Chapter 8

INSTRUMENTAL GENESIS, INDIVIDUAL AND SOCIAL ASPECTS

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Abstract: In Chapter 6, we analyzed didactic phenomena occurring during experiments in integrating symbolic calculators. We then showed how adopting an instrumental approach to analyzing these phenomena helped in understanding the influence of such tools upon mathematical activity and upon knowledge building. It is during the process of instrumental genesis that a calculator becomes a mathematical instrument.

In the first part of this chapter, we analyze the different forms that instrumental genesis takes by studying students' behavior so as to establish a typology of work methods in calculator environments. This typology indicates that the more complex the environment, the more diverse the work methods, and, consequently, the more necessary the intervention of the teacher in order to assist instrumental genesis.

In the second part of this chapter, taking this necessity into account, we introduce the notion of instrumental orchestration, defined by a didactical configuration and its modes of exploitation.

An orchestration is part of a scenario for didactical exploitation which aims to build, for every student and for the class as a whole, coherent systems of instruments.

Key words: Command process, Instrumental orchestration, Metaknowledge, Scenario in use, Work methods.

1. DIFFERENCES IN INDIVIDUAL INSTRUMENTAL GENESIS

Students have very different relationships with their calculator. Several methods help to pinpoint this diversity: conducting surveys of a wide student population to elicit their answers to a few questions posed at a given time, or following the instrumental genesis for a few students over the course of quite a long period of time. These different methods make it possible to bring out, for students' behaviors, several 'types of typologies'.

1.1 Local typologies

These typologies take into account only some aspects of instrumental genesis. In this sense, we speak of local typologies.

1.1.1 A typology linked to calculator learning type

Faure & Goarin (2001), from a survey of 500 10th grade students (most of them using graphic calculators), propose a typology depending on *the calculator learning type*. They take into account three approaches: learning with the teacher, learning through instructions for use, and learning by trial and error. Then the authors distinguish, within the given population, four profiles (Figure 8-1):

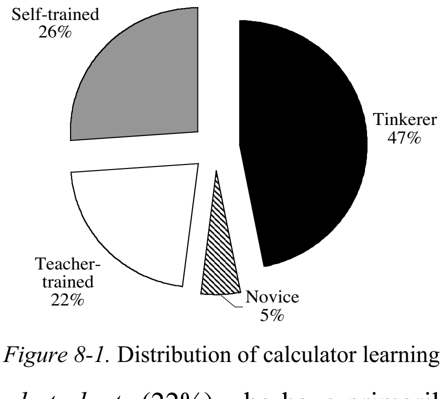


Figure 8-1. Distribution of calculator learning types

- *teacher-trained students* (22%) who have primarily learnt calculator use from the teacher;
- *self-trained students* (26%) who have primarily learnt calculator use from the instructions for use, and not substantially from the teacher;
- *tinkerers* (47%) who have learnt calculator use without guidance from any institutional source (whether teacher or instructions for use);

- *novices* (5%) who have had no training from the teacher, not consulted the instructions for use, and not tried to learn by themselves.

This typology can be related to some questions asked to the students.

i) "Do you know, with your calculator, how to find an approximate value for $\frac{\pi - \sqrt{2}}{\sqrt{2} + 1}$, to define a given function, to use a table of values, to graph a function, to choose an adequate window, to write programs?"

We can see, Figure 8-2, the frequencies for the answers *well* and *very well.*

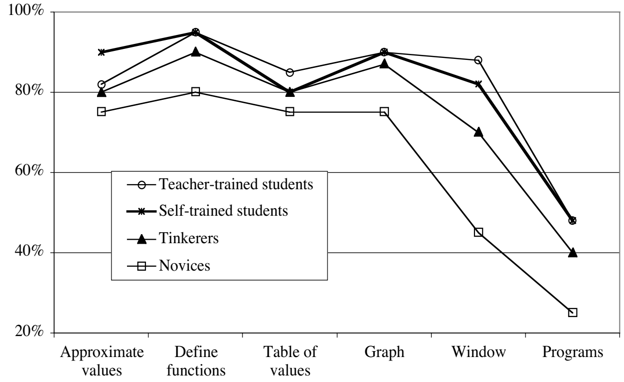


Figure 8-2. Typology and type of calculator knowledge

Techniques related to function graphs seem to be best mastered by all students. Teacher-trained students and self-trained students appear very confident in all the domains, whereas novices, logically, show quite weak competencies.

ii) "Is a calculator *useful* for computation, studying function variation, finding function limits, graphing functions, solving equations, and studying the domains for which functions are defined?"

Figure 8-3 shows the frequencies of the response *very much*:

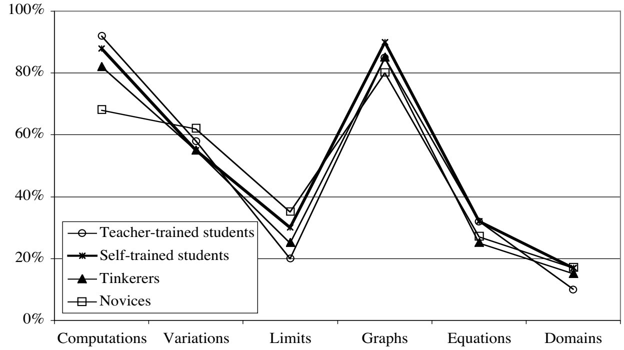


Figure 8-3. Typology and calculator usefulness

Answers for the different profiles are very similar: for all students, calculators are used essentially for computing and graphing functions.

iii) “Is a calculator useful *in the classroom* (for assessment, for the lesson, to help research), *at home* (for exercises, to learn lessons, to explore)?”

Figure 8-4 shows the frequencies of the response *very much*:

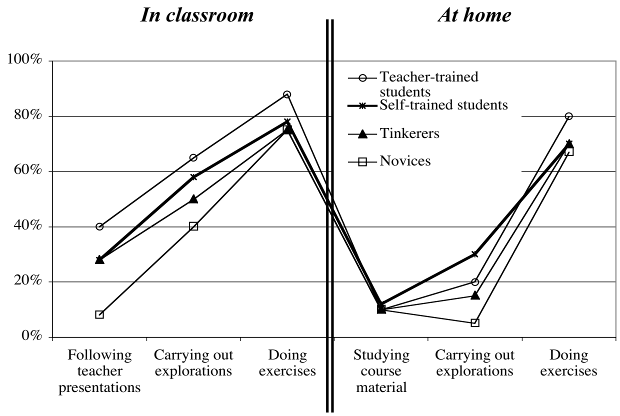


Figure 8-4. Typology and type of calculator work

Teacher-trained students accord a greater importance to calculator use during the lesson, which is to be expected: if the teacher has shown them how to use a calculator, s/he probably uses it in her/his mathematics teaching.

iv) “What is the relative importance of your *notebook*, your *textbook* and your *calculator*?”

Figure 8-5 shows the frequencies of the responses *great* and *essential*:

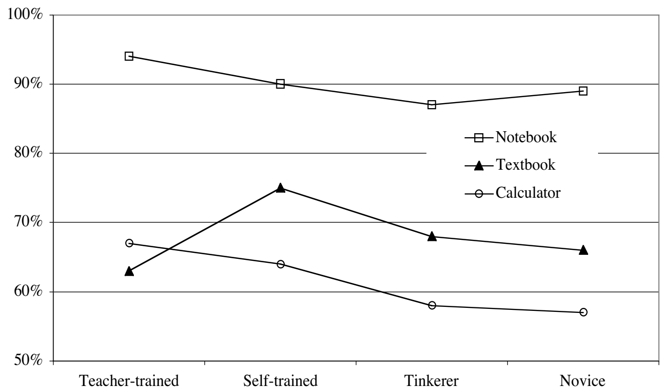


Figure 8-5. Typology and usefulness of mathematical tools

In each case, the notebook appears the essential tool, but, for trained students, calculator use overtakes textbook use. It is also interesting to remark that self-trained students, who learned calculator use from the printed instructions for use, are the most frequent users of textbooks.

To summarize, for the four questions asked, while the typology gives some results it does not identify great differences in calculator use. Some elements can explain this situation:

- the nature of the different typology categories is not the same: the teacher-trained students have not chosen to be trained (they have simply been in a classroom where a teacher took charge of this training), whereas self-trained students, tinkerers and novices are related by being placed in a situation (of no institutional training) which imposes a personal choice (choosing to learn from written instructions for example rather than through a trial and error strategy);
- the nature of tool utilization does not depend only on the type of training. Instrumental genesis is a process: other elements necessarily intervene (mainly individual work methods and learning environments, which are obviously different for the 500 students surveyed).

1.1.2 A typology linked to the privileged frame of work

Defouad (2000) notes that "instrumental genesis is not the same for all students; it depends on their personal relationships with both mathematics and computer technologies". He adds that, at the beginning of instrumental genesis, the relationships with graphic calculators are the most important. He distinguishes thus, in this graphic calculator environment, three profiles, a *numerical* one, a *graphical* one and a *paper-and-pencil* one, according to the frame of work privileged by the student: computation by calculator for the numerical profile, graphing with calculator for the graphical one, and obviously, work mainly with paper and pencil for the paper-and-pencil profile.

These categories are not stable:

- Defouad shows that, over the course of instrumental genesis, the nature of students' relationships with mathematics becomes more and more influential;
- we saw (Figure 6-10) that the applications employed to achieve tasks could change, according to the type of environment (for example, when moving from a graphic calculator environment to a symbolic calculator environment).

1.2 A more global typology

1.2.1. Principles for a typology

Understanding differences in students' behaviors is quite difficult. It needs to take into account, over a long time, more than just their privileged frame of work. For example, Mouradi & Zaki (2001) took into account the importance of paper-and-pencil work, but also the knowledge that students effectively use, interactions between pairs of students, students and teacher, and finally between students and computer. We have also (Trouche 2000) tried to consider various elements:

- *information sources* used, which can be previously built references, resort to paper-and-pencil, to the calculator or to the setting (in particular, during research activity, in practical work for example);
- *time of tool utilization* (both the global time over which the calculator is in use, and the time spent performing each instrumented gesture);
- *relationship of students to mathematics* and in particular *proof methods*: proof can proceed through *analogy*, *demonstration*, *accumulation of corroborating clues* (a particular form of over-checking, Chapter 6, § 1.2.2), from *confrontation* (based on comparison of various results obtained via the different information sources), and last from *cut and paste* (based on the transposition of isolated and not necessarily relevant pieces of proof);
- *metaknowledge* that is to say, knowledge which students have built about their own knowledge (Box 8-1).

Box 8-1.

Metaknowledge

Somebody is never in a wholly 'new' situation when discovering an artifact. S/he has already built knowledge about her/his environment and about her/himself, which is to say *metaknowledge*. Metaknowledge has emerged from several research fields:

- in the field of Artificial Intelligence, Pitrat (1990, p.207) distinguishes, between metaknowledge, knowledge about knowledge, knowledge about one's own knowledge, knowledge necessary to manipulate knowledge;
- in the field of didactics of mathematics, Robert & Robinet (1996) distinguish knowledge linked to mathematics, knowledge linked to gaining access to mathematical knowledge, and knowledge about one's own mathematical functioning (here these authors evoke the notion of control, as a global metaknowledge);
- in the field of cognitive psychology, Houdé & al (2002) also raise the question of control, when speaking of the co-existence, in each person, of both relevant and non-relevant schemes. If rationality, which generally exists for each individual, doesn't appear in her/his cognitive performances, the reason is often that the irrelevant schemes have not been inhibited.

We have stressed (Trouche 2000) the central role of the subject's *control* of her/his own activity. More precisely, we named this control *command process,* defined as the "conscious attitude to consider, with sufficient objectivity, all the information immediately available not only from the calculator, but also from other sources, and to seek mathematical consistency between them" (Guin & Trouche 1999b).

This command process takes place within a chart of essential knowledge (Figure 8-6), which is required in mathematical activity, in particular, when using symbolic calculators. It distinguishes two types of metaknowledge:

- first-level metaknowledge which makes it possible to seek information (*investigation*) from several sources: built references -- both material and psychological --, paper-and-pencil, the calculator, other students -- in particular within group work -- which makes it possible to store this information or to express it;
- second-level metaknowledge which makes it possible to process this information (*semantic interpretation, inference, coordination-comparison* of information coming from one or several sources, from one or several calculator applications or from other students).

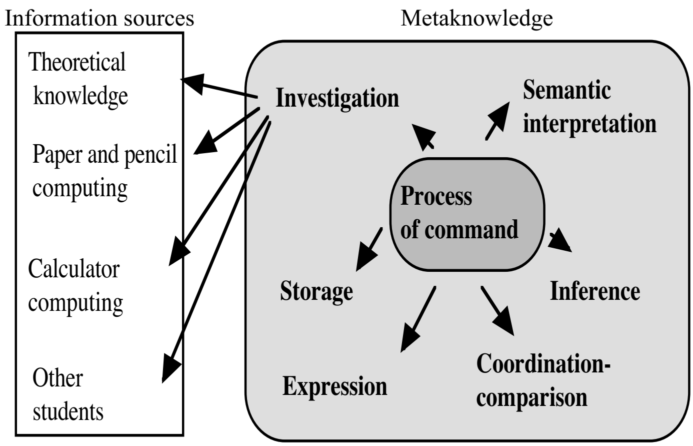


Figure 8-6. Information sources and metaknowledge

This chart itself does not completely describe a subject's behavior:

- each aspect of metaknowledge should be more clearly defined; for example investigation does not have the same character if it is applied only to the calculator or to the textbook, or to the setting as against it unfolding in all directions; the storage of new knowledge can be achieved alongside former knowledge or can lead to a *cognitive reorganization* (we know from Dorfler (1993) that experiences in computerized environments do not easily lead to such a cognitive reorganization);

- it is necessary to give a more precise description of the order in which the different types of metaknowledge are made use of, the respective time attributed to each of them. This precise description could be given when one has to describe the action of a given subject aiming at executing a given task in a given environment. The chart above could then provide us with a grid for analyzing this action.

Considering these different points, we have identified five extreme types of behavior, from observing students' work over a whole year (firstly in a graphic calculator environment, then in a symbolic calculator environment) and from analysis of their written productions and questionnaires, regularly handed out to them:

- a *theoretical* work method[1], characterized by the use of mathematical references as a systematic resource. Reasoning is based essentially on analogy and over-excessive interpretation of facts with occasional use of calculator;
- a *rational* work method, characterized by reduced use of a calculator, and mainly employing a traditional (paper-and-pencil) environment. What is distinctive here is the strong command by the student, with inferences playing an important role in reasoning;
- an *automatistic* work method, characterized by similar student difficulties whether in the calculator environment or in the traditional paper-and-pencil environment. Tasks are carried out by means of cut and paste strategies from previously memorized solutions or hastily generalized observations. The rather weak command by the student is revealed by trial and error procedures with very limited reference to understanding of the tools used, and without strategies for verifying machine results;
- a *calculator-restricted* work method, characterized by information sources more or less restricted to calculator investigations and simple manipulations. Reasoning is based on the accumulation of consistent machine results. Command by the student remains rather weak, with an avoidance of mathematical references;
- a *resourceful* work method, characterized by an exploration of all available information sources (calculator, but also paper-and-pencil work and some theoretical references). Reasoning is based on the comparison and the confrontation of this information, involving an average degree of command by the student. This is revealed in the form of investigation of a wide range of imaginative solution strategies: sometimes observations prevail, at other times theoretical results predominate.

The time devoted to each instrumented gesture is also an important element when discriminating between the various types of work method:

- this time is extremely brief for calculator-restricted and rational work methods. In the first case, because *zapping* behavior (Chapter 6, § 1.2.2) involves going from one image to another without thinking; in the second case, because the calculator is only used for targeted choices (no hesitation before nor adjustment after doing it);

- this time is much longer for theoretical, automatistic and resourceful work methods, for different reasons: for the theoretical and resourceful work methods, this time is necessary in order to analyze and compare one result with others; for the automatistic work method time is necessary to carry out the gesture itself and to understand the calculator result.

We summarize this typology in Figure 8-7.

Various sources of information were used to build this typology. Among them, practicals play an important role. Below we illustrate the typology in relation to a particular task studied during the course of the research.

Work method	**Theoretical**	**Rational**	**Automatistic**	**Calculator restricted to**	**Resourceful**
Privileged information source	Theoretical references	Paper and pencil	No single source	Calculator	No single source
Privileged metaknowledge	Interpretation	Inference	Investigation	Investigation	Comparison
Privileged proof method	Analogy	Demonstration	Copy and paste	Accumulation	Confrontation
Command process	Medium	Strong	Weak	Weak	Medium
Global time for calculator work	Medium	Short	Medium	Long	Medium
Time devoted to each instrumented gesture	Long	Short	Long	Short	Long

Figure 8-7. Five work methods in a calculator environment

1.2.2 Illustration of the typology

We proposed work on this task to an experimental 12^{th} grade class, in a graphic calculator environment: students did not have at their disposal a *Limit* command as in a symbolic calculator environment (Appendix 6-2). This work took place after a detailed lesson on function limits, in particular about polynomials (Trouche 2000).

The function is defined by:

$P(x) = 0.03x^4 - 300.5003x^3 + 5004.002x^2 - 10009.99x - 100100.$

The questions are:

- "determine its limit at $+\infty$;
- determine a calculator window which confirms your result".

This type of function is well known to students. Here the difficulty comes from the distance between the four real roots (– 10/3, 10, 10.01 and 10000), which makes the choice of a relevant window difficult. On the standard window (Figure 8-8), the graph obtained is not easy to interpret. On a fitted window, the graph does not correspond to the students' idea of a limit at +∞ for a function.

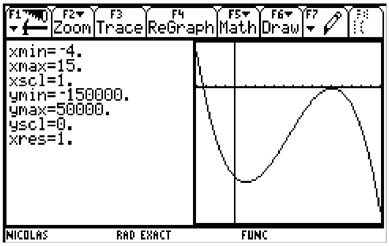
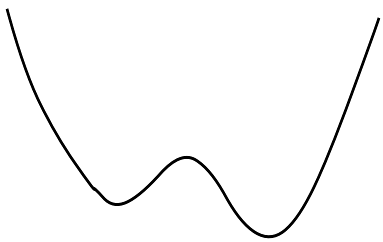


Figure 8-8. A function graphic, which looks quite strange

From observation of students it was possible to identify examples of the different work methods:

- *theoretical work method: student A*

S/he identified a polynomial function and evoked the relevant theorem: this polynomial has the same limit at +∞ as its term of highest degree. Therefore, $\lim_{+\infty} P$ is +∞. Through making a sketch on paper, A indicated that s/he knew the overall shape of a fourth degree polynomial.



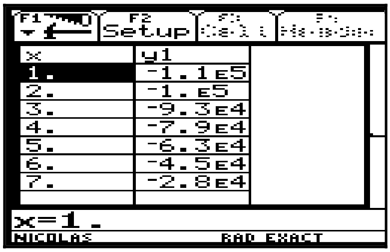
This theoretical result was also used to find a relevant window for the calculator: A chose a wider and wider range for the x–axis [0, Xmax], adjusting Ymax with respect to the value of 0.03 $(Xmax)^4$;

- *rational work method: student B*

B reproduced the method shown during the lesson (in order to demonstrate the theorem). So B factorized the term of highest degree. Then s/he gave the limits of each factor and, by applying the theorems on limit sums and products, found the limit of P. In order to obtain an appropriate window, B undertook a traditional function analysis: finding the derivative of P, then the derivative of P' and the sign of P''(x). Due to lack of time, B could not finish the work;

- *automatistic work method: student C*

C did not recognize a *reference* function form (Chapter 9, § 3.2.1, note 13) and used her/his calculator to form some idea about it. Defining the function on the calculator took rather a long time.



C was unable to analyze the graph shown on the standard window or to obtain a more appropriate window, and was not able to use a more theoretical approach, which seemed too difficult for such a complex object. The only information was obtained from the table of values (see right side). An answer was given from these results: $\lim_{+\infty} P$ seems to be $-\infty$;

- *calculator-restricted work method: student D*

From the beginning, student D started looking for an appropriate window for the graph and carried out various tests involving numerous commands:

- the *Trace* command led to the location of points situated outside the screen and therefore, the student redefined the window to allow the overall graphic representation to be shown;
- secondly, *Zoom* commands permitted quicker searches.

D used all forms of exploration possible on this calculator (using the widest possible range for the x variable). In this way, s/he obtained the required result using only the resources of the calculator, without any reference to theoretical results, and without putting any record of her/his work on paper;

- *resourceful work method: student E*

Using theorems learnt during the lesson, E was able to assert that $\lim_{+\infty} P = +\infty$. Then s/he looked for confirmation through a graphic representation of the function. After some concordant tests, s/he assumed that the graphics invalidated her/his first result obtained with a theoretical argument: the function seemed to be strongly decreasing, even for high values of x. E therefore tried to solve this contradiction and to find a justification for the exceptional status of this function. Observing the expression of P, s/he noticed that the coefficient of x^3 was extremely large while the coefficient of x^4 was very small. For E this point justified the exceptional status of this polynomial:

- *"For standard coefficients, it is the term with the highest degree which counts;*
- *in this case (a great difference between these coefficients), it is the term x^3, which counts. Therefore $\lim_{+\infty} P$ is $-\infty$".*

This makes clear the characterization of this work method in terms of a search for coherence when confronted with various results from different sources.

This typology has also been put to test in other situations, particularly in symbolic calculator environments (Guin & Trouche 2002).

1.2.3 Interest of the typology

Clearly this typology does not aim to (and could not) constitute a partition of the work methods of different students in a given environment. The work methods of most students cannot be classified as one of these types: they generally fall between positions and they can move between one and another. However, this typology does make it possible to establish a geography of the class, which has three-fold interest:

- it gives indicators to mark out, at a given moment, the relationship of a student with the five working styles brought to the fore. Besides, these five poles appear in similar form in other work: Hershkowitz & Kieran (2001) distinguish, for example, two types of behavior, linked to different methods of coordinating representations in a graphic calculator environment: "A *mechanistic-algorithmic way* (where students combine representatives in non-thinking, rote ways) and a *meaningful way"*. The former one is close to our calculator-restricted work method. The latter one looks like the theoretical work method, which we have previously described;
- it helps the teacher to play on the complementarities of the various work methods: we have shown (Trouche 1996) the interest of the association between rational and resourceful work methods for practicals. It thus gives the teacher a means to organize the learning environment. However, these evolutions depend significantly on work situations and arrangements set up by the teacher;
- it gives indicators to mark out the evolution of student approaches and thus to interpret their moves in terms of instrumental genesis. We have shown, for example (Trouche 1996) significant evolutions of the calculator-restricted work method toward the resourceful work method; we have also shown *that the more complex the environment, the more difficult the command process and the greater the diversity of work methods* (Trouche 2004), and, consequently, the more necessary the intervention of the teacher in order to *assist instrumental genesis* and *help the process by which the student exercises command.*

2. INSTRUMENTAL ORCHESTRATIONS

In both Chapter 6 and Chapter 8 (§ 1), the complexity of instrumental genesis is apparent, and so the need to articulate a set of instruments from a set of artifacts. Variability amongst students is also apparent in the instrumental genesis taking place in a given *class*[2]. Until now, we have considered these processes only in their individual dimension. But each utilization scheme has also a social dimension[3], of which Rabardel & Samurçay (2001) point out the importance:

> The world that genetic epistemology is interested in is a world of nature, not of culture. We have moved beyond this limitation by giving utilization schemes the characteristics of social schemes: they are elaborated and shared in communities of practice and may give rise to an appropriation by subjects, or even result from explicit training processes.

The integration of instruments within a class needs to take the process of instrumental genesis into account. Obviously while it does not remove the individual dimension of this process, it reinforces the social dimension, thus limiting individual diversity.

Box 8-2.

Didactic Exploitation System
(Chevallard 1992, p.195)

The successful integration of a technical tool in the teaching process requires complex and subtle work of *didactical implementation*. Chevallard uses a computer metaphor in order to distinguish three levels whose interaction is essential:
1) *Didactical hardware*: didactical environment components, various artifacts (calculators, overhead projectors, teaching software…), but also instructions for use, technical sheets, etc.
2) *Didactical software*: mathematics lessons.
3) *Didactic exploitation system*: essential level concerned with making relevant use of the potential resources of a didactical environment and with achieving both the coordination and integration of first and second levels.
Chevallard underlines the importance of this third level: without it, the didactical hardware components run the risk of being completely excluded from the teaching scene. Within the history of introducing computers into the institution of schooling, account has begun to be taken of the necessity of acknowledging and coordinating these three levels only in the face of failures and under the pressure of disappointments. Available software (word processing, spreadsheets, CAS, etc.) has not generally been conceived with teaching in mind and thus requires exceptional work for didactical implementation, along lines which have hardly been developed in rough.
A didactic exploitation system requires *didactical exploitation scenarios*. A didactical exploitation scenario is composed of a pedagogical resource and its implementation modes (in an ordinary classroom, in a special classroom, etc.) referred to as a *didactical configuration*.
Solving problems of the didactical integration of computerized tools requires the development of a true *didactical engineering* of computerized tools. The didactical engineer, between computer scientist and teacher, does not yet exist (or only as a prototype). Once such

a profession does exist, the teacher will be freed from tasks which s/he cannot carry out (didactical materials production) and will be able to become a specialist in teaching. *Further from machines; closer to students*. More than a teaching evolution, this would be a teaching revolution.

The *institution of schooling* has to take charge of these 'explicit training' processes. These explicit training processes require that a didactic exploitation system be designed (Box 8-2) so as to ensure the integration of tools in a class, and their *viability*.

This design requires models and exemplars of use:

> The degree to which this [CAS] technology is likely to be productive in the classroom will be highly dependent on the availability of proven models and exemplars to guide teachers and students in its use (Ruthven 1997).

Models and exemplars of use must include questions of management of time and space, and organization of tools within the classroom.

In order to take account of this necessity, we have introduced the notion of *instrumental orchestration* (Box 8-3) to refer to an organization of the artifactual environment, that an institution (here the schooling institution) designs and puts in place, with the main objective of *assisting* the *instrumental genesis of* individuals (here students).

An orchestration is part of a didactical exploitation scenario: it is designed in relation both a given environment and to a mathematical *situation* (Brousseau 1997). As states Rabardel (2001): "activity mediated by instruments is always *situated* and situations have a determining influence on activity".

Box 8-3.

The word *orchestration* is often used in the cognition literature. Dehaene (1997) uses this word pointing out an *internal* function of coordination of distributed neural networks. Ruthven (2002) also uses this word, in the mathematical field, pointing out a cognitive internal function (in relation to the construction of the derivative concept): "unifying ideas are careful orchestrations of successive layers of more fundamental ideas around a more abstracted term". In fact, the necessity of orchestrations, in this sense, clearly manifests itself in the learning of mathematical sciences seen as "the construction of a *web* of connections - between classes of problems, mathematical objects and relationships, real entities and personal situation-specific experiences" (Noss & Hoyles 1996, p.105). In our sense, the word orchestration means an *external* steering of student's instrumental geneses.

The word *orchestration* is indeed quite natural when speaking of a set of *instruments*.

The *orchestration*, in the musical register, may indicate two things:

- the work of the composer to adapt, for an orchestra, a musical work originally written for only one instrument or a few;

- *"the art to put in action various sonorities of the collective instrument* which one names orchestra by means of infinitely varying combinations" (Lavignac, French musicographer, 1846-1916).
By choosing this word, we refer here to this second and more general sense.

Both define an *instrumental orchestration*:
- a set of *configurations* (i.e. specific arrangements of the artifactual environment, one for each stage of the mathematical situation);
- a set of *exploitation modes* for each configuration.

These exploitation modes may favor production of activity accounts. These accounts can themselves be integrated as new learning and teaching tools.

An instrumental orchestration may act *mainly* at several levels:
- at the level of the artifact itself;
- at the level of an instrument or a set of instruments;
- at the level of the relationship a subject maintains with an instrument.

These levels correspond to the three levels of artifacts distinguished by Wartofsky (1983):
- "The level of primary artifacts which corresponds to the concept of the artifact as it is commonly used (...), computers, robots, interfaces and simulators;
- (...) [The level of] secondary artifacts, which consists of representations both of the primary artifacts and of modes of action using primary artifacts;
- (...) The level of tertiary artifacts (...) represented, for trained subjects in particular, by simulated situations as well as by reflexive methods of self-analysis of their own or the collective activity".

2.1 A first level instrumental orchestration: guide to mathematical limit

We have proposed such a guide (Trouche 2001) to assist learning of the idea of limit.

In order to define this orchestration, we have to analyze the gap between the mathematical idea to be taught (Box 8-4) and the ways in which the artifact has implemented it (Box 8-5).

From these constraints of the artifact, one can generate some hypotheses about the techniques which students put in place to study limits of functions, and about the operational invariants (Boxes 6-4 & 6-5) likely to be built in such an environment. The constraints of the TI-92 do not favor moving beyond a kinematic point of view on the idea of limits:

Box 8-4.

The limit concept

Several *frames*[4] (geometrical, algebraic and numerical) are involved in studying limits, sometimes creating productive intuitions, sometimes acting as obstacles. Trouche (2003) distinguishes two main points of view:

- a *kinematic* point of view, within a generally geometrical frame: a quantity y (depending on x) tends towards b as x tends towards *a* if, when x becomes closer to *a*, y becomes closer to b. For this definition, movement has a crucial role: one can say that 'variable *pulls* function'. The geometrical frame is also important: the limit idea involves bringing together graphical representatives or geometrical objects (curves and asymptotes for example);
- an *approximation* point of view, in a numerical frame: a quantity y (depending on x) tends towards b as x tends towards a if y can be as close to b as one wants, as long as x is close enough to *a*. It is thus the degree of precision that one wants which *constrains* the variable.

Construction of the limit concept involves going beyond the first point of view, but it is often an *articulation* of the two points of view that permits this notion to be grasped.

- the calculator, through its symbolic application, only gives (if it 'knows' an answer) the value of the limit, which is not sufficient to give sense to the idea;
- through its graphical and numerical applications, the calculator clearly presents a kinematic point of view.

We want to define an orchestration aiming to support instrumental genesis, transforming the TI-92 artifact into an instrument for computation of limits. To achieve this objective, it is necessary to *fit out* the artifact itself (it is a first level orchestration) in order to favor, in this environment, the passage from a kinematic point of view to an approximation point of view.

Box 8-5.

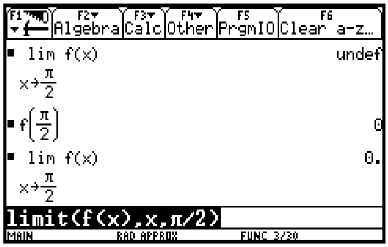
Constraints on limit computations of one symbolic calculator (TI-92) and corresponding potentialities

We use here the typology of constraints presented in Chapter 6, § 2.1.

1) The *internal constraints* (what, by nature, the artifact can do?)
A symbolic calculator contains a CAS application (Computer Algebra System); it can determine an exact limit provided that the corresponding "knowledge" has been entered.
A symbolic calculator can also (as a graphic calculator) give graphic or numerical information on the local behavior of functions. The processing is done by numerical computation.

2) The *command constraints* (what are the available commands?)
Only one *Limit* keystroke exists. It is a formal command, located in the calculator symbolic application.

Its syntax is (see screen) "limit(f (x),x,a)"; it corresponds to the order of the statement "the limit of f(x) as x tends toward a". Nevertheless this command can be combined with the *approximate detour* (Chapter 6, § 2.3.1). In the example shown, the *Limit* command, applied to the function $f(x) = (\cos x)^x$ does not give the result "directly", but gives it by switching to approximate detour (screen copy, 3rd line).



3) The *organization* constraints (how are the available commands organized?)
The different applications (symbolic, graphical or numerical) permitting the study of functions are directly accessible on the keyboard. As a part of graphical or numerical applications, the operation of the calculator requires the interval of x and then the interval of y to be chosen first. This is a natural order for the study of functions, but is not an adequate order for studying limits (Box 8-4): the mathematical organization and tool organization are opposed from a chronological point of view.

This configuration rests on putting in place, within the calculator, three levels for each study of a limit. We present these through an example of a limit, the value of which is not known by the calculator $\lim_{x \to +\infty} \frac{\sqrt{x} + \cos x}{x + \sin x}$ (Appendix 6-2).

These three levels[5] are accessible from a subsidiary menu linked to the *Limit* command (Figure 8-9).

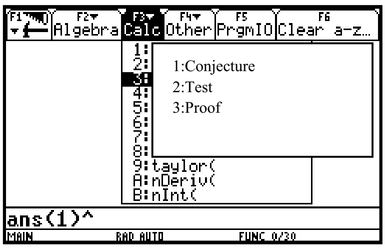


Figure 8-9. Menu and sub-menu allowing access to three levels of study of limits

Level 1. Conjecture searching

The *Conjecture* command gives access to a split screen: on the left side is the *TABLE of values* application, on right side, the *GRAPH* application (Figure 8-10). The split screen allows these two applications to be connected.

One has to choose a table setting and a graph window (here, the study being in the neighborhood of infinity, corresponding to 'large' values of the variable x).

Observation of both tables of values allows a conjecture to be formed: maybe the function limit is equal to 0.

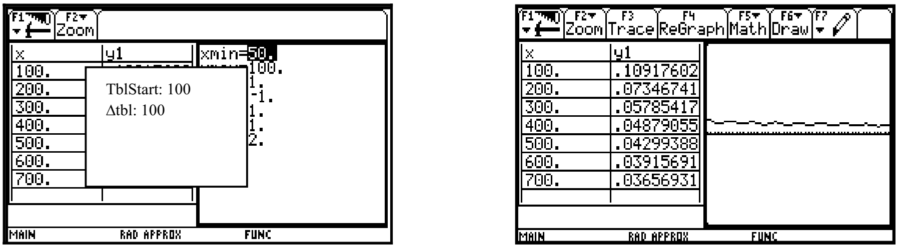


Figure 8-10. Conjecture searching

Level 2. Testing

The *Testing* command gives access to a new split screen, again with the table of values and graphical applications (Figure 8-11). But here there is a fundamental logical reversal: one has to choose first the neighborhood for y, image of the variable x.

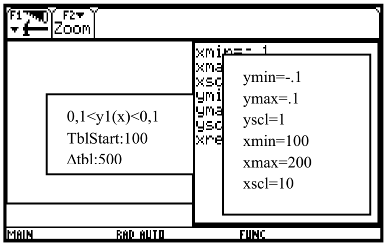


Figure 8-11. Conjecture testing

It is thus the degree of precision one wants which *constrains* the variable. It is a sort of challenge, linked to the approximation point of view (Box 8-4): if one wants y to be in a given neighborhood of 0, in which interval [a, + ∞ [is it sufficient to choose x?

The obtained numerical and graphical results (Figure 8-12) show that the constraints on the variable are not sufficient: the aimed degree of approximation is not achieved. The student thus has to go back to choose a new table set and graph window. These gestures are not only gestures of exploration: they are preparing the passage from a kinematic point of view to an approximation point of view.

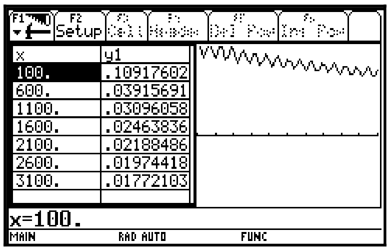


Figure 8-12. Not sufficient constraints on the variable

Level 3. Proof

The *Proof* command gives access to a new split screen: on the left side, the symbolic application, on the right side a work sheet dedicated to proof (Figure 8-13). The symbolic application can give the limit value (although this is undefined in the case shown here).

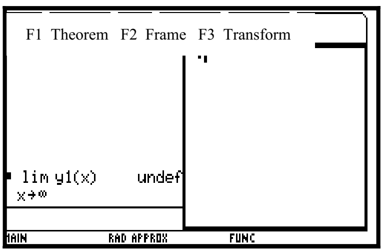


Figure 8-13. Proof screen

The right work sheet gives access to menus which allow the "guiding of thinking about problems and tool classification principles" (Delozanne 1994):

- the *F1* menu gives access to a set of theorems. It leads to work on characterizing the functions studied (for example, "is it possible to apply theorems on limits of rational functions to the given function?");
- the *F2* menu allows framing strategies to be tried out, with the help of calculator numerical and graphical applications. It leads to a comparison with *reference functions* (Chapter 9, § 3.2.1, note 13);
- the *F3* menu gives access to symbolic functionalities (factoring, expanding, etc.).

We have thus defined an orchestration *configuration*. Defining orchestration involves choosing some *exploitation modes.* Several modes are possible:

- this limit math guide can be always available or available only during a specific teaching phase;
- students can be free to use this guide, when available, as they want, or they can be obliged to follow the order of the three given levels;
- the list of stored theorems can be fixed, or it can be progressively established, linked to the mathematical lessons, built in the classroom and collectively stored in each calculator;
- activity accounts can be required, describing all the steps of instrumented work, or not.

The orchestration defined in this way, modifying the artifact itself, is not a matter of building the explication module of an expert system (besides, Delozanne (1994) indicates that this task is quasi-impossible if the software designer has not initially taken this development into account). It could only constitute a *guide*[6], an assistant for instrumental genesis, in the study of limits, making it possible to move from one frame to another, and providing balance between the two points of view constituting this notion (Box 8-4). Designing such an orchestration involves analyzing precisely both the notion to be taught (from an epistemic point of view) and the way in which the artifact has implemented it. It does not solve, in itself, the problem of the learning process of the limit idea: one also has to choose a field of critical functions, more generally a field of problems nested in didactic situations which have to be elaborated. Defining a didactical exploitation scenario requires then the choosing of an orchestration which is well adapted to each stage of this problem treatment.

2.2 A second level orchestration: around the sherpa-student

The utilization of individual tools within the school, in the form of calculators fitted with a small screen, raises the issue of the socialization of students' actions and productions. This socialization requires particular arrangements. Since the beginning of the 1990s, there has been a particular artifact -- a view-screen -- which allows one to project the calculator's small screen[7] onto a big screen, which the entire class can see. Guin & Trouche (1999a) presented an instrumental orchestration, which exploits this arrangement with the main objective of socializing -- to a certain extent -- students' instrumental genesis.

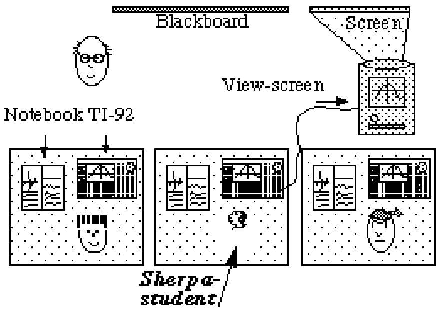


Figure 8-14. The sherpa-student, part of an instrumental orchestration

The *configuration* of this orchestration (Figure 8-14) rests on the devolution of a particular role to one student: this student, called the *sherpa-student*[8], pilots the overhead-projected calculator. S/he will thus be considered, for both class and teacher, as a reference, a guide, an auxiliary and a mediator. This orchestration favors the collective management of a part of the instrumentation and instrumentalization processes (Chapter 6, § 2.2): what a student does with her/his calculator -- traces of her/his activity -- is seen by all. This allows one to compare different instrumented techniques and gives the teacher information on the schemes of instrumented action being built by the sherpa-student.

It also has other advantages:

- the teacher is responsible for guiding, through the student's calculator, the calculators of the whole class (the teacher does not perform the instrumented gesture but checks how it is performed by the sherpa-student). The teacher thus fulfils the functions of an *orchestra conductor* rather than a *one-man band*[9];

- for his/her teaching, the teacher can combine paper-and-pencil results obtained on the board, and results obtained by the sherpa-student's calculator on the class screen. This facilitates, for students themselves, the combination of paper-and-pencil work and their calculator work at their own desk.

Several *exploitation modes* of this structure may be considered. The teacher may first organize work phases of different kinds:

- sometimes calculators are shut off (and so is the overhead projector): it is then a matter of work in a paper-and-pencil environment;
- sometimes calculators are on as well as the overhead projector and work is strictly guided by the sherpa-student under the supervision of the teacher (students are supposed to have exactly the same thing on their calculator-screens as is on the big screen for the class). Instrumentation and instrumentalization processes are then strongly constrained;
- sometimes calculators are on as well as the overhead projector and work is free over a given time. Instrumentation and instrumentalization processes are then relatively constrained (by the type of activities and by referring to the sherpa-student's calculator which remains visible on the big screen);
- sometimes calculators are on and the projector is off. Instrumentation and instrumentalization processes are then only weakly constrained.

These various modes seems to illustrate what Healy (2002) named *filling out* and *filling in*[10], in the course of classroom social interaction:

- when the sherpa-student's initiative is free, it is possible for mathematically significant issues *to arise out* of the student's own constructive efforts (this is a filling out approach);
- when the teacher guides the sherpa-student, it is possible for mathematically significant issues to be *appropriated during* children's own constructive efforts (filling in approach).

Other variables must also be defined: will the same student play the role of the sherpa-student during the whole lesson or, depending on the results proposed, should such and such a student's calculator be connected to the projector? Must the sherpa-student sit in the front row or must she/he stay at her/his usual place? Do all students have to play this role in turn or must only some of them be privileged?

Depending on the didactic choices made, secondary objectives of this orchestration can arise:

- to favor debates within the class and the making explicit of procedures: the existence of another point of reference distinct from the teacher allows new relationships to develop between the students in the class and the teacher, between this student and the teacher -- about a result, a conjecture, a gesture or a technique --;
- to give the teacher means through which to reintegrate remedial or weak students into the class. The sherpa-student function actually gives remedial

students a different status and forces the teacher to tune his/her teaching procedures to the work of the student who is supposed to follow her/his guidelines. Follow-up of the work by this student shown on the big work-screen allows very fast feed-back from both teacher and class.

This instrumental orchestration involves coordination of the instruments of all students in the class and favors the connection, by each individual, of different instruments within her/his mathematics work.

2.3 Another second level orchestration: practicals

Guin & Trouche (1999a) present an organization of students' research work in a calculator environment: *practicals.*

This orchestration aims to:

- make it possible for instrumental genesis to proceed at its own rhythm;
- develop social interactions between peers;
- favor establishment of relationships between different tools (calculator and paper-and-pencil) within a research process.

The configuration is this one: each student has at her/his disposal a calculator, paper-and-pencil. Students work in pairs (work groups are small, because of the smallness of the calculator screen) to solve a given problem. These problem situations (Appendix 8-1) are created with the aim of promoting interaction between calculators, theoretical results, and handwritten calculations as an aid to conjecture, test, solve and check. After working on these problem situations, each pair has to explain and justify their observations or comments, noting discoveries and dead-ends in a written research report. The role of this report is twofold:

- it focuses the student activity on the mathematics and not on the calculator, forcing students to give written explanations for each stage undertaken in their research (a very important step);
- it gives the teacher a better understanding of the various steps of the students' work method, and makes it possible to follow the instrumental genesis of students.

There is only one notebook for each pair. This choice is an important one: each research team is thus obliged to find a consensus, or to explain divergences.

Several *exploitation modes* are possible:

- students can be free (or not) to form themselves into pairs. We showed (Trouche 1996) the value of some specific pairings, for example a student with a quite *rational* work method and a student with a quite *calculator-restricted* work method: the interaction allows an evolution of each work method and some enrichment of instrumentation processes;

- students can be free (or not) to choose which one will write the research report;
- the teacher can offer appropriate assistance to help students out of deadlocks, to reinitiate reflection during the practical, or only at the end of it, or a week after;
- written research reports can be given to the teacher at the end of practicals, or a week later. In the first case, research reports are more faithful (showing what happens during practical, moment by moment, step by step). In the second case, students can have more time to read their own report, to think about their own work, to criticize their own research;
- after reading students' research reports, the teacher can give a problem solution, in relation to the students' results, or give only some partial indications opening up new strategies for students to pursue during further practicals.

In the frame of this orchestration, teachers and students play a new role, as stated by Monaghan (1997): "the teacher is viewed as a technical assistant, collaborator, facilitator and as a catalyst, and students have to cooperate in group problem solving".

2.4 A third level orchestration: mirror-observation

In the previous orchestration (§ 2.3), a research notebook constitutes an essential tool for students (for making explicit their own calculator and paper-and-pencil approaches, evaluation of the relevance of results, etc.). This notebook thus appears to be a tool for *activity self-analysis* (Rabardel & Samurçay 2001). We have presented (Trouche 2003) another arrangement, so called *mirror-observation* (Figure 8-15).

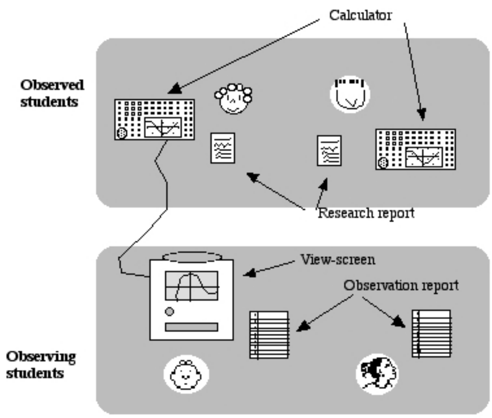


Figure 8-15. The *mirror-observation* configuration

This device aims at analysis of a work period during the course of the year. The *configuration* is this one: students work in pairs. Half of the pairs (Figure 8-15, *observed students*) carry out a given mathematical task. The other half observe and note the actions carried out, with the help of two artifacts:

- a palette for overhead projection makes it possible to capture the calculator screen of one of the students from the pair (the one who is not in charge of the written research report);
- observation sheets are used to note, every fifteen seconds[11], the whole of the students' actions. These sheets (Figure 8-16) appear as grids in which are located different types of tasks: *paper-and-pencil* tasks, *calculator* tasks (distinguishing the different applications involved), tasks relating to *interactions* (with the teacher, other students, or oneself: *hazy gaze*) and last the *other* actions that have nothing to do with the problem dealt with.

		15	*30*	*45*	*1mn*	*15*	*30*	*45*	*2mn*
Paper-and-pencil	Reading text of problem	x							
	Reading lesson								
	Reading accomplished work								
	Reading neighboring work								
	Drawing								
	Computation							x	x
Calculator	Machine condition (on/off)		x	x	x	x	x		
	Computation		x	x					
	Y Editor				x				
	Graph								
	Table of values								
	Neighboring machine								
Interactions	Hazy gaze					x	x		
	With teacher								
	With neighbor								
Other									

Figure 8-16. A timed observation sheet
(During the first 15 seconds, student reads problem text, then s/he uses her/his calculator for some computations, etc.)

Examination of the grids, corresponding to observation of five student pairs during the first five minutes is presented in chronological order (Figure 8-17). One may observe the large dispersion of work methods: pair number 1 takes a very short time reading the problem text, and rushes toward the calculator. Pairs number 3 and 4 spend a certain time on various irrelevant actions.

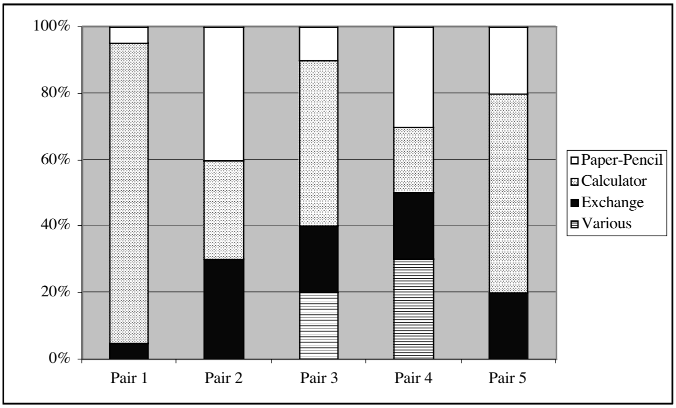


Figure 8-17. Compared synthesis of actions performed by the five pairs of students during the first five minutes of the activity

This instrumental orchestration highlights, not so much the results of the activities, but the various forms that these can take. It enables students, as they themselves noticed, *"while observing others to observe oneself"* (hence the suggestion of the term *mirror* for this type of observation) offering the possibility of an auto-analysis of the action, the construction by a *reflexive mediation*: for the majority of the observed subjects, the organization of action revealed by the chronological synthesis caused major surprise (for instance: *"How come, I haven't spent more than 15 seconds reading the text of the problem?"*). The gap between what the students actually did and what they remembered doing, as well as the gap between the written traces of the research report and the written traces of the observation sheet, allow a profound reflection on the shape of activity, allow the understanding of certain defaults and the rectification of certain failings.

Various exploitation modes are possible. Among them:

- this orchestration may be used only exceptionally, or may be a regular tool for regulation of students' instrumented activity;
- one may fix, or not, the role of each observed student (for example one can be in charge of the calculator, the other in charge of the research report);
- the type of tasks noted on the timed observation sheet can be modified, in relation to the type of mathematical problem set;
- each observation sheet analysis can be done within the group of four students (the students observed and the two observing), or all these observation sheets can be made public with the whole class.

Other devices can arouse students' thinking about their own activity[12]: for example some experiments (Trouche 1998) incorporated some form of "barometer" of the integration of instruments, i.e. questionnaires asking students about their instrumented activity. These accounts, giving information about what students think about their own activity, are complementary to observations from their peers.

3. DISCUSSION

Common elements can be recognized within these various instrumental orchestrations: a favoring of interaction between students, the publication and use of accounts of activity (sherpa-student's screen, research reports, or timed observation sheets), thus giving the teacher means to understand and guide the instrumental genesis of students. Instrumental orchestration combines all these elements to reinforce the *social dimension* of instrumented action schemes and to assist students in the process of command.

The need for a strong process of command is linked to the practice of mathematics; mathematics seen as "a web of interconnected concepts and representations which must be mastered to achieve proficiency in calculation and comprehension of structures" (Noss & Hoyles 1996). It is also linked to the tools available.

The necessity of taking the tools of the environment into account is not new. Proust (2000) notes, for example, recurrent mistakes in Babylonian numerical texts, in computation involving numbers composed of more than five figures. Her hypothesis is that these mistakes came from sticking together two computations realized with a tool linked to the five fingers of the hand. More exactly, it is a matter of the bad *articulation* of two types of artifacts: artifacts for material computation, and for writing, fingers being involved in both types of gesture.

What is true for 'old' computation environments is all the more true in a computerized environment (Basque & Doré 1998). Very sophisticated artifacts such as those available in a symbolic calculator environment give birth to a *set of instruments*. The articulation of this set demands from the subject a strong process of command, allowing her/him to build coherent *systems of instruments.* As Rabardel (2000) notes, this is a crucial point:

> This question seems to us particularly crucial in view of the current context of technological expansion. What artifacts should be proposed to learners and how should we guide them in their instrumental genesis and through the evolution and adjustment of their systems of instruments?

Instrumental orchestrations seem to give some elements of an answer to this question. They take into account artifacts in the learning environment, at three levels (tool level, instrument level, meta-level). They take place (Figure 8-18) within a didactical exploitation scenario (Box 8-2). According to the metaphor, we could say that designing an orchestration requires a musical frame. The following chapter will treat this point, i.e. the design of mathematical situations and problems.

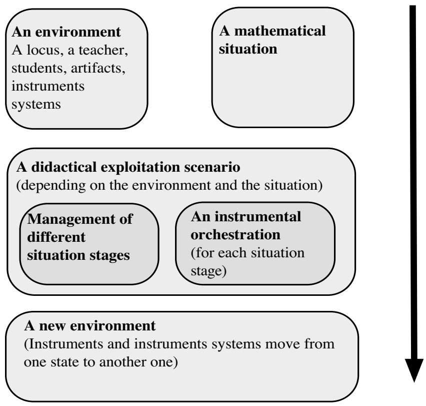


Figure 8-18. Evolution of a learning environment

After choosing a mathematical situation and defining the management of successive situation stages, designing an instrumental orchestration implies defining a didactical configuration and its exploitation modes.

A didactical exploitation scenario has effects on a learning environment:

- obviously it has effects on the knowledge built, via the treatment of mathematical situations;
- it has effects on the *didactical contract* (Box 8-6). For example, the devolution of a particular role to a sherpa-student (§ 2.2) enables parts of this contract to be made explicit;
- it has effects on instrumental genesis, i.e. on instruments and systems of instruments.

Box 8-6.

Didactical contract
(Brousseau 1997)

Brousseau evokes the *contract*, i.e. "the relationships determining - explicitly for a small part, but mainly implicitly, what each partner, teacher and learner, has to manage and what s/he will have the responsibility for". Part of this contract, which is related to the content, mathematical knowledge, is the *didactical contract*.
Brousseau points out the importance of the points at which this contract breaks down: "knowledge is precisely what will solve the crisis related to these breakdowns (…). The surprise of the student, when s/he can't solve the problem and rebels against the teacher who has not made her/him able to solve it, the surprise of the teacher who estimated her/his lessons to be sufficient… revolt, negotiation, search for a new contract depending on the new state of knowledge". The essential notion is therefore not the contract itself, but, through the breakdowns, the *process of searching for a hypothetical contract.*

Designing such scenarios of didactical exploitation is a complex task, calling for various competencies (Chevallard 1992): computer engineers, didactical engineers, curriculum designers, etc. Such work certainly exceeds the possibilities of a teacher, alone in her/his class.

In the context of ICT distance learning, some experimentations (Joab & al 2003) allowed teachers to work collaboratively and gave birth to a new type of pedagogical resource, including a *scenario in use* (Allen & al 1994, 1996) taking the management of artifacts into account. This seems to be a way to make orchestrations explicit (see Conclusion).

APPENDIX

Appendix 8-1
About practicals
(Trouche 1998, p.16)

Work method proposed to students

You have to study this problem as a mathematical researcher. The most important thing therefore is the writing of a research report, in which you will note your approaches (even dead-ends), methods, tools used (if you use a calculator, you will specify which applications, which gestures, etc.).
If you solve the problem, so much the better! If not, the research done will not be useless! One learns from failures as well as from successes. What will be assessed will be more the relevance of your methods than the results themselves.
Your research report obviously has to be readable. But don't think of it as a final examination: it will necessarily bear marks of hesitation linked to each research process.
Working in pairs calls for collaboration, putting ideas in common and sharing tasks: a team enterprise. Avoid becoming too specialized in function (for example: always the same student using the calculator, always the same writing the research report): prefer task rotation!
During practicals, you have to study the following *main question*. You will probably have no time to study extra questions: you will tackle this problem later, in order to go further than the main question.

A example of practical text

Main question
How many figures 0 are there at the end of the numerical value of these expressions: 10!, 100!, 1000!, 1997!
Extra question
n being a given positive integer, a sequence u(n) is defined by "the number of figures 0 at the end of n!". Can you define this sequence for your calculator (and so answer easily the main question above)?

NOTES

1. It has been difficult to find good labels for each work method. Other difficulties appear when translating them. Firstly we chose the five French labels *théorique*, *rationnel*, *scolaire*, *bricoleur* and *expérimentateur*. The label *scolaire*, criticized, has been replaced by *automate* in our most recent papers (Trouche 2004). At first, our English labels were *theoretical*, *rational*, *random*, *mechanical* and *resourceful* (Guin & Trouche 1999b). The new labels chosen here seem to us better adapted, even if a single expression cannot summarize the whole description itself: for example, the fourth work method cannot be only characterized by the fact the work is mainly restricted to the calculator.
2. In this paragraph, the word *class* means basic schooling structure. Our propositions are based on experimentations carried out in 10^{th}, 11^{th} and 12^{th} grade classes.
3. What is true for instrument use is also true, more generally, for mathematical practice. Brousseau (1997) writes: "… doing mathematics is first, for a child, a social activity, not only an individual activity".
4. The notion of *frame*, in this sense, was introduced by Douady (1986, p.11): it is "made of the objects of a branch of mathematics, the relationships between these objects, their eventually diverse formulations and the mental images associated with these objects and relationships".
5. These are theoretical proposals, not yet implemented on a calculator and not experimented.
6. With the same goal, Texas Instrument has developed a *Symbolic Math Guide* for its symbolic calculators "to help students learn algebra and some aspects of calculus by guiding them as they develop correct text-book-like solutions. SMG can be used when a student first learns a topic or as quick review" (http://education.ti.com).
7. Some constraints of this artifact can be analyzed:
   - the connection with a calculator requires a special plug on it, available only on some calculator types;
   - the cable linking this artifact to a calculator is only 2 m long.

   The consequence of these two constraints is that this device is probably designed for the teacher's use. Bernard & al (1996) showed indeed that it is, when available in a classroom, connected to the teacher's calculator.
8. On the one hand, the word sherpa refers to the person who guides and who carries the load during expeditions in the Himalaya, and on the other hand, to diplomats who prepare international conferences.
9. This advantage is not a minor one. Teachers, in complex technological environments, are strongly prone to perform alone all mathematical and technical tasks linked to the problem solving in the class (Bernard & al 1996).
10. Healy (2002) identified a major difference between instructional theories drawing from constructivist perspectives and those guided by sociocultural ideologies, which related to the primacy assigned to the individual or the cultural in the learning process. Constructivist approaches emphasise a *filling-outwards* (FO) flow in which personal understandings are moved gradually towards institutionalized knowledge. A reverse *filling-inwards* (FI) flow of instruction described in sociocultural accounts stresses moving from institutionalized knowledge to connect with learners understandings. Teaching interventions in Healy's study were therefore designed to allow investigation of these two different instructional approaches: the FO approach aimed to encourage the development

of general mathematical models from learners' activities; and the FI approach intended to support learners in appropriating general mathematical models previously introduced.

11. Meinadier (1991) thus estimates the size of the "task unit" in the context of computer use.
12. Vasquez Bronfman (2000, p.227) defined the *reflexive practicum*, an arrangement of quite the same nature, as "a frame, a way, aiming helping learners to acquire the art of working in uncertain (or undetermined) domains of their practice".

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