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PROBABILITY IN TEACHER EDUCATION AND DEVELOPMENT

Chapter 14

> The success of any probability curriculum for developing students' probabilistic reasoning depends greatly on teachers' understanding of probability as well as a much deeper understanding of issues such as students' misconceptions (Stohl, p. 351, this chapter).

1. INTRODUCTION

The purpose of this chapter is to investigate issues concerning the nature and development of teachers' probability understanding. The chapter begins with a discussion of central issues that affect teachers' efforts to facilitate students' probabilistic understanding. I then examine teachers' knowledge and beliefs about probability, their ability to teach probabilistic ideas, and lessons learned from programs in teacher education that have aimed at developing teachers' knowledge about probability.

Stochastics (probability and statistics) has become an area of emphasis in school curricula in the past 10-15 years (e.g., National Council of Teachers of Mathematics [NCTM], 1989, 2000). However, most teachers have little or no prior experience with many of these topics in their own schooling and teacher preparation programs. In the 1990's, efforts at professional development for practicing teachers began while teacher preparation programs started to include some attention to probability or statistics in mathematics methods courses. There is evidence (Vacc, 1995) that many teacher education professionals doubted the appropriateness or usefulness of statistics and probability in the K-4 standards as recommended by the NCTM (1989). This most likely had an effect on the attention given to probability in some elementary teacher education programs. Many middle grades/secondary teacher education programs typically require a course in statistics that includes some probability, but it is presented mainly from a purely theoretical perspective. Nonetheless, many middle grades/secondary level teachers are asked to teach substantial content in statistics and

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probability while having very little meaningful understanding of the skills and concepts they are expected to teach.

Many studies have shown that adults and college-level students typically have a variety of misconceptions related to probability (e.g., Fischbein & Schnarch, 1997; Konold et al., 1993; Shaughnessy, 1977). Without specific training in probability and statistics, preservice and practicing teachers (and perhaps some teacher educators) may rely on their beliefs and intuitions, and have similar misconceptions as reported in these studies. Almost every study reported in earlier chapters of this book includes implications for teaching and teacher education. These implications require teachers (and teacher educators) to:

- understand probability concepts,
- understand students' conceptions of probability, and
- critically think about the research results related to students' development of probabilistic ideas as a means of informing their instruction.

This is a tall order for teachers and teacher educators who may hold the same beliefs, intuitions, and misconceptions as their students.

This chapter addresses teachers' understanding of probability, beliefs and misconceptions, classroom practices in teaching probability, and effects of teacher development projects on teaching and learning probability. In order to frame the discussions within this chapter, it is useful first to consider central issues that affect teachers' learning to teach probability.

2. ISSUES IN LEARNING TO TEACH PROBABILITY

Many teachers have a computational orientation towards teaching mathematics that most likely stems from an underlying belief that doing mathematics is a rule-driven, right or wrong endeavor (Thompson, 1984; Thompson, Philipp, Thompson, & Boyd, 1994). This type of orientation to teaching mathematics tends to focus on procedures and skill-based activities. Some teachers are transitioning to a reform-oriented perspective that expands their vision of teaching mathematics beyond computation and encompasses a shift in beliefs towards a constructivist model of learning. Such a model of learning dually considers the individual as she/he constructs meaning (von Glasersfeld, 1995) and the social interactions and negotiations among individuals (Voigt, 1996). These social interactions enable and constrain an individual and are necessary to develop taken-as-shared meanings that allow a group of individuals to effectively operate and communicate collectively (Cobb, 1994). However, Heinz, Kinzel, Simon, and Tzur (2000) have noted that teachers in transition to more reform-oriented ideas may have a "perception-based" approach to teaching that influences their goals and activities for students; that is, they teach in order to have students perceive the mathematics as the teacher understands it. Because the teacher's perspective directs her or his instructional practice, a teacher with a perception-based perspective may not attend carefully to students' understanding. Teachers may plan activities and use tools or representations that make sense to their own mathematical understanding, rather than crafting activities to build on students' current and developing understandings.

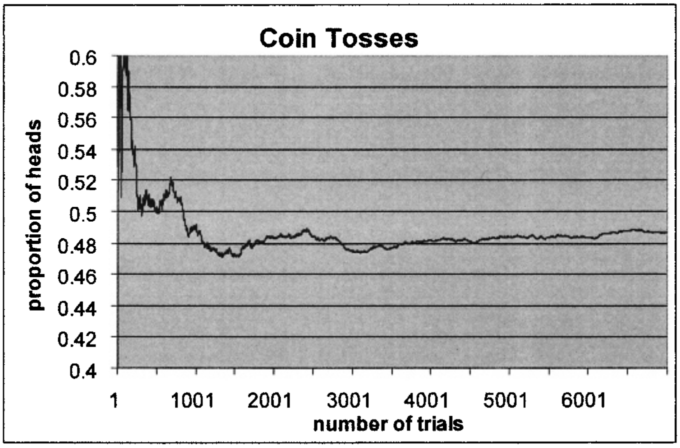
Often, teachers with a computational orientation are likely to assume a deterministic view when teaching (and learning) probability. That is, teachers may assume that the purpose of teaching and learning about probability is to use procedures to calculate theoretical probabilities in the absence of considering the real world application of these probabilities. The study of probability is fundamentally different from deterministic situations considered in the study of other areas of mathematics (e.g., functions, numerical operations, geometry). The theoretical field of mathematics called "probability theory" has as many procedures and structures as any other field of mathematics. However, directly linking this structure (and accompanying theoretical exercises) to real situations, like rolling dice or predicting the weather, is not nearly as straightforward as in other areas of mathematics studied in school.

The inability to predict whether or not a "four" will occur on a roll of a die is due to the inability of humans to directly account for the complex nature of the physics involved (e.g., air resistance, speed, friction). Thus, one cannot exactly determine the actual probability of rolling a "four" on a regular six-sided die. However, we can use a classical Laplacean approach to *embody* the complexities of the physics and apparent (and probably imperfect) symmetry of the die and assume the probability of rolling a "four" is 1/6. This theoretically derived probability of 1/6 is an *estimate* of the actual probability that is unknown to us. If one rolls a die a given number of times, a frequentist approach can be used to state the experimental probability in terms of the proportion of "fours." But again, this experimental estimate only describes the probability of getting a "four" based on that set of random tosses. A repeated set of die rolls would most likely produce a different experimental estimate of the actual probability. Given the uncertainty of experimental results, it is not surprising that many teachers favour a classical approach to probability wherever possible. Such an approach relies on counting techniques, leads to a single theoretical answer to the probability of an event, and avoids a realistic interpretation of that value.

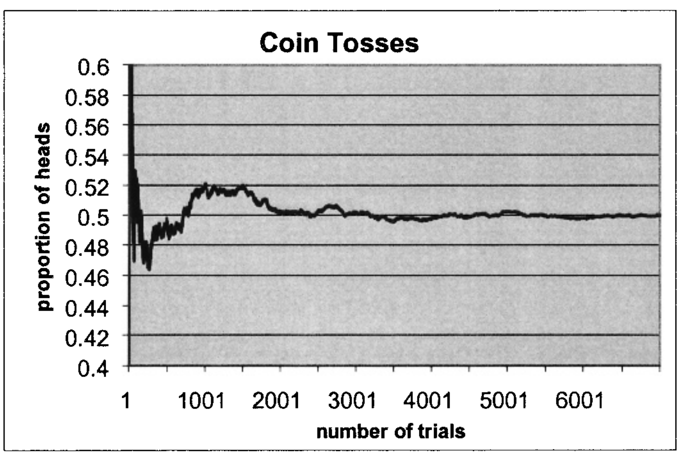
Only in an approach to teaching that embraces both a classical and frequentist approach for estimating probability can students develop appropriate probability intuitions and avoid the types of misconceptions described in several earlier chapters (see Batanero & Sanchez; Jones & Thornton; Watson; this volume). The key mathematical theorem used to interpret empirical results as compared to theoretically derived probabilities is the law of large numbers. Unfortunately, one source of many misconceptions (e.g., gambler's fallacy, law of small numbers) may be due to an incorrect interpretation of this law as implying that experimental probabilities *limit* to the theoretical probability. This law actually tells us that the *probability* of a large difference between the empirical probability and the theoretical probability limits to zero as more trials are collected. Thus, it is possible, although unlikely, to have an empirical probability substantially different from the theoretical probability, even after a large number of trials.

Teachers may misinterpret this law of large numbers and misguide students into expecting a necessary convergence of empirical probabilities with a large number of trials. This issue is exacerbated with an expectation that this "convergence" should be quick. Some teachers (and textbooks) may only note that with a large number of trials, the experimental probabilities will be close to the theoretical probability. Or they may slightly change the words in this statement to say that the experimental probability approaches, or gets closer and closer to, the theoretical probability. This language is used in calculus with the concept of limit and implies that with a large numbers of trials it is not possible to have an experimental probability that is significantly different from a theoretical probability. Thus, in a student's mind, and perhaps the teacher's mind, there is some modest sample size at which experimental probability will be very close to the theoretical probability, and it will not stray henceforth.

Consider the graphs in Figure 1 that show two sets of 7000 trials of a simulated fair coin toss. The results in Figure 1A may be confusing to a student who has the conception that the empirical probability should converge to the theoretical probability. After about 500 trials, the proportion of heads is close to 0.5 but then gets closer to 0.52 by 600 trials. By 1000 trials, the proportion of heads is now about 0.48 and tends to stay near 0.48 until about 6000 trials when it becomes slightly closer to the expected 0.5. Even though the likelihood of the experimental probability being significantly different from the theoretical probability of 0.5 gets smaller with larger trials, it is still quite possible to obtain an experimental probability of 0.475 after 3000 trials. Unfortunately the graph shown in Figure 1B is often used in textbooks to illustrate the law of large numbers since the empirical probability in this set of trials *does* stay very close to an expected theoretical probability of 0.5. It would be much more illustrative to include several graphs of sets of simulated trials that demonstrate different possibilities.



A



B

Figure 1. Proportion of heads in two sets of 7000 fair coin tosses

Some teachers may intuitively understand the complexity of the law of large numbers and altogether avoid any classroom discussions concerning the interplay between empirical and theoretical probability. However, a tendency to organize teaching of probability as theoretical constructs results in consequences that are: "even worse than in other branches of mathematics education as stochastic knowledge has a specific theoretical character" (Steinbring, 1991a, p. 139).

Steinbring claims teachers' difficulty in teaching probability in classrooms is rooted in the nature of probability which he believes includes circularities in definitions that cannot be well understood from either a classical or experimental approach alone. He argues there is a mutual dependence between the object that a theoretical probability can describe and the concept of probability as a relative frequency that emerges from an experiential situation. In addition, he emphasizes that neither of these can alone characterize the meaning of probability and there is a need to develop them together. Steinbring (1991b) refers to an epistemological triangle whose vertices are concept, sign, and object. He further elaborates that there is not an unambiguous definition of probability as a "sign" (Laplacean theoretical probability) or "object" (empirical frequencies), but that the relationship between the two is "open and subject to development" (p. 507) as a learner comes to understand a concept of probability (see also, Jones & Thornton, this volume).

The conceptual complexity of probability is a major issue for the development of teachers' knowledge. However, teachers' difficulties in understanding and teaching probability may also be related to the disconnect that exists between probability and statistics. Often times, probability is addressed as a subset of concepts addressed within statistics and little connection is made between data analysis, descriptive statistics, and probability in school mathematics. Some of the most powerful and useful ways to use probability involve making sense of a statistic derived from samples and claims that are made about a population. For example, suppose you interview 950 professors in the US and ask them "Do you drink coffee?" and then determine the following statistic: the percentage of these 950 people who drink coffee. Suppose 68% of those interviewed answered yes. With what justification can the claim that 68% of all U.S. professors drink coffee be made? To answer this, one has to understand how to develop statistical meaning with the application of probability. If teachers emphasize a classical-based single-answer approach to probability, they or their students may only interpret a statistic like "68% of 950 professors" as the correct proportion to describe all professors. Instead, performing experiments (or simulations) where students collect data to conjecture or make inferences about a probability (e.g., the fairness of a die) or population distribution (e.g., proportion of green marbles in a bag with unknown amount of marbles), can help them to make deeper connections between statistics and probability, including use of confidence intervals instead of single-value estimates.

In 1992, Shaughnessy advocated research on teachers' understanding of stochastics; however, to date, very little research has been reported. The core issues discussed here will surface throughout this chapter in summarizing what is known about teachers' probability knowledge, results of projects for developing teachers' knowledge of stochastics as a content domain, and their instructional effectiveness for teaching stochastics. This chapter is an attempt to summarize what we do know about teachers' knowledge of probability and issues concerning teacher education for preparing teachers to effectively develop students who can reason under uncertainty.

3. KNOWLEDGE OF PROBABILITY FOR TEACHING

In accord with Shulman (1986), it is important to consider teachers' content knowledge about probability and their pedagogical content knowledge that goes beyond knowledge of probability to an understanding of the issues of teaching and learning probability. Ball (2000) expands this idea further to encourage teacher educators to be concerned with teachers' knowledge of mathematical content as well as how this knowledge must be expanded to include knowledge for teaching as they will use it in their daily practice (e.g., choosing tasks, highlighting students' responses during classroom discussions, creating appropriate assessment questions). The success of any probability curriculum for developing students' probabilistic reasoning depends greatly on the teachers' understanding of probability in addition to their deeper understanding of issues such as students' misconceptions (which are discussed in-depth in many other chapters in this volume) and the use of representations and tools (e.g., Pratt; Pfannkuch; this volume). Teachers also need a repertoire of tasks that can enhance non-deterministic thinking and connections to statistics.

Compared to the many chapters in this volume dedicated to students' understanding of probability, there has been significantly less research on teachers' knowledge *of* probability and their knowledge *for* teaching probability. The review of the research fits into four broad categories:

- teachers' beliefs and content knowledge of probability,
- teachers' understanding of students' understanding of probability,
- teachers' implementation of probability lessons, and
- teachers' use of simulation tools in their own learning and in their teaching.

Teacher's Beliefs and Content Knowledge of Probability

In a study of 22 practicing and 12 preservice elementary teachers, Begg and Edward (1999) found that teachers had a weak understanding of probability, with only about two-thirds understanding equally likely events and very few understanding the concept of independence. The teachers tended to believe that order or pattern would not be associated with random events and often used the representative heuristic assuming that every sample or series of outcomes needs to be representative of the expected population. For example, they judged an outcome of 1, 2, 3, 4, 5, 6, from a Lotto draw or HTHTH from a coin flip as less likely than events that appeared "more random" such as 2, 13, 19, 27, 30, 38, or TTHTH. These same teachers also reported a lower confidence in their ability to teach probability as compared to graphing and statistical calculations. In addition, these teachers were more concerned with getting ideas and activities for use in their classroom than increasing their own knowledge of probability and statistics concepts.

Carnell (1997) studied 13 preservice middle grades teachers' understanding of conditional probability. In accordance with the difficulties noted by Falk (1988) in relation to understanding conditional probability (e.g., defining the conditioning event, the temporal order of the conditioning event and the target event, and confusing conditionality as causality; see Batanero & Sanchez; Jones & Thornton; Tarr & Lannin; this volume), each of these preservice teachers demonstrated evidence of holding these misconceptions. In general, Carnell characterized some of their misconceptions by noting that they often used inferred events as the conditional event, disregarded conditional events that occurred after a target event in real time, used independence improperly, and inappropriately applied computational procedures for calculating probability. If teachers have the same misconceptions as their students, how can they develop appropriate lessons and tasks to facilitate students' understanding of conditional probability?

In the mid 1990's, Watson (2001) did a substantial amount of work developing a profile instrument to gather information about teachers' content and pedagogical content knowledge of probability and statistics. She administered the instrument as a survey to 15 primary (elementary) and 28 secondary teachers throughout Australia. Watson used teachers' answers to the questions on the profile instrument to look for patterns and to describe general characteristics of the teachers. There are several interesting points that arise from the part of her report relating to teachers' knowledge about probability.

When asked to choose a topic in chance or data and write a brief lesson plan, Watson (2001) reported that primary teachers most commonly chose topics such as surveys, graphing, general ideas of chance, and probability in a specific context (e.g., die tosses). Secondary teachers most commonly chose general ideas of chance with some choosing probability distributions. In addition, 53% of the teachers reported enjoying the same topics that they chose for their lesson. In general, the teachers were more familiar and comfortable with the concept of "average" than they were with the concept of "sample." This may be an indication of their unfamiliarity with the concept of sample or of its critical role in the study of probability and statistics. It may also be an indication of their preference for concepts with computational components. For example, teachers may feel they understand the concept of average since they can compute it. In contrast, the concept of sample may represent too much uncertainty for their comfort level. The teachers also reported a low confidence in their ability to teach "odds." In addition, Watson reported that the secondary teachers were significantly more confident than the primary teachers in their ability to teach equally likely outcomes, basic probability calculations, and sampling.

Several authors note the difficulty teachers have in understanding and applying the difference between deterministic reasoning under situations with certainty and non-deterministic reasoning in situations with uncertainty. Nicholson & Darnton (2003) argue that teachers with strong mathematical backgrounds but weak statistical understanding have a tendency to focus on "procedural aspects of calculating the correct answer" (p. 1) and are uncomfortable studying and teaching random processes that lead to inference and decision making. With regard to elementary level teachers, Pereira-Mendoza (2002) argues that teachers' mathematical experiences have a negative impact on their view of stochastical ideas and inhibit their development as teachers of stochastics. These points are certainly aligned with the issue discussed earlier of teachers employing a deterministic mindset that relies heavily on a classical approach to calculating probabilities a priori (before any trials are even done).

Gattuso and Pannone (2002) reported that secondary teachers (n=91) in Italy considered statistics worthwhile but that it took away from other aspects of the mathematics curriculum. This is not surprising, and teachers in other countries most likely have similar opinions. The Italian teachers also reported being insecure in how to best teach statistical topics, not because of their lack of statistics knowledge, but their lack of preparedness in appropriate pedagogy for teaching statistical ideas. Perhaps these teachers intuitively understand the points advocated by Ball (2000), Shulman (1986) and others, and recognize they need additional pedagogical content knowledge for teaching probability and statistics.

Teachers' Understanding of Students' Understanding of Probability

Knowing students' possible difficulties with a topic is an important aspect of pedagogical content knowledge. There is very little research on teachers' understanding of students' difficulties with probability.

Watson's (2001) instrument also assessed teachers' knowledge of difficulties students may have with probability and statistics. The teachers' low confidence in teaching the concept of "odds" was also demonstrated when they were asked to interpret "7:2" and give examples of student responses that would be appropriate and inappropriate. Some teachers admitted they did not know how to interpret this ratio. In addition, while most teachers could give examples of inappropriate responses students might give, only 15 high school teachers could give examples of appropriate responses. The inability of the majority of teachers to interpret correctly this ratio as odds and to transition between a part-part odds (7:2) interpretation and a part-whole probability (7/9 and 2/9) interpretation suggests an extreme disconnect in these teachers' understanding of fraction, ratio, proportion, and probability.

When reporting what their students have difficulty with, only two primary teachers in Watson's study mentioned "finding probabilities," while 13 secondary teachers mentioned students' difficulties with mostly procedural aspects (e.g., calculating probabilities, permutations, tree diagrams) and some conceptual aspects (e.g., theoretical probability, inference, conditional probability). Although these data suggest that the teachers can identify students' difficulties with probability, it also appears that the teachers are focusing primarily on procedural aspects of probability and may have a computational teaching approach to probability that focuses on classically derived single answers. Watson (2001) made the following observation:

> At the senior secondary level, where some teachers had previously taught theoretical aspects of probability and statistics and traditional programs were well documented, there was a recognition of the difficulty of the topics for students but little effort to introduce activity-based aspects, such as simulation or actual sampling, that would reinforce theory (p. 325).

In addition, although many types of activity-based lessons on chance were occurring in primary teachers' classrooms, there did not seem to be a coherent approach to the study of chance concepts. This is certainly consistent with teachers' lower confidence rating in teaching many probabilistic concepts and suggests a critical need for professional development.

Teachers' Implementation of Probability Lessons

Very little research has been reported on teachers' instructional practices as they implement probability lessons in their classrooms. Two studies, Steinbring (1991b) and Haller (1997) demonstrate the profound effect of teachers' probability knowledge on the complex process of teaching students to reason probabilistically.

Working in German classrooms, Steinbring (1991b) described overall patterns observed in several teaching episodes that focused on basic concepts of probability. In general, the teachers and students perform some chance experiments and discuss the results. There is an attempt to explain the outcomes of the experiment by a simple rule and differences between the theoretical expectation based on this rule and the actual experimental results are noted. In most cases, the differences are accounted for by "chance" as a prevailing factor in the experiment and the differences between theoretical and experimental probabilities. In other words, "chance" is used as a magical notion like "luck" to explain something that cannot be controlled. He discussed an example of a teaching episode in detail and noted that "the original openness of the classroom interaction is increasingly narrowed down in the course of teaching" (p. 512). He further noted that the teacher's actions in this episode may develop, for students and teacher, the concept of chance as the third relational vertex in a triangle where the "object" level of experimental results and the "symbol" level of calculating theoretical probabilities are the other vertices. In such a way, students who have this sort of understanding of chance may not ever question the validity or independence of events in an experiment with results that significantly differ from any theoretical model. It seems that the social context of the classroom as well as the teacher's understanding of both empirical and theoretical probability can contribute to such an oversimplification of the relationship between the two.

Haller (1997) conducted classroom observations of middle school teachers' probability lessons as part of a follow-up for four teachers who had participated in a summer institute on rational numbers. This institute included three days of instruction on probability and pedagogical issues on teaching probability, including common student misconceptions. The results from the classroom observations indicated that teachers who had a low-level content knowledge (based on pre- and post-tests during the institute) made content errors, demonstrated misconceptions in their lessons, depended highly on their textbooks, and missed opportunities to develop relationships between fractions, decimals, and percents within the context of probability. By way of contrast, teachers with higher content knowledge made no mathematical errors, made connections between decimals, fractions, and percents, and substantially supplemented the textbook with questions and activities. Teaching experience did not appear to have as great an impact on teachers' probability instruction as did their understanding of probability.

Teachers' Use of Simulation Tools

Although many have advocated the use of technology for teaching statistics and probability for over a decade (e.g., Biehler, 1991; Ben-Zvi, 2000; NCTM 2000; CBMS, 2001), little research has been done on how teachers make sense of probability concepts using such tools or how they implement such tools in their own classrooms. In her work with elementary preservice teachers, Dugdale (2001) observed that access to computer software allowed teachers to design a pair of die such that there was an equally likely chance of an even product and an odd product when the numbers on the rolled die were multiplied. The teachers were able to simulate a large number of trials, compute relative frequencies and convince themselves that they had created a fair game. Dugdale noted that the preservice teachers were able to use the software as a tool to foster discourse and develop insights into probability that often do not occur when a limited number of trials is performed with physical die. She also emphasized that the preservice teachers were not satisfied with merely observing relative frequencies from the computer simulation; they transitioned to reasoning about the theoretical probabilities to verify their computer-generated results.

High school teachers in Sanchez's (2002) study also gained better understanding about concepts such as variability and the usefulness of simulating probability situations. One caution that Sanchez noted was that the teachers did not seem to have an understanding of how to use the software tools effectively to foster students' conceptual understanding. The teachers believed computer simulations were useful after theoretically examining probability outcomes or physically simulating a situation. These practicing teachers did not appear to value the role of computer tools in helping analyze results of a simulation or prompting the validation of results through a theoretical model, as did the preservice elementary teachers in Dugdale's (2001) study. The high school teachers generally chose to simulate typical textbook problems; they ignored fundamental concepts of distribution and focused instead on frequencies.

Although the literature base on teachers' understanding and use of technology in their own probability learning and teaching is scarce, I am struck by the differences between the elementary and high school teachers' approaches to using computer tools to simulate chance events. I hypothesize that, whereas the high school teachers desired a more formal theoretical approach that was in accord with their strong mathematical background, the elementary preservice teachers were more open to an experimental approach within the two dice game context. For many high school teachers, teaching counting techniques, combinatorics, and theoretical probability is

> more mathematical than supervising students setting up a computer simulation or discussing the law of large numbers, sampling in election polls, or what it means when the weather forecaster says there is a 40% chance of rain (Scheaffer, Watkins, & Landwehr, 1998, p. 16)

A reliance on the theoretical nature of probability can certainly influence teachers' use of computer simulations and would help to explain why teachers in Sanchez's (2002) study viewed simulation as a process that follows a theoretical approach. That is, they viewed simulation as an appropriate way of comparing or confirming the previously determined theoretical value.

In a current study, Stohl (2004) is examining how 35 middle school teachers interpret students' interactions with a simulation tool (*Probability Explorer*, Stohl, 1999-2002). The teacher interpretations will then be compared to the research analysis of these students' work with the simulation tool (Stohl & Rider, 2003) to determine similarities or differences in the interpretations made. In this study, teachers first solve a task using the simulation tool and then watch and interpret three video examples of students solving the same task. Preliminary results of the teachers' analysis of student work indicate that the teachers are attuned to students' decisions about how to collect data using the simulation tools (e.g., determining sample size), and students' use of representations (e.g., bar graphs, pie graphs, and data tables) to make sense of empirical data. However, the teachers often miss subtleties in students' actions and language: subtleties that indicate students' developing understanding of meaningful probability ideas. Instead, they are often quick to criticize students' lack of understanding of formal probability ideas (e.g., theoretical probability and independence). Accordingly, the teachers do not seem to have a strong sense that the development of probabilistic ideas (e.g., the law of large numbers) is a complex process and is difficult to assess.

4. TEACHER DEVELOPMENT PROJECTS

The development of new curriculum materials, based on the reform efforts of organizations such as the NCTM (1989, 2000), has increased the need for education of teachers who can effectively implement probability and statistics lessons in their classrooms. As new curriculum materials are placed in schools that have an increased emphasis on probability, teachers need opportunities to increase their content and pedagogical content knowledge. Haller (1997) advocates that professional development on the use of new curricula should "involve opportunities to critically analyze the text[book] for opportunities to capitalize on situations presented within instructional units as well as potential student questions" (p. 200). Teachers' abilities to capitalize on tasks posed in curricula are dependent on the robust nature of their knowledge of both probability and the teaching of probability.

Although many professional development and teacher education materials have been developed and implemented in the past 15 years, there is little research on the effect of these programs on teachers' knowledge and classroom practices vis-à-vis probability. Many programs have focused on teachers' understanding of statistics and data analysis but there has been little focus on probability (e.g., Teach-Stat, Friel & Joyner, 1997; Chance and Data for Luddites, Watson, 1997; Alabama Quantitative Literacy Workshop; Yarbrough, Daane, & Vessel, 1998). Moreover, only a few of the programs have produced research results concerning teacher's knowledge of statistical ideas (e.g., Friel & Bright, 1993; McClain, 2002a, 2002b).

In reviewing the research program by McClain, one finding is useful to consider in the context of probability. McClain (2002b) found that teachers applied more sophisticated understandings when analyzing univariate data sets than those typically drawn upon by their students. Thus, teachers may approach statistics and probability tasks significantly differently than their students. If not made aware of this difference, teachers may be led to believe that students should use more advanced understandings with a particular task: ones that closely resemble the teacher's perspective on the task. If teachers take the same task they learned within a professional development setting, and implement the task with students in their classroom, the teachers may assume a perception-based perspective (Heinz et al, 2000) without the pedagogical content knowledge to critically think about how students will approach the task. Consequently, teachers may make pedagogical decisions based on their own understanding of the task rather than their students' current understandings.

Yarbrough, Daane, and Vessel (1998) report that elementary teachers in the Alabama Quantitative Literacy Workshop showed evidence that their workshop experiences had a positive effect on their instructional practices. The workshop was conducted in 9 days over a 10-month period and included 60 elementary and secondary teachers. The goal of the workshop was to provide hands-on instruction in probability and statistics that would increase teachers' knowledge as well as expand their repertoire of instructional strategies. The timing of the sessions also allowed for concentrated time (5 days) over a summer and several follow-up sessions during the school year. Classroom observations of 10 of the elementary teachers and follow-up surveys and interviews showed that teachers implemented several of the workshop activities in their classroom and were able to adapt easily the activities according to the grade level of the students they taught. The teachers also implemented probability and statistical concepts across the curriculum rather than in an isolated unit.

Discourse patterns in the observed classrooms included a focus on students' reasoning about data and multiple representations, and on making sense of probability and experimental results. Although there were a few observed incidents of mathematical errors during class discussions and on teacher-produced handouts, the teachers reported an improved self-confidence in teaching probability and statistics.

Haller (1997) reported on middle grades teachers' growth in probability knowledge as they participated in the 4-week Rational Number Project Middle Grades Teacher Enhancement summer institute. A major goal of the institute was to prepare teachers to be able to use NSF-sponsored (National Science Foundation) middle grades curricula that had been adopted in schools. In fulfilling this goal the institute included a concurrent focus on content and pedagogical content knowledge. Three days of this institute were focused on probability. Pre-assessment results indicated that most teachers did not possess the probability knowledge that was required to answer the questions they and their students would encounter in the NSF-sponsored curricula. Only about one-third of the teachers could accurately calculate multi-stage probabilities and they exhibited typical misconceptions related to small sample sizes, representativeness, and negative-recency effects. Post-instructional assessment given immediately following the three days of instruction indicated growth in teachers' probability knowledge as well as an increased confidence in their knowledge.

As a follow-up to the summer institute, Haller (1997) chose a subgroup of four teachers whom she observed while they were teaching probability. Even though 30 teachers had attended the summer institute, Haller had difficulty sampling teachers who would be actually teaching probability *and* were confident enough to allow videotaping in their classroom. A few teachers commented that they would teach probability only if there was enough time at the end of the school year—an all too common phenomenon. Recall that Haller found teachers' content knowledge, not their teaching experience, to be a critical aspect in their ability to effectively teach probability, capitalize on students' responses in class discussions, and maximize the curricula materials they had available.

The 35 teachers in Stohl's (2004) study took a graduate level course on teaching and learning data and probability in the middle school as part of the North Carolina Middle Mathematics Project. In this course, teachers videotaped a classroom episode when they were teaching a data or probability topic. A sample of these teachers will be studied to examine how they implement the use of simulation tools (both hands-on manipulatives and technological tools) in their classroom teaching of probability. These results will complement the work by Haller (1997) and add to the knowledge base of teachers' pedagogical content knowledge and their instructional practices with technology tools.

All the teachers in Haller's (1997) study agreed that "probability is hard to teach" (p. 204). In taking this position, the teachers provided a litany of difficulties: students misunderstand many concepts, there is an unpredictability of activity outcomes from any randomly generated experiment, and teachers have difficulty assessing whether students learn concepts simply from doing hands-on activities. These teachers' concerns about teaching probability echo some of the very core issues discussed at the beginning of this chapter—unpredictability is uncomfortable for teachers and hands-on activities are not viewed as being as "mathematical" as a more theoretical approach. Both of these views may stem from teachers' beliefs that include fundamentally deterministic views, a computational orientation to mathematics in general, and more specifically, a classical approach to probability. The concern about whether students learn from hands-on activities also seems related to the perception-based orientation that Heinz et al (2000) attribute to many teachers who are in transition from traditional to reform approaches in mathematics instruction.

5. SUGGESTIONS FOR TEACHER EDUCATION

The central issues discussed at the beginning of this chapter helped frame the lens for reporting the various research results related to probability in teacher education. In making suggestions for the future preparation of teachers of probability, we obviously need to consider teachers' pedagogical content knowledge (Shulman, 1986) as well as content knowledge specific to teaching and learning probability. First and foremost, teachers' content knowledge of probability is critical, as seen by research results from several studies in this chapter (e.g., Haller, 1997; Watson, 2001).

Improvement in the teaching of probability needs to include teachers simultaneously developing their own understandings in stochastics (including a deep understanding of the law of large numbers) and reflecting about the deterministic and non-deterministic nature of our world as it applies in various contexts (Lopes & de Moura, 2002). Kvatinsky and Even (2002) proposed a framework for teachers' subject matter knowledge of probability. This framework extends Even's (1990) general framework for mathematics teachers' subject matter knowledge that has been applied to other content domains. The Kvatinsky and Even (2002) framework includes seven aspects of subject matter knowledge teachers should have for probability:

- The essential features of probability as a non-deterministic phenomenon, the classical and frequentist approaches to probability, and the subjective approach where probability is interpreted as strength of judgment.
- The strength of probability as an integral part of natural phenomena where new fields such as quantum physics have emerged from a probabilistic perspective on our world.
- How to use and interpret different representations and models (e.g., Venn diagrams, tree diagrams) for computing or interpreting probability.
- How and when to use alternative ways of approaching probability (i.e., the classical or the frequentist approaches).
- A basic repertoire of examples that can be used for certain concepts (e.g., examining consecutive outcomes of rolling a die to discuss independence).
- Different forms of knowledge and understanding so one can distinguish between intuitive knowledge and formal theoretical probability; especially knowing that intuitive knowledge may lead one astray in probability.
- Which aspects about mathematics are supporting and withholding in probability knowledge (e.g., axiomatic theorems in probability such as probabilities of events in a sample space summing to 1, the central issue of the law of large numbers being a limit of a probability, instead of the limit of a point estimate).

Organizations such as the Conference Board of Mathematical Sciences (CBMS, 2001) have further elaborated the specific content knowledge that should be learned by teachers at elementary, middle, and high school levels. Taken together, the aspects of Kvatinsky and Even (2002), the CBMS (2001), and the central issues emphasized earlier in this chapter, should inform the content of courses developed for preservice and practicing teachers.

The pedagogical content knowledge needed for appropriately planning and implementing probability lessons is just as complex as the study of probability itself and cannot be left out of teachers' education. Teachers need to understand how to use different representations and tools (e.g., technology, manipulatives) to help students collect and analyze data from experiments and to know the properties of different representations that make a concept salient (e.g., we may not want to simplify a fraction describing a probability if the numerator and denominator have specific meanings connected to the context). Teachers should also be able to use various examples from research on students' misconceptions as the starting place for the creation of student tasks and for classroom discussions. And more importantly, teachers need to know the common student intuitions related to those examples and be able to craft activities to get students to experience an effect that may cause them to question their initial intuition. However, as emphasized in the earlier part of this chapter, teachers must develop their own understanding of the complexities of probability concepts (e.g., law of large numbers) and resist falling back on a deterministic mindset to craft activities that emphasize only a classical and single-answer approach to probability.

Part of the call from Shaughnessy (1992) was for more research on teachers' knowledge of probability that could inform professional development and teacher education. I am not convinced that we have enough knowledge about teachers' content or pedagogical content knowledge to convincingly design the most effective educational opportunities for them. I do know, however, that our current efforts in teacher education are not sufficient. I echo the need for more research, especially research related to the effectiveness of any newly developed teacher education or professional development efforts. Teacher educators should take what is known and make careful decisions about the creation of educational opportunities for teachers. Moreover they should critically examine the effects of these developmental opportunities on teachers' knowledge and instructional practices.

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