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AGENCY AND CREATIVITY IN THE SEMIOTICS OF LEARNING MATHEMATICS

Abstract. Semiotics provides a way of conceptualising the teaching and learning of mathematics driven by a primary focus on signs and sign use. It considers patterns of sign use and production, and the contexts and social rules underlying sign use. It attends to agency in the learner's personal appropriation of signs and the meaning structures embodying the relationships between signs. Learner agency is manifested in communicative activity involving sign 'reception' (listening, reading) and sign production (speaking, writing, sketching). It is most marked in individual creativity in sign use, which is manifested at all levels in schooling and in the activities of the working mathematician.

Key words: Semiotics, semiotic systems, mathematical activity, agency, creativity, teaching and learning of mathematics, sign transformations, appropriation, conventionalization .

INTRODUCTION

A semiotic perspective of mathematical activity provides a way of conceptualising the teaching and learning of mathematics driven by a primary focus on signs and sign use. In providing this perspective it offers an alternative to any psychological perspective that focuses exclusively on mental structures and functions. It also rejects any straightforwardly assessment or performance focussed perspective concerned only with student behaviours. Instead it offers a novel synthesis that encompasses but also transcends these two types of perspective, driven by a primary focus on signs and sign use in mathematics. Beyond the traditional psychological focus on mental structures and functions it considers the personal appropriation of signs and the underlying meaning structures embodying relationships between signs. Beyond behavioural performance it is concerned with patterns of sign use and production, including individual creativity in sign use, and the underlying social rules and contexts of sign use. Thus a semiotic approach draws together the individual and social dimensions of mathematical activity which are understood as mutually dependent and constitutive aspects of the teaching and learning of mathematics.

The primary focus in a semiotic perspective is on communicative activity in mathematics utilising signs. This involves both sign 'reception' and comprehension via listening and reading, and sign production via speaking and writing or sketching. While these are conceptually distinct, in actualisation these two activities overlap and are mutually shaping in conversations (semiotic exchanges between persons within a social context). Sign production or utterance is primarily an agentic and often a creative act. For the speaker has to choose and construct texts to utter on the

M. H. G. Hoffmann, J. Lenhard, F. Seeger (Eds.), Activity and Sign – Grounding Mathematics Education, 23 – 34.

basis of their appropriated and learned repertoire of signs. In so doing, speakers are taking risks in exposing themselves to external correction and evaluation against the rules of appropriate utterances. 'Text' denotes more than a piece of writing here. As is widespread in semiotics, it is a compound sign made up of constituent signs, and can be uttered or offered in a conversation in many ways. It may be spoken, written, drawn, represented electronically and may include gestures, letters, mathematical symbols, diagrams, tables, etc., or some combination.

Texts, signs and their use need to be understood as part of more complex systems. First of all, sign use is always socially located and is a part of social and historical practice. In Wittgensteinian (1953) terms sign use comprises 'language games' embedded in social 'forms of life' (Ernest 1998). Second, signs are never used individually. Signs are always manifested as part of semiotic systems, with reference implicitly or explicitly, to other signs. The term semiotic system is used here to comprise the following three components:

1. A set of signs, the tokens of which might possibly be uttered, spoken, written, drawn, or encoded electronically.
2. A set of relationships between these signs based on an underlying meaning structure (or structures) embodying these relationships,
3. A set of rules of sign production, for producing or uttering both atomic (single) and molecular (compound) signs. (These rules are in most cases implicit, acquired by 'case law'.)

The social and historical embedding of semiotic systems concerns both their structural dimension (Saussure's *langue*) and in their functional role (Saussure's *parole*). These dimensions, while theoretical separable, are woven together in historico-social practice. The evolution of semiotic systems can be examined historically in terms both of these dimensions. Such developmental processes result in knowledge systems, such as school mathematics, that provides the underlying structure to the planned learning environments for students. However, just as semiotic systems change and develop over history, so too the semiotic systems mastered by learners develop and change over the course of their learning careers, becoming more elaborated and providing the basis for more complex and abstract systems. Mastering these enlarging semiotic knowledge systems constitutes learning. This feature is also a basis for a some learning difficulties, for the semiotic systems mastered by learners are never static. As they near mastery of a particular system, the teacher extends the system with new signs, relationships, rules or applications. For example, for a young child mastering elementary calculation $3 - 4$ is impossible. But later $3 - 4 = -1$. Similarly 3 divided by 4 (3/4) is at first impossible. Later it is not only possible but ¾ names the answer. These, together with more complex changes in the rules that occur (are imposed) as semiotic systems are extended, and the problems they cause, have been named epistemological obstacles (Bachelard 1951, Sierpinska 1987). Thus a structural view of semiotic systems can provide only a freeze-frame picture of a growing and life-like entity. Indeed in practice it is difficult to clearly distinguish and demarcate the range of semiotic systems encountered in school mathematics because of their growth and their mutually constitutive inter-relationships.

Successful mathematical activity in school requires at least partial mastery of some of the semiotic systems involved in schooling at the appropriate level. A number of different but interrelated and overlapping semiotic systems are important in learning, and mastery of the following systems usually constitute significant stages in learning school mathematics the way it is currently organised: Numbers and counting; Numerical computation, Fractions (rational numbers) and their Operations, Elementary linear algebra (solving equations), Analysis (calculus) and Abstract (axiomatic) group theory. Clearly, the semiotic systems chosen from university mathematics are more arbitrary than those chosen from the earlier years of schooling, in the sense that some university students could study mathematics but not analysis or abstract groups. The topic areas could be identified differently, but nevertheless they constitute a central part of taught mathematics straddling the years from kindergarten to university study. Naturally, there are further overlapping semiotic systems in school mathematics learned in parallel with these (e. g., geometry and probability) and even from this perspective the mathematics curriculum could be 'cut up' into different semiotic systems.

Semiotic systems are incorporated in all human communicative activities, and are inextricably woven into the fabric of all social activities and institutions. So the question can be posed: what is unique about their nature and deployment within school mathematics? A number of mathematically specific systems (topic areas) with dedicated sign systems, meanings and rules of use are mentioned above. But more than this, I want to suggest that there is an underlying characteristic shared by most if not all semiotic systems in school mathematics and more widely, by mathematics itself. In brief, my claim is that these systems are fundamentally sequential and procedural. In a sense this is an empty or superficial description, because at the heart of mathematics are its meanings, its purpose as a device for meaning-making, and this is driven by its social and human aims and context. But to treat these further issues in addition to its means of signification requires an in-depth discussion of historical and philosophical issues that are not only too complex and elaborate for the space here, but which are also clouded by centuries of metaphysical and ideological preconceptions about mathematics. However, the view of mathematical signs as sequential and procedural in nature of helps explain a well-known pathological outcome of education in which learners only appropriate surface characteristics without managing to transform them into part of a larger system of personal meanings.

My claim is that texts in the semiotic systems of mathematics are representative of sequences of actions (physical or textual), and the signs stand for steps (the individual results of procedures), actions on these steps (the procedures themselves), sequences of steps linked by procedures, and collections of these entities. My description includes the so-called entities involved themselves, whereas in mathematics and school mathematics we have almost nothing but the signs that stand for these steps, procedures and collections. Physical actions (such as enumerating a sequence of ordinals in counting a collection of tangible objects: 1, 2, 3, 4, 5, ...) which have an extended temporal existence become rapidly replaced by spatially extended sequences of signs, which themselves can become embodied into truncated 'super'-signs (cardinal numbers in the example). Such a replacement of process signs by product signs in mathematics (the reification of constructions) is discussed in the philosophy of mathematics (Machover 1983, Davis 1974, Ernest 1998), and in mathematics education (Dubinsky 1988, Ernest 1991, Sfard 1993). In linguistics, there is a well known parallel in the process of nominalisation, in which verbs designating actions and activities are transmuted into nouns, which representing the names of entities (Chomsky 1965). What is unique in mathematics is the great height to which these towers of abstraction rise, with each level reifying actions on lower level entities and processes into new entities. My claim is that all that there is (above the very basic ground floor of physical actions) is signs or names, and actions upon them.

The claim that mathematics is fundamentally procedural is lent some support by the philosophical position of intuitionism, which regards the objects and sentences of mathematics as representing constructions (Troelstra and van Dalen 1988, Heyting 1956). Although the intuitionist philosophy only has a minority of mathematicians as adherents, one of its achievements has been to translate a very significant part of mathematics including the content of all elementary (i. e., school) mathematics and much of advanced mathematics into transparently constructive (i. e. procedural) form Bishop (1967). Of course the Intuitionists do not accept that virtually all is signs or actions on them, for they posit some transcendent subjective (but universal) domain of meanings. Supporting, and in large part inspiring my account, Rotman's (1993) semiotic theory of mathematics also interprets mathematical inscriptions as recipes, instructions, or claims about the outcomes of procedures, without the need to posit entities beyond our social and cultural constructions.

What I am claiming (fully aware of the ontological implications) is that the so-called objects of mathematics are themselves the products of sequential actions and procedures. However, the tendentious nature of this statement is neutralised by the adoption of a semiotic perspective, rather than a philosophical one (for the moment). For my universe of discourse here is populated primarily by signs (and the persons who use them) rather than any abstract objects of mathematics.

A valuable feature of semiotics is that it is neutral towards representationalism. No assumption need be made that a sign must mirror the world or some mathematical reality. Semiotics regards signs, symbols, texts and all of language as constitutively public. However, meanings and imagery can be and are appropriated, elaborated and created by individuals and groups as they adopt, develop and invent sign-uses in the contexts of teaching, learning, doing and reflecting on mathematics, and all of the other important activities of life. Thus semiotics rejects the simple subjective/objective dichotomy that consigns mathematical knowledge to 'in here' or 'up there.' It provides a liberating perspective from which to study mathematics and education. It opens a new avenue of access to the concepts that have been developed for mathematics education in the social sciences and the other sciences, including psychology, but it also allows access to the intellectual resources and methods of the arts and humanities.

AGENCY AND CREATIVITY

Learners are human beings with all the complexity and moral aspects this involves. Human beings are constitutively social beings and this entails a widespread range of capacities concerning interpretation and sense-making in social or interpersonal situations. Focussing on the classroom, learners understand in their own ways the roles and asymmetric power relations of the teacher-student relationship, the aims and purposes of school mathematical activity and tasks (both espoused and enacted, where these differ[1]), and many other relevant aspects of micro-social context. Into the shifting and multifaceted context of the classroom learners brings their own historically formed subjectivity, sense of self, and capacities for meaning making. Acknowledging this formative background, the features I wish to focus on here are the central ones of agency and creativity.

Agency is the central capacity all human beings have for initiating (and continuing) activities, including the possibility of inaction. In focussing on learner agency I am not assuming that students or persons in general are rational beings making rational choices. All sorts of psychological factors can drive choices and behaviour, but this is irrelevant to the present discussion. In learning mathematics, the activities involved are primarily communicative involving mathematical sign systems, notably sign 'reception' (listening, reading) and sign production (speaking, writing, sketching). Creativity in such activities or conversations may be conceptualised as the ultimate expression of agency. In a minimal sense, almost any semiotic sign production can be classified as creative, because it involves first making a selection from the semiotic repertoire available, which includes signs and modes of expression, and then putting together and making a new public utterance. In practice the selection, combination and utterance of signs may very well be woven inseparably into a single action. By definition, any sign utterance is new because of its unique temporal and contextual location in conversation. However such usage trivialises the term creativity through making it universally applicable. By analogy with problem solving (a significant analogy, especially in the domain of mathematics) routine utterances can be distinguished from non-routine utterances. In the latter, semiotic elements (including the context) are combined in a novel and non-routinised way in the utterance. It is cases like this that are better characterised as creative.

Manifestations of agency in sign system usage are understood here, based here on a Wittgensteinian (1953) perspective, participation in *language games* embedded within social *forms of life*. Thus communicative activity involving mathematical sign systems is always encompassed within the social. Furthermore, the component activities of sign reception and production involved in language games are woven together within the larger epistemological unit of *conversation* (Ernest 1991, 1994, 1998). The way in which these two activities are mutually shaping in is shown in the model (Figure 1) of sign appropriation (reception) and sign use (production).

Figure 1 is based on Harré's (1983) model of 'Vygotskian space', previously applied to mathematics in Ernest (1998).[2] Evidently it embodies the well known dictum of Vygotsky 1978, 128)

> Every function in the child's cultural development appears twice, on two levels. First, on the social and later on the psychological level; first between people as an interpsychological category, and then inside the child as an intrapsychological category.

In the figure these two levels are represented, at least in part, first by the top left corner, for the socio-cultural is both public and collective, and secondly, by the bottom right corner, for the (intra)psychological is both individual and private. The other two corners are crossing points on the boundary between the two levels, and these are the locations where learner semiotic agency is acted out.

SOCIAL LOCATION

MANIFESTATION	Individual		Collective
Collective	Learner's public utilisation of sign to express personal meaning (Public & Individual)	*Conventionalization* →	Social (teacher & others) negotiated and conventionalised (via critical acceptance) sign use (Public & Collective)
	Publication ↑		↓ *Appropriation*
Private	Learner's development of personal meanings for sign and its use (Private & Individual)	← *Transformation*	Learner's own unreflective response to and imitative use of new sign utterance (Private & Collective)

Figure 1. Model of Sign Appropriation and Use

Following the processes in the model, signs and sign systems become adopted by the individual learner first in the process of *appropriation*. This leads to the learner's own unreflective response to and imitative use of a single sign, be it atomic or compound, or of a set of sign utterances. The learner has thus appropriated a collective sign into something for herself that is private. This is also the route by means of which learners appropriate the rules of sign-use, mostly through observing their exemplification in practise. Agency is manifested in several ways at this stage, including attending to the public sign utterance, becoming aware, to a greater or lesser extent, of the immediate context and associations of the sign use, and using the sign in an imitative way. The privately initiated uses of the sign, albeit possibly in response to another's request or command, are a public manifestation of learner agency. In such use the whole cycle is brought into play in miniature, because the sign as utilised in a personal performance is manifested publicly, and would normally be subject to social acceptance or correction (conventionalization). Such use corresponds in great part to Skemp (1976) and Mellin-Olsen's (1981) notion of *instrumentalism*, because of the simple imitative performativity involved. I avoid the term 'instrumental understanding' here, because of the commonly associated ideological assumption that locates knowledge and understanding 'inside' the private minds of individuals rather than as primarily manifested in public performances (which can also be rehearsed in private thought). Through the conventionalization of performance (applied to sign utterances) at this stage the learner also can become aware of restraints and restrictions applying to sign use, that is some of the rules of sign production that constitute part of the overall sign system.

When the next stage is achieved for a particular sign, which may follow a whole sequence of related appropriations, performances and conventionalization s in the mini-cycle described above, the learner will usually develop personal meanings for the sign and its use. This transforms it into something that is individual as well as private, because of the personal meanings associated with the sign. This will typically include a whole nexus of associations including a sense of where and how the sign is to be used acceptably. Such associations are primarily tacit, manifested in usage, but can include rationalisations and explanations about the limits, nature and purposes of sign usage. These may be appropriated from teacher and peer explanations prior to transformation into the meaning nexus, very likely tested and corrected by further mini-cycles involving publication and conventionalization . The successful appropriation and transformation of a sign, with its nexus of associated meanings and meta-discourse, finds a parallel Skemp's (1976) notion of 'relational understanding' in mathematics. This involves not only being able to use the sign correctly, that is, mostly corresponding to conventionally accepted usage within the micro-community of the classroom under the authority of the teacher, but also being able to offer a rationale or explanation for the usage. It may be inappropriate to describe the transformational process in which a meaning nexus is elaborated privately by the individual as manifestation of agency, as many of the processes are unconscious and involuntary. However the attention, persistence, and repeated performances in both sign utterances and explanatory meta-discourse evidently are manifestations of agency.

The third phase illustrated in Figure 1 is that of *publication*. In this process the individual learner engages in a conversational act in publicly performing or making a sign utterance. Mathematically this could vary from a quick, spontaneous verbal, gestural or written response to a question or other stimulus, through to constructing an extended text elaborated and revised over a period of time, prior to offering it to others. A group of learners can elaborate such a text co-operatively, but this process will have subsumed many sub-cycles in which individuals have communicated (offered signs) to others in the group in an extended conversation giving rise to a jointly elaborated, negotiated and agreed text.

It is in the publication stage of the overall cycle that agency is manifested most evidently and clearly. For the individual must initiate and produce a public sign utterance. At the simplest level this is an act of participation or even will, mediated through semiotic and social capabilities. More complex sign productions and utterances involve an elaborate series of meaning-attentive and meaning-driven voluntary actions. Agency is involved in interpreting the context and in choosing the mode, type and particular sign response and in making it. However, many psychological and social factors can inhibit, distort or enhance this performance, including such things as the learners self-confidence, perception of the surrounding others, classroom climate and so on.

Finally, the overall cycle is completed through the process of *conventionalization*. In this phase learner sign productions having been fed into the social milieu (the classroom conversation) are subjected to attention, critique, negotiation, reformulation and acceptance, or sometimes rejection, by the teacher and others. The outcome is an agreed or imposed conventionalization which is both public and collective. Because of the power and authority asymmetry in the classroom (and indeed in virtually all interpersonal contexts, but especially in socially sanctioned learner-teacher relationships) teacher approval will normally be the final arbiter of acceptance, rather than majority or learner agreement. Typically the conventionalized sign that is accepted will need to satisfy the following criteria.

1. **Relevance**. The sign or text is perceived to be a relevant response or putative solution (or possibly an intermediary stage to one) to a recognized (i. e., sanctioned) starting sign which has the role of a task, question or exercise. This might be teacher imposed or otherwise shared and authorized.
2. **Justification**. The mode of and steps in the derivation of the sign from the authorised 'starting point' will normally be exhibited as a semiotic transformation of signs, that is employing accepted or acceptable rules or means of sign transformations within the semiotic system, or justified meta-linguistically.[3]
3. **Form**. Both the signs and their transformations (where offered) will normally exhibit teacher-acceptable form, thus conforming to the rhetoric of the semiotic system involved as realized and defined in that classroom. This system could be that of spoken verbal comments, drawn and labeled diagrams, numerical calculations, algebraic derivations, or some combination of these or other sign types.[4]

These criteria primarily apply at the object language level, that is they directly concern mathematical tasks or contents. However they can also be applied meta-linguistically as comments on rather than as additions to object language level utterances in the classroom conversation.

If the public sign utterance deviates in relevance, justification or form a central aspect of the conventionalization stage will be the criticism, rejection or correction of the sign for its lack of acceptability in these dimensions. Such a process may involve 'degoaling', i. e., switching to a new goal, target or task (Hughes 1986) which could be intended to serve as an intermediate step towards the original goal, or which might be a shift in the discourse to a new subject matter. Conversation, even in its formal and controlled manifestation as it occurs in the mathematics classroom can be fluid and shifting in its actualisation, just as it can be rigid and one-sided. It can be 'live' in which near spontaneous verbal responses as well as other modes of response are sought and encouraged by the teacher and expressed by learners, or it can be highly formalised and regulated with the teacher directing attention to written tasks and requiring (and allowing) only formal written responses to them at determined moments.

The process of conventionalization is the stage in the cycle that is most public. For it often acts on a sign uttered or presented by the learner and involves the critical acceptance, correction or rejection of the sign. This is where the teacher's agency is at work, directed at the capabilities involved in skilled sign production. Indeed, the teacher may initiate the semiotic cycle at this point by introducing her own sign or text. (Mostly this will refer to previously introduced signs and conversations, but it may have a variety of functions beyond task setting, including explanation or scene setting to aid learners in the creation of meaning.) Skemp (1979) has described a central aspect of the teachers' aim as being the development of *logical understand-*

ing in the learner, to cap the instrumental (performance orientated) and relational (meaning elaboration and justification production) capabilities. The learner manifests logical understanding in this sense through being able to utilise and produce signs using the correct mode of expression and 'grammatical form', thus demonstrating a growing mastery of relevant aspects of the rhetoric of school mathematics. Through participation in and experience of conventionalization the learner first appropriates and then transforms into a personal aspect of her individual agency the capability of a critical and corrective perspective on signs. This involves not only the ability to produce signs in accordance with the (growing) set of rules of sign production manifested in the classroom, but also the capability to critically review and correct signs to conform to these rules. Ultimately the successful learner develops and adopts the aspect of agency corresponding to the role of the critic; the ability to make judgements concerning the correctness of sign utterances (with respect to relevance, justification or form) as is appropriate to the context. This involves the appropriation of a social role, a mode of 'voice', first experienced in the actions of others in conversation.

Traditionally in linguistic research two modes of sign usage are distinguished: listening/reading and speaking/writing. From the perspective of mathematical learner agency we might also distinguish two levels of functioning: lower level (responsive) and higher level (autonomous). Lower level functioning involves responding to signs or texts 'literally'. In listening/reading in school mathematics this means taking the signs as simply presenting routine tasks or instructions, or less commonly, as informational. In speaking/writing this usually involves simply offering an utterance in a response to some semiotic stimulus (spoken or written) delimited by the perceived constraints of the social context of utterance. In mathematics typically this involves simply performing a routine task. This usually necessitates applying one or more semiotic transformations to a sign, resulting in a sequence of signs (e. g., counting vocally or subvocally, performing column addition, solving a linear equation) resulting in a terminal sign, the 'answer'. Underpinning this is the ability to make sense of mathematical signs and texts, to interpret them as tasks and to apprehend their object, purpose and goals, within a variety of contexts, most notably, in the school context. Where these abilities are lacking or not fully developed it is the role of conversations directed by the teacher or more capable others, following the model in Figure 1, to further develop them.

Higher level or autonomous functioning means responding to signs in a more reflective way. In listening/reading this means spending time and making more effort to explore and create meanings for signs and also engaging in self-monitoring and self-reflection in the process. As the term reflection suggests, this involves elements of inward or self-directed dialogue. The metaphor of examining one's image in a mirror suggests stepping outside oneself and viewing oneself from the perspective of another, adopting an outsider's viewpoint. In dialogue, a person can adopt two opposite roles. First there is the role of proponent (or friendly listener) presenting (or following) sympathetically a text, a line of uttered or privately rehearsed argument or thought experiment, for exploratory or understanding purposes (Peirce 1931 – 58, Rotman 1993). By 'sympathetic' I mean adopting the point of view of the proponent or utterer and attempting to construct and enter into the sense of the utterance as it is (understood to be) intended. This is attempting to 'share' the constructor's meaning, rather than looking for grounds on which to dismiss it for failure of relevance, justification or form (this, taken to extremes, can pre-empt fully developed and elaborated sense-making). However, the role of proponent is not intrinsically reflective or higher order, for it can also be adopted at a lower, passively attentive level.

Secondly, there is the role of critic, in which a text, a sequence of signs, which could be an argument, a mathematical derivation, and so forth, is examined for weaknesses and flaws. This involves having appropriated and transformed into personal capabilities at least some of the context-specific criteria of acceptability manifested by others (primarily the teacher). These criteria typically pertain to the relevance, justifiability or rhetorical form of the text or sign utterance in question, and are meta-linguistic criteria when made explicit. Being able to adopt the role of critic to apply to others' or one's own texts is an intrinsically reflective and higher order capacity. It cannot be done meaningfully in an automatic or thoughtless way. This fits with the tradition in educational psychology that classifies evaluation, defined as making judgements using internal (i. e., textual) evidence and external criteria, as belonging to the highest level of intellectual functioning (Bloom 1956). It also evidently encompasses a dimension of agency since it constitutes the adoption of a specific agentic role.

In speaking/writing, higher level or autonomous functioning means constructing and elaborating signs or texts in a thoughtful and reflective way. Typically in school mathematics this involves the transformation of tasks presented as mathematical texts into further more manageable representations and in doing so applying a variety of textual and symbolic transformations to representations and their parts to complete the tasks. Different modes of representation can be employed singly or together in a school mathematics text, including any combination of symbols, written language, labelled diagrams, tables, sketches, models and arrayed objects (and even gestures where the text is spoken). It is common in school mathematics for problem solution processes to use more modes of representation than the starting text (task), or the final text (answer). The procedures of problem solving include the active processes of imagining, writing, drawing or making sequences of representations (not necessarily either monotonic or single branched sequences) progressing from the initial text (given task) to a final (in terms of fulfilling task demands) and permissible (derived by allowed transformations), often simple, textual representation (the potential task 'solution'). To carry through a multi-step process of this type successfully requires the student to be attentive to and in control of the purpose, direction and outcomes of subsidiary procedures and transformations. Where the construction and concatenation of the sequence of semiotic actions deployed is not automatic, that is has not been practised on similar tasks until it has become routinised for this particular student, it is appropriate to call it creative. It corresponds to non-routine problem solving and involves the student or person in constructing and combining in novel ways (new to herself, at least) different signs and procedures.

Carrying out tasks individually or in groups may be the most common higher level activity in speaking/writing in school mathematics. However, other activities can also occur such as the students writing mathematical questions and tasks, or posing mathematical problems themselves, with some sense of what the solution processes entail. Either way, speaking/writing at this level involves the most obvious and explicit manifestation of learner agency, since the activities are internally initiated and conducted. They are, of course, also texts uttered in response to antecedent texts in a conversation; but then so is all semiotic and communicative activity. Once again, the higher level agentic functioning involved in writing questions and tasks, and posing problems in the mathematical classroom is creative activity since it involves the construction of imaginative new texts.

Studies comparing novice and expert problem solvers in mathematics have shown that the latter successfully combine (and alternate between) the two higher level roles distinguished above, namely proponent and critic. Schoenfeld (1992), for example, found that novices typically spent most of their time in aimless exploration of problems, seeking to solve without any conscious design. This can be valuable for enriching understanding, but when persisted in, as in the study, it usually led to failure. The expert problem solvers and mathematicians cycled through a variety of activities directed at the problem, including reading, analysing, exploring, planning, implementing, and verifying. Furthermore, they repeatedly asked self-directed questions, typically at the points of transition between the different types of activity. These were higher level, critical and self-regulative questions asking what was being sought, what was being found, etc. This illustrates how higher level creative activity in mathematics needs to combine the roles of proponent and critic in an internalised, self-directed dialogue. Thus following the model shown in Figure 1 it is not just signs that become appropriated by persons, but the whole cyclic conversational process ultimately must become internalised for high level creative activity in school mathematics and in mathematics itself.

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NOTES

[1] I make this distinction, because as is well known the overt purpose of a classroom mathematical task and what the students come to learn is the teacher's actual focus of attention or emphasized outcome may differ (e. g., working an exercise vs. writing its solution in a certain style).

[2] In Ernest (1998) I utilize this model explicitly to account for the acquisition of language, mathematics and mental powers by young learners, as well as using a parallel model for the creation of shared mathematical knowledge in and by the mathematics research community. However, I view this model as showing the interplay between public vs. private and collective vs. individual in the role and meanings attributed to signs and texts (as well as in the construction of signs and texts) in conversation in general. This has particular relevance to the years of formal schooling, which I focus on here.

[3] Sign transformations do not always mean the replacement of just one (or more) part(s) of a compound sign by another part(s), with the retention of the unreplaced parts. It may involve replacement of the whole sign complex by another. For example, in a logical proof (a classic transformational sequence in advanced mathematics) some proof steps involve the insertion of a new sign with no components shared or overlapping with the previous step, e. g., in axiom use.

[4] The rhetoric of school mathematics concerns the standards, norms and rules (possibly tacit) of grammatical and expressional correctness, as well as stylistic and genre appropriateness, in the presentation and modes of expression of signs. These norms and rules are primarily applied to formal written texts (including symbols, diagrams, etc.), although spoken expressions are also rhetorically constrained, but usually more loosely. In contrast to logic the rhetoric of school mathematics is highly local and context-

bound, and for contingent and historical reasons varying rules and norms are applied across different institutions and locations (as well as at different ages).

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