such as conditional probability and independence but we have scarce knowledge about the evolutions of these intuitions during instruction and even less data about the kinds of instruction that might lead to more normative thinking. With respect to connections between statistics and probability, the field is wide open. Although there has been insightful historical and theoretical research into cognitive and pedagogical aspects of probability and statistical inference, there is need for empirical research to investigate the evolution of key ideas such as random variable, probability distribution, and classical and experimental statistical inference. Moreover, it is now possible for this research to be undertaken in learning environments that incorporate sophisticated simulation and sampling distribution software.

The final section of the book examined the vital role of the teacher and concomitant issues such as pedagogy, assessment, and teacher education and development. With respect to pedagogy, an argument was made for treating probability as a multidisciplinary study with emphasis on the mathematical modeling of students' lived experiences. The notion of lived experiences was also a strong component of the research we have presented on assessment. In particular, a contrast was made between traditional assessment and authentic assessment where students are assessed on tasks that go beyond the often sterile culture of the classroom. The teacher is the key to everything we have discussed about teaching and learning in probability and that means that research dealing with teachers and classrooms is critical to the whole enterprise. So far this research is largely embryonic and our documentation has focused mostly on teachers' own knowledge of probability and to a lesser extent on their knowledge of students' probabilistic thinking.

Much of what we have documented about the teacher and the teacher's role in teaching and assessing probability has highlighted the need for further investigation. There is almost no research on a modeling approach to the teaching of probability, especially a modeling approach that incorporates cultural, social, and political contexts. In the same way, the relatively recent introduction of probability as a mainstream area in the mathematics curriculum has not provided much opportunity for research into authentic assessment. Although there have been numerous assessment instruments constructed and administered by researchers, there is a striking need for both written and interview assessment that can be used on large scale populations. Mention has already been made of the fact that teacher research in probability is in its infancy. In addition to the emerging research on teachers' knowledge of probability *per se* and teachers' knowledge of students' probability cognitions, it is essential that researchers investigate the effects of professional development and enhancement programs on various kinds of teacher knowledge.

As we leave this diverse and demanding discussion on the teaching and learning of probability, Deborah Bennett's (1999) words offer a poignant message:

> In the short run chance may seem volatile and unfair. And while experience with long-run frequencies can help to modify some of our maladaptive behaviors based on a misunderstanding of randomness and probability, a very long run may be required. Considering the misconceptions, inconsistencies, paradoxes, and counter-intuitive aspects of probability, it should be no surprise that, as a civilization, we took a long time to develop correct intuitions. Indeed, every day we can see evidence that the human species does not have a very highly developed probabilistic sense. (pp. 187-188).

In a very real sense, Bennett's words encapsulate much of what we have discussed in this book; on the one hand, the beguiling complexity of chance and probability and on the other hand, the challenges of learning it somewhat against the odds of our own intuitions. We trust that our contribution to the field of learning and teaching probability may contribute to the development of a more "highly-developed probabilistic sense" in future generations of children and adults.

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