Chapter 6

OTHER OUTRANKING APPROACHES

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Abstract In this chapter, we shortly describe some outranking methods other than ELECTRE and PROMETHEE. All these methods (QUALIFLEX, REGIME, ORESTE, ARGUS, EVAMIX, TACTIC and MELCHIOR) propose definitions and computations of particular binary relations, more or less linked to the basic idea of the original ELECTRE methods. Beside them, we will also describe other outranking methods (MAPPAC, PRAGMA, IDRA and PACMAN) that have been developed in the framework of the Pairwise Criterion Comparison Approach (PCCA) methodology, whose peculiar feature is to split the binary relations construction phase in two steps: in the first one, each pair of actions is compared with respect to two criteria a time; in the second step, all these partial preference indices are aggregated in order to obtain the final binary relations. Finally, one outranking method for stochastic data (the Martel and Zaras' method) is presented, based on the use of stochastic dominance relations between each pair of alternatives.

Keywords: Multiple criteria decision analysis, outranking methods, pairwise criteria comparison approach.

1. Introduction

The outranking methods constitute one of the most fruitful approach in MCDA. They main feature is to compare all feasible alternatives or actions by pair building up some binary relations, crisp or fuzzy, and then to exploit in an appropriate way these relations in order to obtain final recommendations. In this approach, the ELECTRE family and PROMETHEE methods (see Chapters 4 and 5 in this book) are very well known and have been applied in a lot of real life problems. But beside them, there are also other outranking methods, interesting both from theoretical and operational points of view. All these methods propose definitions and computations of particular binary relations, more or less linked to the basic idea of the original ELECTRE methods, i.e. taking explicitly into account the reasons in favor and against an outranking relation (concordance-discordance analysis using appropriate veto thresholds). Some of these methods, moreover, present also a peculiar way to build up final recommendations, by exploiting the relations obtained in the previous step. In this chapter, we shortly describe some outranking methods other than ELECTRE and PROMETHEE. In Section 2 we present some outranking methods dealing with different kind of data (QUALIFLEX, REGIME, ORESTE, ARGUS, EVAMIX, TACTIC and MELCHIOR). Some of these methods are based on concordance-discordance analysis between the rankings of alternatives according to the considered criteria and the comprehensive ranking of them; others on direct comparison of each pair of alternatives, more or less strictly linked to the concordance-discordance analysis of ELECTRE type methods. In Section 3 some outranking methods (MAPPAC, PRAGMA, IDRA and PACMAN) are described. They have been developed in the framework of the Pairwise Criterion Comparison Approach (PCCA) methodology. Its peculiar feature is to split the binary relations construction phase in two steps: in the first one, each pair of actions is compared with respect to two criteria a time, among those considered in the problem, and partial preference indices are built up. In the second step, all these partial preference indices are aggregated in order to obtain the global indices and binary relations. An appropriated exploitation of these indices gives us the final recommendations. Finally, in Section 4 one outranking method for stochastic data (the Martel and Zaras' method) is presented. The main feature of this method is that the concordance-discordance analysis is based on the use of stochastic dominance relations on the set of feasible alternatives, comparing their cumulative distribution functions associated with each criterion. Some short conclusions are sketched in final Section.

2. Other Outranking Methods

The available information is not always of cardinal level; some times the evaluations of alternatives are ordinal scales, especially in social sciences. These evaluations may take the form of preorders. Several methods were been developed to aggregate this type of local evaluation in order to obtain a comprehensive comparison of alternatives. For example, we can mention Borda, Condorcet, Copeland, Blin, Bowmam and Colantoni, Kemeny and Snell, etc. (see [31]). Some methods that we will present in this Section drawn inspiration by some of them.

We present some outranking methods consistent with ordinal data, since they do not need to convert ordinal information to cardinal values, as it is the case, for example, in [15]. We will present some methods frequently mentioned in the literature on MCDA, where the general idea of outranking is globally implemented: QUALIFLEX, REGIME, ORESTE, ARGUS, EVAMIX, TACTIC and MELCHIOR, these methods are not too complex and do not introduce the mathematical programming within their algorithm as it is the case, for example, in [3]. We present also EVAMIX even if it was been developed for ordinal and cardinal evaluations.

2.1 QUALIFLEX

The starting point of QUALIFLEX [28, 27] was a generalization of Jacquet-Lagrèze's permutation method [8].

It is a metric procedure and it is based on the evaluation of all possible rankings (permutations) of alternatives under consideration. Its mechanism of aggregation is based on Kemeny and Snell's rule.

This method is based on the comparison among the comprehensive ranking of the alternatives and the evaluations of alternatives according to each criterion from family F (impact matrix). These evaluations are ordinal and take the form of preorders. For each permutation, one computes a concordance/discordance index for each couple of alternatives, that reflects the concordance and the discordance of their ranks and their evaluation preorders from the impact matrix. This index is firstly computed at the level of single criterion, after at a comprehensive level with respect to all possible rankings. One tries to identify the permutation that maximizes the value of this index, i.e. the permutation whose ranking best reflects (the best compromise between) the preorders according to each criterion from F and the multi-criteria evaluation table.

The information concerning the coefficients of relative importance (weights) of criteria may be explicitly known or expressed as a ranking (for example a preorder). In this case, [27] has show that one can circumscribe the exploration to extreme points (the vertex) of polyhedron formed by the feasible weights.

Given the set of alternatives A, the concordance/discordance index for each couple of alternatives $(a, b), a, b \in A$, at the level of preorder according to the criterion $g_j \in F$ and the ranking corresponding to the k^{th} permutation is:

$$I_{jk}(a,b) = \begin{cases} 1 & \text{if there is concordance} \\ 0 & \text{if there is ex aequo} \\ -1 & \text{if there is discordance.} \end{cases}$$

There is concordance (discordance) if a and b are ranked (not ranked) in the same order within the two preorders, and *ex aequo* if they have the same rank. The concordance/discordance index between the pre-order according to the criterion g_j and the ranking corresponding to the k^{th} permutation is:

$$I_{jk} = \sum_{a,b \in A} I_{jk}(a,b).$$

The comprehensive concordance/discordance index for the k^{th} permutation is:

$$I_k = \sum_j \pi_j I_{jk}(a,b),$$

where π_j is the weight of criterion g_j, $j = 1, 2, \ldots, n$. The number of permutations k (Per_k) is $m!$ where $m = |A|$. The best compromise corresponds to the permutation that maximize I_k. If π_j are not explicitly known, but expressed by a ranking, then the best compromise is the permutation that:

$$\max_{P(\pi_j)} I_k,$$

where $P(\pi_j)$ is the set of feasible weights

EXAMPLE 6 *Given 3 alternatives* $a_1, a_2, a_3 \in A$; *3 criteria* g_1, g_2, g_3 *and the evaluation table (see Table 6.1 where a rank number 1 indicates the best outcome, while a rank 3 is assigned to the worst outcome with respect to each criterion), there are 3! possible permutations:*

$$\begin{aligned} Per_1 &: \ a_1 > a_2 > a_3 \\ Per_2 &: \ a_2 > a_1 > a_3 \\ Per_3 &: \ a_2 > a_3 > a_1 \\ Per_4 &: \ a_3 > a_2 > a_1 \\ Per_5 &: \ a_3 > a_1 > a_2 \\ Per_6 &: \ a_1 > a_3 > a_2. \end{aligned}$$

One index is computed for each pair (g_j, Per_k)*, that, for our example, give a total of 18 concordance/discordance indices. For example for the pair* (g_1, Per_1)*, we have for the criterion* g_1*:* $a_1 > a_2$, $a_2 \approx a_3$, $a_1 > a_3$, *and for*

Table 6.1. **Rank evaluation of alternatives (impact matrix).**

		Criterion		
		g_1	g_2	g_3
Alternative	a_1	1	2	1
	a_2	2	1	3
	a_3	2	3	2

Table 6.2. **The concordance/discordance indices.**

		Criterion		
		g_1	g_2	g_3
Permutation	Per_1	2	1	1
	Per_2	0	3	-1
	Per_3	-2	1	-3
	Per_4	-2	-1	-3
	Per_5	0	-3	1
	Per_6	2	-1	3

the Per_1*:* $a_1 > a_2$, $a_1 > a_3$, $a_2 > a_3$, *that gives* +1 *for the couple* (a_1, a_2), +1 *for the couple* (a_1, a_3) *and* 0 *for the couple* (a_2, a_3). *Thus, the value of the index* I_{11} *is equal to 2.*

The concordance/discordance indices are given in the Table 6.2.

Concerning the weights, for example:

1 *If the three criteria have the same importance, i.e.* $\pi_j = \frac{1}{3}, j = 1, 2, 3,$ *then we obtain that the maximum value of the index is* $\frac{4}{3}$ *for the permutations* Per_1 *and* Per_6.

2 *If we know that* $\pi_1 \geq \pi_2$, $\pi_2 \geq \pi_3$ *and* $\pi_j \geq 0$ *for all j, then* $\pi_3 = 1 - \pi_1 - \pi_2$ *(see Figure 6.1).*

Then, to obtain the permutation that maximizes the index I_k*, we must check for the three vertices* $(1, 0)$, $(\frac{1}{2}, \frac{1}{2})$ *and* $(\frac{1}{3}, \frac{1}{3})$*. The maximum value of the index is equal to 2 for the permutations* Per_1 *and* Per_6*, for the weights (1,0,0).*

The result of this method is a ranking of alternatives under consideration. QUALIFLEX is based on pairwise criterion comparison of alternatives, but no outranking relation is constructed. An important limitation of this method concerns the fact that the number of permutations increases tremendously with the number of alternatives. This problem may be solved. Ancot [1] formulated this problem as a particular case of Quadratic Assignment Problem; this algorithm is implemented in the software MICROQUALIFLEX.

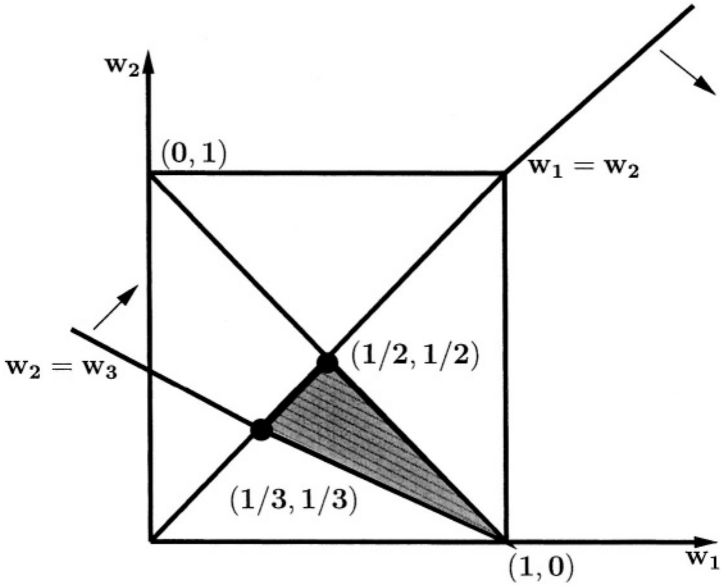


Figure 6.1. Set of feasible weights.

2.2 REGIME

The REGIME method [9, 10] can be viewed as an ordinal generalization of pairwise comparison methods such as concordance analysis. The starting point of this method is the concordance C_{il} defined in the following way:

$$C_{il} = \sum_{j \in \hat{C}_{il}} \pi_j,$$

where $\hat{C}_{il}$ is the concordance set, i.e. the set of criteria for which a_i is at least as good as a_l, a_i and $a_l \in A$ and π_j is the weight of criterion $g_j \in F$. The focus of this method is on the sign of $C_{il} - C_{li}$ for each pair of alternatives. If this sign is positive, alternative a_i is preferred to a_l; and the reverse if the sign is negative.

The first step of the REGIME method is the construction of the so-called regime matrix. The regime matrix is formed by pairwise comparison of alternatives in the multi-criteria evaluation table. Given a and $b \in A$, for every criterion we check whether a has a better rank than b, then on the corresponding place in the regime matrix the number +1 is noted, while if b is a better position than a, the number – 1 is the result.

More explicitly, for each criterion $g_j, j = 1, 2, \cdots, n$, we can defined an indicator $c_{il,j}$ for each pair of alternatives (a_i, a_l).

$$c_{il,j} = \begin{cases} +1 & \text{if } r_{ij} < r_{lj} \\ 0 & \text{if } r_{ij} = r_{lj} \\ -1 & \text{if } r_{ij} > r_{lj}, \end{cases}$$

where r_{ij} (r_{lj}) is the rank of the alternative a_i (a_l) according to criterion g_j. When two alternatives are compared on all criteria, it is possible to form a vector

$$c_{il} = \left(c_{il,1}, \cdots, c_{il,j}, \cdots, c_{il,n}\right)$$

that is called a regime and the regime matrix is formed of these regimes. These regimes will be used to determine rank order of alternatives.

The concordance index, in favor of the alternative a_i, is given by:

$$C_{il} = \sum_j \pi_j c_{il,j},$$

If the π_j are explicitly known, we can obtain a concordance matrix $\mathbf{C} = [C_{il}]$, with zero on the main diagonal (Table 6.3).

Table 6.3. Concordance matrix.

$$\begin{array}{c} \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccccccccc} a_1 & \cdots & \cdots & \cdots & \cdots & \cdots & a_l & \cdots & \cdots & a_m \end{array} \\ \begin{bmatrix} 0 & & & & & & \vdots & & & \\ & & & & & & \vdots & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & C_{il} & \cdots & \cdots & \cdots \\ & & & & & & \vdots & & & \\ & & & & & & \vdots & & & 0 \end{bmatrix} \end{array}$$

One half of this matrix can be ignored, since $C_{il} = -C_{li}$.

In general the available information concerning the weights is not explicit (not quantitative) and then it is not possible to compute the matrix $\mathbf{C}$. If the available information concerning the weights is ordinal, the sign of C_{il} may be determined with certainty only for some regimes [30]. For others regimes a such unambiguous result can not be obtained; such regime is called critical regime.

EXAMPLE 7 *We can illustrated this method on the basis of multi-criteria evaluation table with 3 alternatives and 4 criteria (Table 6.4, [10]).*

Table 6.4. Rank evaluation of alternatives (impact matrix).

		Criterion			
		g_1	g_2	g_3	g_4
Alternatives	a_1	3	1	1	2
	a_2	2	2	3	1
	a_3	1	3	2	3

Table 6.5. Regime matrix.

		Criterion			
		g_1	g_2	g_3	g_4
Comparison	(a_1,a_2)	−1	+1	+1	−1
	(a_1,a_3)	−1	+1	+1	+1
	(a_2,a_1)	+1	−1	−1	+1
	(a_2,a_3)	−1	+1	−1	+1
	(a_3,a_1)	+1	−1	−1	−1
	(a_3,a_2)	+1	−1	+1	−1

For this example, the regime matrix is presented in the Table 6.5.

If we make the hypothesis that $\pi_1 = \pi_2 = \pi_3 = \pi_4 = \frac{1}{4}$, wefind $C_{12} = 0$, $C_{13} > 0$, $C_{21} = 0$, $C_{23} = 0$, $C_{31} < 0$ and $C_{32} = 0$. Thus a_1 is preferred to a_3, but we can not conclude between a_1 and a_2, a_2 and a_3. If we know for example that:

$$\pi_2 \geq \pi_4 \geq \pi_3 geq \pi_1, \sum_j \pi_j = 1 \text{ and } \pi_j \geq 0,$$

then we find that $C_{12} =$ - $\pi_1 + \pi_2 + \pi_3$ - $\pi_4 \geq 0$ in all cases, which means that, on the basis of a pairwise comparison, a_1 is preferred to a_2. In a similar way it can be shown that, given the same information on the weights, a_1 is preferred to a_3, and that a_2 is preferred to a_3. Thus we arrive at a transitive rank order of alternatives.

It is not possible to arrive at such definite conclusions for all rankings of the weights. If we assume that:

$$\pi_1 \geq \pi_2 \geq \pi_3 \geq \pi_4, \sum_j \pi_j = 1 \text{ and } \pi_j \geq 0,$$

it is easy to see that from the first regime may result both positive and negative values of C_{il}'s. For example if π = (.40, .30, .25, .05), $C_{12} > 0$, whereas for π = (.45, .30, .15, .10), $C_{12} < 0$. Therefore, the corresponding regime is called a critical regime. The main idea of regime analysis is to circumvent these difficulties by partitioning the set of feasible weights so that for each region a final conclusion can be drawn about the sign of C_{il}.

Let the ordinal information available about the weights be:

$$\pi_1 \geq \pi_2 \geq \pi_3 \geq \pi_4, \sum_j \pi_j = 1 \text{ and } \pi_j \geq 0.$$

The set of weights satisfying this information will be denoted as T. We have to check, for all regimes c_{il}, if c_{il} may assume both positive and negative values, given that π is an element of T. The total number of regimes to be examined is $2^n = 2^4 = 16$. For our example, the number of critical regimes is equal to four:

$$\begin{matrix} -1 & +1 & +1 & -1 \\ +1 & -1 & -1 & +1 \\ -1 & +1 & +1 & +1 \\ +1 & -1 & -1 & -1 \end{matrix}$$

The number of critical regimes is even, since we known that if c_{il} is a critical regime then $c_{li} = -c_{il}$ is critical. The subsets of T can be characterized by means of the structure of the critical regimes. The four critical regimes of our example give two critical equations:

$$\begin{aligned} f_1(\pi) = \pi_1 - \pi_2 - \pi_3 + \pi_4 &= 0 \\ f_2(\pi) = \pi_1 - \pi_2 - \pi_3 - \pi_4 &= 0. \end{aligned}$$

The following subsets of T can be distinguished by means of these equations:

$$\begin{aligned} T_1 &= T \cap \{\pi : f_1(\pi) > 0 \text{ and } f_2(\pi) > 0\}, \\ T_2 &= T \cap \{\pi : f_1(\pi) > 0 \text{ and } f_2(\pi) < 0\}, \\ T_3 &= T \cap \{\pi : f_1(\pi) < 0 \text{ and } f_2(\pi) < 0\}, \\ T_4 &= T \cap \{\pi : f_1(\pi) < 0 \text{ and } f_2(\pi) > 0\}. \end{aligned}$$

An examination of $T_1, \ldots, T_4$ reveals that T_4 is empty, so that ultimately three relevant subsets remain. The subsets T_1, T_2 and T_3 are convex polyhedra, as it is the case for the set T. The extreme points of these polyhedra can be determined graphically in the case of four criteria. The extreme points for T are:

$$\begin{aligned} A &: \quad (1, 0, 0, 0) \\ B &: \quad \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right) \\ C &: \quad \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right) \\ D &: \quad \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) \end{aligned}$$

In addition to these four points, the extreme points

$$E: \quad \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\right) \text{ and}$$
$$F: \quad \left(\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$$

are needed to characterize T_1, T_2 and T_3. The characterization of T_1, T_2 and T_3 by means of the extreme points are for T_1: A, B, E, F; for T_2: B, D, E, F and for T_3: B, C, D, E.

Once the partitioning of the weight set has been achieved, for each subset of T it is possible to indicate unambiguously the sign of C_{il} for each pair of alternatives. Let ν_{il} be defined as follows:

$$\begin{aligned} \nu_{il} &= +1 \text{ if } C_{il} > 0, \\ \nu_{il} &= -1 \text{ if } C_{il} < 0. \end{aligned}$$

Then a pairwise comparison matrix **V** can be constructed consisting of elements equal to +1 or -1, and zeros on the main diagonal. A final ranking of alternatives can be achieved on the basis of **V**.

For example, take an interior point of subset T_1 (e.g. the centroid computed as the mean of the extreme points). Determine the sign of C_{il} for all regimes occurring in the regime matrix (Table 6.5). Thus we find for the pairwise comparison matrix $\mathbf{V_1}$:

$$\mathbf{V_1} = \begin{bmatrix} 0 & -1 & -1 \\ +1 & 0 & -1 \\ +1 & +1 & 0 \end{bmatrix}$$

On the basis of $\mathbf{V_1}$ we may conclude that a_3 is preferred to a_2 which in turn is preferred to a_1. For the two other subsets of weights we find:

$$\mathbf{V_2} = \begin{bmatrix} 0 & -1 & +1 \\ +1 & 0 & -1 \\ -1 & +1 & 0 \end{bmatrix} \qquad \mathbf{V_3} = \begin{bmatrix} 0 & +1 & +1 \\ -1 & 0 & -1 \\ -1 & +1 & 0 \end{bmatrix}.$$

The second pairwise comparison matrix does not give a definitive ranking of alternatives, but on the basis of $\mathbf{V_3}$ we may conclude that a_1 is preferred to a_3 which is again preferred to a_2.

The relative size of subsets T_1, T_2 and T_3 are not equal. If we assume that the weights are uniformly distributed in T, the relative size of the subsets of T can be interpreted as the probability that alternative a_i is preferred to a_l. Probabilities are aggregated to produce an overall ranking of alternatives. The relative sizes of the subsets can also be estimated using a random generator. This is recommended if there are seven criteria or more, since the number of subsets increases exponentially with the number of criteria [30].

The relevant subsets given an arbitrary number of criteria can be found in [10]. The REGIME method can be applied to mixed evaluations (ordinal and cardinal criteria) without losing the information contained in the quantitative evaluation. This requires a standardization of the quantitative evaluation. Israels and Keller [12] has been proposed a variant of REGIME method where the incomparability is accepted. The REGIME method is implemented in a system to support Decision on a finite set of alternatives: DEFINITE [13].

2.3 ORESTE

ORESTE (see [32, 33]) has been developed to deal with the situation where the alternatives are ranked according to each criterion and the criteria themselves are ranked according to their importance. In fact the ORESTE method can deal with the following multi-criteria problem. Let A be a finite set of alternatives a_i, $i = 1, 2, \cdots, m$. The consequences of the alternatives are analysed by a family F of n criteria. The relative importance of the criteria is given by a preference structure on the set of criteria F, which can be defined by a complete preorder S (the relation $S = I \cup P$ is strongly complete and transitive, the indifference I is symmetric and the preference P is asymmetric). For each criterion g_j, $j = 1, 2, \cdots, n$, we consider a preference structure on the set A, defined by a complete preorder. The objective of the method is to find a global preference structure on A which reflects the evaluation of alternatives on each criterion and the preference structure among the criteria.

The ORESTE method operates in three distinct phases:

First phase. Projection of the position-matrix.

Second phase. Ranking the projections.

Third phase. Aggregation of the global ranks.

We start from n complete preorders of the alternatives from A related to the n criteria, (for each alternative is given a rank with respect to each criterion). Also for each criterion is given a rank related to its position in the complete preorder among the criteria. The mean rank discussed by Besson [2] is used. For example, if the following preorder is given for the criteria $g_1 P g_2 I g_3 P g_4$, then $r_1 = 1, r_2 = r_3 = 2.5$ and $r_4 = 4$, where r_j is the Besson-rank of criterion g_j; idem for the alternatives, $r_j(a)$ is the average (Besson) rank of alternative a with respect to the criterion g_j. Given $\{r_j(a), r_j\}$, ORESTE tries to build a preference structure $O = \{I, P, R\}$ on A such as:

- $a_i P a_l$ if a_i is comprehensively preferred to $a_l (O_{il} = 1, O_{li} = 0)$,

- $a_i I a_l$ if a_i is indifferent to $a_l (O_{il} = O_{li} = 1)$,
- $a_i R a_l$ if a_i and a_l are comprehensively incomparable $(O_{il} = O_{li} = 0)$.

Projection. Considering an arbitrary origin 0, a distance $d(0, a_j)$ is defined with the use of $\{r_j(a), r_j\}$ such that $d(0, a_j) < d(0, b_j)$ if aP_jb, where $a_j = g_j(a)$ is the evaluation of alternative a with respect to criterion g_j. When ties occur, an additional property is: if $g_j I g_k$ and $r_j(a) = r_k(b)$ then $d(0, a_j) = d(0, b_k)$. For the author, the "city-block" distance is adequate:

$$d(0, a_j) = \alpha r_j(a) + (1 - \alpha) r_j,$$

where α stands for a substitution rate $(0 < \alpha < 1)$. The projection may be performed in different ways [29, 33].

EXAMPLE 8 *Given the following example with 3 alternatives and 3 criteria (without ties). The complete preorders of alternatives are:* aP_1bP_1c, bP_2cP_2a *and* cP_3aP_3b, *and for the criteria:* $g_1 P g_2 P g_3$. *This example may be visualized by a position matrix (Table 6.6).*

Table 6.6. Position-matrix.

$$r_j(.) : \quad \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \end{array}$$

Being $r_1 = 1$, $r_2 = 2$, $r_3 = 3$, *the city-block distance for this example is given in Table 6.7.*

Table 6.7. City-block.

$$d(0, a_j) : \quad \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 2+\alpha & 3-\alpha \\ 1+\alpha & 2-\alpha & 3 \\ 1+2\alpha & 2 & 3-2\alpha \end{bmatrix} \end{array}$$

Ranking. Since it is the relative position of projections that is important and not the exact value of $d(0, a_j)$, the projections will be ranked. To rank the projections a mean rank $R(a_j)$ is assigned to a pair (a, g_j) such that $R(a_j) \leq R(b_k)$ if $d(0, a_j) \leq d(0, b_k)$. These ranks are called comprehensive ranks and are in the closed interval $(1, mn)$. For our example $R(a_1) < R(b_2)$ since $1 < 2 - \alpha$ $(0 < \alpha < 1)$.

Aggregation. For each alternative one computes the summation of their comprehensive ranks over the set of criteria. For an alternative a this yields the final aggregation

$$R(a) = \sum_j R(a_j).$$

For our example, if $\frac{1}{3} < \frac{\alpha}{2} < \frac{1}{2}$ we obtain:

$$\begin{array}{ccccccccccccc} 1 & < & 1+\alpha & < & 2-\alpha & < & 1+2\alpha & < & 2 & < & 3-2\alpha & < & 2+\alpha \\ R(a_1) & < & R(b_1) & < & R(b_2) & < & R(c_1) & < & R(c_2) & < & R(c_3) & < & R(a_2) \\ 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 \\ & < & 3-\alpha & < & 3 & & & & & & & & \\ & < & R(a_3) & < & R(b_3) & & & & & & & & \\ & & 8 & & 9 & & & & & & & & \end{array}$$

$$R(a) = 16, R(b) = 14, R(c) = 15.$$

In the ORESTE method, the following index is also computed:

$$C(a, b) = \sum_{j:aP_jb} [R(b_j) - R(a_j)].$$

It is easily shown that $C(a, b) - C(b, a) = R(b) - R(a)$. Moreover, the maximum value of $R(b) - R(a)$ equals $n^2(m - 1)$.

For our example with $\frac{1}{3} < \alpha < \frac{1}{2}$, we obtain: $C(c, b) = 3$, $C(a, b) = 2$ and $C(a, c) = 3$. Thus, we may obtain the preference structure $O = \{I, P, R\}$ in such way that if $R(a) \leq R(b)$ then aIb or aPb or aRb, where β stands for an indifference level and γ for an incomparability level (see Figure 6.2).

For our example with $\frac{1}{3} < \alpha < \frac{1}{2}$, we have $\frac{C(c,b)}{R(c)-R(b)} = 3$, $\frac{C(a,b)}{R(a)-R(b)} = 1$ and $\frac{C(a,c)}{R(a)-R(c)} = 3$. Thus, if $\beta = \frac{1}{18} = \frac{1}{n^2(m-1)}$ and $1 \leq \gamma \leq 3$ we obtain bPa, aRc and cRb.

These thresholds are interpreted in [29]. When $\gamma = \infty$, the outranking relation is a semi-order which becomes a weak order if $\beta = 0$.

The global preference relation P built by ORESTE is transitive [29]. The axiom known as the Pareto principle or citizen's sovereignty holds if $\beta < \frac{1}{n(m-1)}$, but the axiom of independence of irrelevant alternatives is generally violated [33].

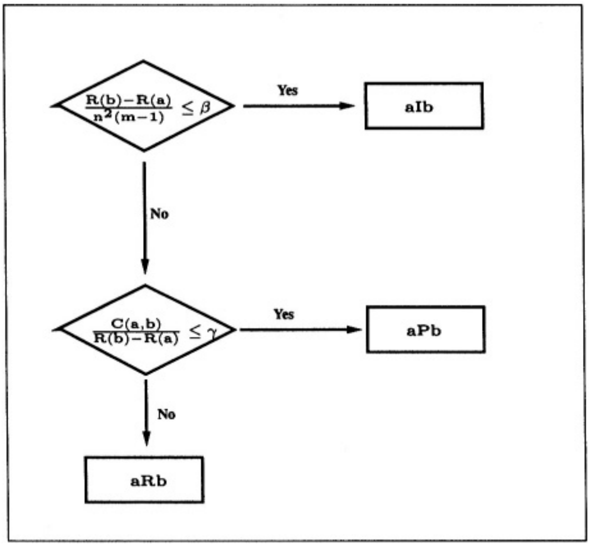


Figure 6.2. ORESTE flow chart.

2.4 ARGUS

The ARGUS method [14] uses qualitative values for representing the intensity of preference on an ordinal scale. They express this intensity of preference between two alternatives $a, b \in A$ by selecting one of the following qualitative relations: indifference, small, moderate, strong or very strong preference. All evaluation on the criteria are treated as evaluations on an ordinal scale, but the evaluations of each alternative with respect to each criterion can be quantitative (interval or ratio scale) or qualitative (ordinal scale).

The way of obtaining the required information from the decision maker (DM) to model his/her preference structure, depends on the scale of measurement of each criterion. If the scale is ordinal, we may use the following possible values: very poor, poor, average, good, very good. To model the preference structure of the DM on this criterion, the DM must indicate his/her preference for each pair of values. He must construct a preference matrix (Table 6.8).

In fact the DM must fill only the lower triangle of this matrix. The number of rows and columns of this matrix depends on the number of different values the ordinal criterion can have. The preference of the DM on an interval scale criterion will depend on $d = g_j(a) - g_j(b)$, while his/her preference on a ratio scale criterion will depend either on d only or on d, $g_j(a)$ and $g_j(b)$. For example, if his/her preference depends on d only, this means that only

Table 6.8. Preference matrix for a criterion with ordinal evaluation.

$g_i(b)$	very poor	poor	average	good	very good
$g_i(a)$ very poor	indiff.				
poor		indiff.			
average			indiff.		
good				indiff.	
very good					indiff.

the absolute difference determines his/her preference. The preference structure of the DM for an interval scale criterion can be modeled by determining for which absolute difference d the DM is indifferent, for which d he/she has a moderate preference, for which d he/she has a strong and for which d he has a very strong preference. For a ratio scale criterion, he/she can also consider the relative difference δ (see Table 6.9). We must indicate if the criterion must be MIN or MAX.

Table 6.9. Preference matrix for a criterion (Max) with evaluation on a ratio scale.

$g_i(a) \geq g_i(b) > 0$	$d = g_i(a) - g_i(b)$	$\delta = \frac{g_i(a)-g_i(b)}{g_i(b)}$
indifferent	$0 \leq d < d_1$	$0\% \leq \delta < \delta_1\%$
small preference	$d_1 \leq d < d_2$	$\delta_1\% \leq \delta < \delta_2\%$
moderate preference	$d_2 \leq d < d_3$	$\delta_2\% \leq \delta < \delta_3\%$
strong preference	$d_3 \leq d < d_4$	$\delta_3\% \leq \delta < \delta_4\%$
very strong preference	$d_4 \leq d$	$\delta_4\% \leq \delta$

The following ordinal scale may be used to reflect the importance of a criterion: not important, small, moderately, very and extremely important. The DM must indicate for each criterion, by selecting a value from this ordinal scale, how important one criterion is for him/her.

When the preference structure of the DM for each criterion is known as well as the importance of each criterion, the comparison of two alternatives a and b with respect to the criterion g_j leads to a two-dimensional table (Table 6.10).

In a cell, f_{st} stands for the number of criteria of a certain importance for which a certain preference between the alternatives a and b occurs, $\sum_s \sum_t f_{st} = n$.

In order to get one overall appreciation of the comparison between the alternatives a and b, the DM must rank all cells of Table 6.10 where $g_j(a) > g_j(b)$. A ranking in eight classes is proposed to DM. Through this ranking a one dimensional ordinal variable is created. In fact there is a combined preference with respect to difference on evaluations and importance of weights where $g_j(a) > g_j(b)$ and where $g_j(a) < g_j(b)$ (see Table 6.11).

Table 6.10. Preference importance table for g_j, a, b.

	criteria preference	not imp.	little imp.	moderate imp.	very imp.	extremly imp.	w_j
$g_j(a) > g_j(b)$	very strong	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	a
	strong	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	b
	moderate	f_{31}	f_{32}	f_{33}	f_{34}	f_{35}	$\vdots$
	small	f_{41}	f_{42}	f_{43}	f_{44}	f_{45}	$\vdots$
$g_j(a) = g_j(b)$	no	f_{51}	f_{52}	f_{53}	f_{54}	f_{55}	$\vdots$
$g_j(a) < g_j(b)$	small	f_{61}	f_{62}	f_{63}	f_{64}	f_{65}	$\vdots$
	moderate	f_{71}	f_{72}	f_{73}	f_{74}	f_{75}	$\vdots$
	strong	f_{81}	f_{82}	f_{83}	f_{84}	f_{85}	b
	very strong	f_{91}	f_{92}	f_{93}	f_{94}	f_{95}	a

Table 6.11. Combined preferences with weights variable.

	$g_j(a) > g_j(b)$	$g_j(a) < g_j(b)$
1	$u_1 = f_{15}$	$\nu_1 = f_{95}$
2	$u_2 = f_{14} + f_{25}$	$\nu_2 = f_{85} + f_{94}$
3	$u_3 = f_{13} + f_{24} + f_{45}$	$\nu_3 = f_{75} + f_{84} + f_{93}$
4	$u_4 = f_{12} + f_{23} + f_{34} + f_{45}$	$\nu_4 = f_{65} + f_{74} + f_{93} + f_{92}$
5	$u_5 = f_{11} + f_{22} + f_{33} + f_{44}$	$\nu_5 = f_{64} + f_{73} + f_{82} + f_{91}$
6	$u_6 = f_{21} + f_{32} + f_{43}$	$\nu_6 = f_{63} + f_{72} + f_{81}$
7	$u_7 = f_{31} + f_{42}$	$\nu_7 = f_{62} + f_{71}$
8	$u_8 = f_{41}$	$\nu_8 = f_{61}$

The decision maker can alter this ranking (by moving a cell from one class to another, by considering more or less classes) until it matches his/her personal conception. Based on those two variables an outranking (S), indifference (I) or incomparability (R) relation between two alternatives is constructed:

$$\text{if } \sum_{k=1}^{h} u_k = \sum_{k=1}^{h} v_k \text{ for all } h = 1, \ldots, 8, \text{ then } aIb;$$

$$\text{if } \sum_{k=1}^{h} u_k \geq \sum_{k=1}^{h} v_k \text{ for all } h = 1, \ldots, 8, \text{ then } aSb;$$

$$\text{if } \sum_{k=1}^{h} u_k \leq \sum_{k=1}^{h} v_k \text{ for all } h = 1, \ldots, 8, \text{ then } bSa.$$

in all other cases aRb.

According to the basic idea of outranking, if alternative a is much better than alternative b on one (or more) criteria and b is much better than a on other criteria, there can be discordance between alternative b and alternative a, and b will not outranking a. The DM must explicitely indicate for each criterion when there is discordance between two evaluations on that particular criterion. For an ordinal criterion he/she can indicate in the upper triangle of the preference matrix (Table 6.8) when discordance occurs. For an interval or ratio criterion, the DM must indicate from which difference (absolute or relative), between the evaluations of two alternatives on that criterion, there is discordance.

EXAMPLE 9 *We have 4 alternatives, 4 criteria and the evaluation table (Table 6.12). In this example, the criteria g_1, g_2, g_3 are ordinal scales, and criterion g_4 is a ratio scale to be minimized.*

Table 6.12. **Evaluation of alternatives*.**

	g_1	g_2	g_3	g_4
a_1	□	⊕	−	13
a_2	⊕	−	□	10
a_3	□	−	−	17
a_4	+	□	□	17

* ⊕ : *very good*; +: *good*; □ : *acceptable*; − : *moderate*.

The following dominance relation can be observed from the data: $a_4 D\, a_3$, so that after deleting a_3, the set of alternatives is $A = \{a_1, a_2, a_4\}$. It is necessary to make this pre-processing step.

The preference modeling ofalternatives with respect to the criteria are given in Tables 6.13, 6.14, and 6.15.

Table 6.13. **Criteria g_1 and g_3 (ordinal scales).**

$g_2(b)$	⊖	−	□	+	⊕
$g_2(a)$ ⊖	indifferent			discordance	discordance
−	moderate	indifferent			discordance
□	strong	moderate	indifferent		
+	very strong	strong	moderate	indifferent	
⊕	very strong	very strong	strong	small	indifferent

The preference structure of weights of the criteria is given in Table 6.16.

Suppose that the ranking in eight classes ofthe combined preference with weight oftwo alternatives presented in Table 6.11 is approved. Table 6.17 gives an example ofa pairwise comparison between a_1 and a_4.

Table 6.14. Criterion g_2 (ordinal scale).

$g_6(b)$	$\ominus$	$-$	$\square$	$+$	$\oplus$
$g_6(a)$ $\ominus$	indifferent				discordance
$-$	small	indifferent			
$\square$	moderate	small	indifferent		
$+$	strong	moderate	small	indifferent	
$\oplus$	very strong	strong	moderate	small	indifferent

Table 6.15. Criterion g_4 (ratio scale MIN).

Preference (a above b)	$d = g_j(a) > g_j(b)$
Indifferent	$0 \leq d < 1$
small	$1 \leq d < 3.5$
moderate	$3.5 \leq d < 6$
strong	$6 \leq d < 9$
very strong	$9 \leq d < \infty$
discordance	$d < -\infty$

Table 6.16. Preference structure of weights.

Weight	
not important	
little important	g_1, g_3
moderately important	g_4
very important	g_2
extremely important	

Table 6.17. Pairwise comparison between a_1 and a_4.

	$g_j(a_1) > g_j(a_4)$	$g_j(a_1) < g_j(a_4)$
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0
6	0	2
7	0	0
8	0	0

The pairwise comparison of all pair of alternatives from A permits to construct the following binary relations: $a_1 S a_4$, $a_1 R a_2$ *and* $a_2 S a_4$ *(see Figure 6.3).*

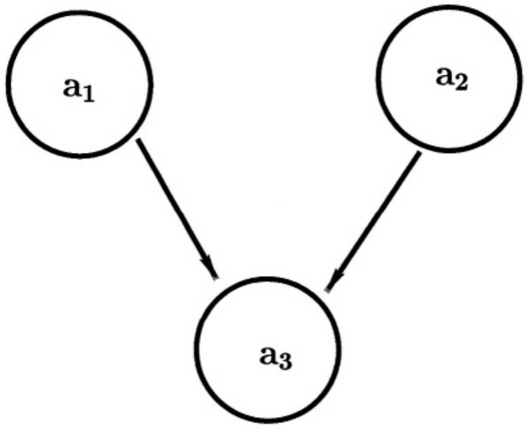


Figure 6.3. Outranking graph.

The ARGUS method demands a relatively great effort from the DM to model his/her preferences.

2.5 EVAMIX

The EVAMIX method [30, 39, 40] is a generalization of concordance analysis in the case of mixed information on the evaluation of alternatives on the judgment criteria. Thus a pairwise comparison is made for all pairs of alternatives to determine the so called concordance and discordance indices. The difference with standard concordance analysis is that separate indices are constructed for the qualitative and quantitative criteria. The comprehensive ranking of alternatives is the result of a combination of the concordance and discordance indices for the qualitative and quantitative criteria.

The set of criteria in the multi-criteria evaluation table is divided into a set of qualitative (ordinal) criteria O and a set of quantitative (cardinal) criteria C. It is assumed that the differences between alternatives can be expressed by means of two dominance measures: a dominance score $\alpha_{ii'}$ for the ordinal criteria, and a dominance score $a_{ii'}$ for the cardinal criteria. These scores represent the degree to which alternative a_i dominates alternative $a_{i'}$. They have the following structure:

$$\begin{aligned} \alpha_{ii'} &= f(e_{ij}, e_{i'j}, \pi_j), \text{ for all } j \in O, \\ a_{ii'} &= g(e_{ij}, e_{i'j}, \pi_j), \text{ for all } j \in C, \end{aligned}$$

where e_{hj} represents the evaluation of alternative a_h on the criterion g_j and π_j the importance weight associated to this criterion. These scores can be defined as follows:

$$\alpha_{ii'} = \left[\sum_{j \in O} \{\pi_j sgn(e_{ij} - e_{i'j})\}^c \right]^{\frac{1}{c}},$$

where

$$\text{sgn}(e_{ij} - e_{i'j}) = \begin{cases} +1 & \text{if } e_{ij} > e_{i'j} \\ 0 & \text{if } e_{ij} = e_{i'j} \\ -1 & \text{if } e_{ij} < e_{i'j}. \end{cases}$$

The symbol c denotes an arbitrary scaling parameter, for which any positive odd value may be chosen, $c = 1, 3, 5, \ldots$. In a similar manner, the quantitative dominance measure can be made explicit:

$$a_{ii'} = \left[\sum_{j \in C} \{\pi_j (e_{ij} - e_{i'j})\}^c \right]^{\frac{1}{c}}.$$

In order to be consistent, the same value for the scaling parameter c should be used as in formula for $\alpha_{ii'}$. It is assumed that the quantitative employed evaluation e_{ij} have been standardized $(0 \le e_{ij} \le 1)$. Evidently, all standardized scores should have the same direction, i.e., a 'higher' score should (for instance) imply a 'larger' preference. It should be noted that the rankings e_{ij} $(j \in O)$ of the qualitative criteria also have to represent 'the higher, the better'. Since $\alpha_{ii'}$ and $a_{ii'}$ will have different measurement units, a standardization into the same unit is necessary. The standardized dominance measures can be written as:

$$\delta_{ii'} = h(\alpha_{ii'}) \text{ and } d_{ii'} = h(a_{ii'}),$$

where h represents a standardization function.

Let us assume that weights π_j have quantitative properties. The overall dominance measure $D_{ii'}$ for each pair of alternatives $(a_i, a_{i'})$ is:

$$D_{ii'} = \pi_o \delta_{ii'} + \pi_c d_{ii'},$$

where $\pi_o = \sum_{j \in O} \pi_j$ and $\pi_c = \sum_{j \in C} \pi_j$. This overall dominance score reflects the degree to which alternative a_i dominates alternative $a_{i'}$ for the given set of criteria and the weights. The last step is to determine an appraisal score s_i for each alternative. In general the measure $D_{ii'}$ may be considered as function k of the constituent appraisal scores, or:

$$D_{ii'} = k(s_i, s_{i'}).$$

This expression represents a well-known pairwise comparison problem. Depending on the way function k is made explicit, the appraisal scores can be calculated. The most important assumptions behind the EVAMIX method concern the definition of the various functions. It is shown in [40], that at least three different techniques can be distinguished which are based on different definitions of $\delta_{ii'}$, $d_{ii'}$ and $D_{ii'}$. The most straightforward standardization is probably the additive interval technique. The overall dominance measure $D_{ii'}$ is defined as:

$$D_{ii'} = \frac{s_i}{s_i + s_{i'}},$$

which implies that $D_{ii'} + D_{i'i} = 1$. To arrive at such overall dominance measures with this additivity characteristic, the following standardization is used:

$$\delta_{ii'} = \frac{(\alpha_{ii'} - \alpha^-)}{(\alpha^+ - \alpha^-)} \text{ and } d_{ii'} = \frac{(a_{ii'} - a^-)}{(a^+ - a^-)}$$

where α^- (α^+) is the lowest (highest) qualitative dominance score of any pair of alternatives $(a_i, a_{i'})$ and a^- (a^+) is the lowest (highest) quantitative dominance score of any pair of alternatives $(a_i, a_{i'})$. The resulting appraisal score is:

$$s_i = \left[\sum_{i'} \frac{D_{i'i}}{D_{ii'}}\right]^{-1}.$$

This expression means that the appraisal scores add up to unity, $\sum_i s_i = 1$.

In the previous elaboration, quantitative weights π_j, $j = 1, 2, \ldots, n$, were assumed. In some circumstances, only qualitative priority expressions can be given. If only ordinal information is given, at least two different approaches may be followed: an expected value approach (see [30, Appendix 4.I]) or a random weight approach. The random weight approach implies that quantitative weights are created by a random selection out an area defined by the extreme weight sets. These random weights γ_j, $j = 1, \cdots, n$, have to fulfill the following conditions:

1 for each $\gamma_j, \gamma_{j'}, \omega_j \leq \omega_{j'} \Rightarrow \gamma_j \geq \gamma_{j'}$,

2 $\sum_j \gamma_j = 1$,

where ω_j denotes a ranking number expressing a qualitative weight with "lower" means "better". For each set of metric weights γ_j, $j = 1, \cdots, n$, generated during one run of the random number generator, a set of appraisal scores can be determined. By repeating this procedure many times a frequency matrix can be constructed. Its element f_{ri} represents the number of times, alternative a_i was placed in the $r - th$ position in the final ranking. A probability matrix with element p_{ri} can be constructed, where:

$$p_{ri} = \frac{f_{ri}}{\sum_i f_{ri}}.$$

So, p_{ri} represents the probability that a_i will receive an $r-th$ position. We can make a comprehensive ranking of the alternatives in the following way:

$$\begin{aligned} a_i &= 1, \text{ if } p_{1i} \text{ is maximal}, \\ a_{i'} &= 2, \text{ if } p_{1i} + p_{2i'} \text{ is maximal and } i' \neq i, \\ a_{i''} &= 3, \text{if } p_{1i} + p_{2i'} + p_{3i''} \text{ is maximal and } i'' \neq i' \neq i, \end{aligned}$$

and so forth.

The EVAMIX method is based on important assumptions: 1) the definition of the various functions f, g, h and k; 2) the definition of the weights of the sets O and C and 3) the additive relationship of the overall dominance measure.

2.6 TACTIC

In the TACTIC method, proposed by Vansnick (see [37]), the family of criteria F is composed of true-criteria or quasi-criteria (criteria with an indifference threshold $q > 0$) $g_j, j = 1, \ldots, n$, and the preference structures correspondent are (P, I) or (P, I, R), where R is the incomparability relation, if no veto-threshold $v_j(\cdot), j \in \mathcal{J}$ is considered or at least one v_j is introduced respectively.

To each criterion $g_j \in F$ an importance weight $\lambda_j > 0$ is associated, as in the ELECTRE methods (see chapter 4 in this book). To model the preferences, the following subset of $\mathcal{J}$ is defined, $\forall a, b \in A, a \neq b$:

$$\mathcal{J}_T(a,b) = \{j \in | : g_j(a) > g_j(b) + q_j[g_j(b)]\},$$

where $q_j[g_j(b)]$ is the marginal indifference threshold as a function of the worst evaluation between $g_j(a)$ and $g_j(b)$, and therefore in this case we have aP_jb.

If in the set F only true criteria are considered, the statement aPb is true if and only if the following *concordance condition* is satisfied:

$$\sum_{j \in \mathcal{J}_T(a,b)} \lambda_j > \rho \sum_{j \in \mathcal{J}_T(b,a)} \lambda_j, \text{ i.e. } \frac{\sum_{j \in \mathcal{J}_T(a,b)} \lambda_j}{\sum_{j \in \mathcal{J}_T(b,a)} \lambda_j} > \rho \text{ if } \mathcal{J}_T(b,a) \neq \emptyset, \quad (6.1)$$

where the coefficient ρ is called required concordance level (usually, $1 \leq \rho \leq \frac{\sum_{j \in I} \lambda_j}{\min_{j \in I} \lambda_j} - 1$) and the two summations represent the absolute importance of the coalition of criteria in favor of a or b respectively.

If also some quasi-criterion is in the set F, in the preference structure (P, I, R) aPb is true if and only if both concordance condition 6.1 and the following *non-veto condition* are satisfied:

$$\forall j \in \mathcal{J}, g_j(a) + v_j[g_j(a)] \geq g_j(b), \tag{6.2}$$

where $v_j[g_j(a)]$ is the marginal veto threshold.

If the condition (6.2) is not satisfied by at least one criterion from *F*, we have aRb. On the other hand, we have aIb if and only if both pairs (a, b) and (b, a) do not satisfy condition (6.1) and no veto situation arises.

We remark that if $\rho = \rho^* = \frac{\sum_{j \in I} \lambda_j}{\min_{j \in I} \lambda_j} - 1$, the condition (6.1) is equivalent to the complete absence of criteria against the statement aPb, i.e. $\mathcal{J}_T(b, a) = \emptyset$ (and therefore in this case, (6.2) automatically holds). If $q_j = 0$ for each criterion g_j, the relation P is transitive for $\rho > \rho^*$. When ρ is decreasing from level ρ^*, we can have two types of intransitivity:

- aPb, bPc, aIc (or aRc),
- aPb, bPc, cPa.

If in equation (6.1) $\rho = 1$, we obtain the basic concordance-discordance procedure of Rochat type:

- for structures (*P, I*) (see [35]),

$$aPb \quad \text{iff} \quad \sum_{j \in \mathcal{J}_T(a,b)} \lambda_j > \sum_{j \in \mathcal{J}_T(b,a)} \lambda_j;$$
$$aIb \quad \text{iff} \quad \sum_{j \in \mathcal{J}_T(a,b)} \lambda_j = \sum_{j \in \mathcal{J}_T(b,a)} \lambda_j;$$

- for structures (*P, I, R*),

$$\begin{aligned}
aPb \quad \text{iff} \quad & \sum_{j \in \mathcal{J}_T(a,b)} \lambda_j > \sum_{j \in \mathcal{J}_T(b,a)} \lambda_j \text{ and} \\
& g_j(b) - g_j(a) \leq v_j[g_j(a)], \forall j \in \mathcal{J}; \\
aIb \quad \text{iff} \quad & \sum_{j \in \mathcal{J}_T(a,b)} \lambda_j = \sum_{j \in \mathcal{J}_T(b,a)} \lambda_j \text{ and} \\
& g_j(b) - g_j(a) \leq v_j[g_j(a)], \forall j \in \mathcal{J}, \text{ and} \\
& g_j(a) - g_j(b) \leq v_j[g_j(a)], \forall j \in \mathcal{J}. \\
aRb \quad \text{iff} \quad & \text{non}(aPb), \text{ non}(bPa) \text{ and non}(aIb).
\end{aligned}$$

The main difference between the ELECTRE I and TACTIC preference modeling is that the latter method is based on the binary relation aPb, while the former aims to build up the outranking relation aSb, $a, b \in A$. Moreover, the validation of the preference relation is now based on a sufficiently large ratio between the importance of criteria in favor and against the statement aPb. Roy and Bouyssou [35] show that this second difference is actually just a formal one. They also remark that, as a consequence of the peculiar characterization of the statement aPb, in TACTIC method is difficult to split indifference and incomparability situations. No particular exploitation procedure is suggested in TACTIC method.

2.7 MELCHIOR

In the MELCHIOR method [16] the basic information is a family F of pseudo-criteria, i.e. criteria g_j with an indifference threshold q_j and a preference threshold p_j ($p_j > q_j \geq 0$) such that, $\forall j \in \mathcal{J}$ and $\forall a, b \in A$:

- a is strictly preferred to b (aP_jb) with respect to g_j iff $g_j(a) > g_j(b) + p_j[(g_j(b)]$,
- a is weakly preferred to b (aQ_jb) with respect to g_j iff $g_j(b)+p_j[(g_j(b)] \geq g_j(a) > g_j(b) + q[(g_j(b)]$,
- a and b are indifferent (aI_jb) iff there is no strict or weak preference between them.

No importance weights are attached to criteria, but a binary relation M is defined on F such that g_iMg_j means "criterion g_i is as least as important as criterion g_j". In order to state the comprehensive outranking relation aSb, the Author proposes to "match" in a particular way the criteria in favor and the criteria against the latter relation (concordance analysis) and to verify that no discordance situation exists, i.e. no criterion g_j from F exists such that $g_j(b) > g_j(a)+v_j$, where v_j is a suitable veto-threshold for criterion g_j (absence of discordance). In this method, a criterion $g_j \in F$ is said to be in favor of the outranking relation aSb if one of the following situations is verified:

- aP_jb (marginal strict preference of a over b) (1st condition)
- aP_jb or aQ_jb (marginal strict or weak preference of a over b) (2nd condition)
- $g_j(a) > g_j(b)$ (3rd condition).

A criterion $g_j \in F$ is said to be against the outranking relation aSb if one of the following situations is verified:

- bP_ja (marginal strict preference of b over a) (1st condition)
- bP_ja or bQ_ja (marginal strict or weak preference of b over a) (2nd condition)
- $g_j(b) > g_j(a)$ (3rd condition).

The *concordance analysis* with respect to the outranking relation aSb, $a, b \in A$, is made by checking if the family of criteria G in favor of this relation "hides" the family of criteria H that are against relation aSb. These subsets of criteria are compared just using the binary relation M on F. A subset G of criteria is said to "hide" a subset H of criteria $(G, H \subset F, F \cap G = \emptyset)$ if, for each criterion g_i from H, there exists a criterion g_j from G such that

- $g_j M g_i$ (1st condition) or

- $g_j M g_i$ or not$(g_i M g_j)$ (2nd condition),

where the same criterion g_j from G is allowed to hide at most one criterion from H.

By choosing two suitable combinations (see [16]) of the above conditions, the first stricter than the other, and verifying the concordance and the absence of discordance, a strong and a weak comprehensive outranking relation can be respectively built up. Then these relations are in turn exploited as in ELECTRE IV method (see chapter 4 in this book). We remark that the latter in fact coincides with MELCHIOR if the same importance is assigned to each criterion.

We finally observe that in both TACTIC and MELCHIOR methods no possibility of interaction among criteria (see Chapter 14 in this book) is taken into consideration, since the first one considers additive weights for the importance of each coalitions of criteria and the last one just "matches" one to one criteria in favor and against the comprehensive outranking relation aSb.

3. Pairwise Criterion Comparison Approach

In this approach, after the evaluations of potential alternatives with respect to different criteria, the phase of building up the outranking relations is split in two different steps, making comparisons at first level (partial aggregation) with respect to each subset of criteria $G_k \subset F$ $(|F| = m, G_k \neq \emptyset, |G| = k, k = 2, 3, \cdots, m-1)$ and then aggregating at the second level these partial results (global aggregation).

With respect to weighting, this way of aggregating preferences allows to take into consideration the marginal *substitution rate* (trade-off) of each criterion from subset G_k at the first step and the *importance* of each coalition of criteria G_k at the second step, with the possibility to explicitly modeling the different meaning of these "weights" and the eventual *interaction* among criteria from each G_k (see chapter 14 in this book). Moreover, peculiar preference attitudes with respect to compensation, indifference and veto relations may be usefully introduced at each step of preference aggregation process; therefore, these particular options may be modelled at "local" and global level, when the partial and aggregated preferences indices respectively are built up.

For $k = 2$, (i.e. when two criteria a time are considered in the first phase of aggregation), we speak of Pairwise Criterion Comparison Approach (PCCA), that is therefore a methodology in which first all the feasible actions are compared with respect to pairs of criteria from *F*, and then all the partial information so obtained are suitably aggregated.

Given $a, b \in A$, in the Multiple Attribute Utility Theory (see chapter 7 in this book) the partial utility functions $u_i[g_i(a)]$, $i \in \mathcal{J}$, are aggregated in different ways to obtain the global utility $u(a)$ of each alternative and then the final recommendation.

In the outranking ELECTRE and PROMETHEE (see Chapter 4 in this book) families methods, from the evaluations of each action with respect to each criterion $g_i \in F$, some (crisp or fuzzy) marginal outranking or preference relations $\phi_i(a, b)$ are built up as elementary indices, or relations, with respect each criterion $i \in \mathcal{J}$ and each (ordered) pair of actions (a, b); then, using these marginal relations and other inter-criteria information, a comprehensive outranking relation or index $\phi(a, b)$ is obtained. In PCCA, in the first stage for each pair of actions (a, b) a fuzzy binary preference index $\delta_{ij}(a, b)$, $i, j \in \mathcal{J}$, is built up as elementary index taking into consideration two different criteria a time; then, by suitable aggregation of these partial indices, a global index $\delta(a, b)$ is obtained, expressing the comprehensive fuzzy preference of a over b.

As in all the other outranking methods, the exploitation of the indices expressing the comprehensive relation allows to obtain the recommendation for the decision problem at hand.

The main reasons that suggest this two levels aggregation procedure are the following:

- limited capacity of the human mind to compare a large number of elements at the same time, taking into consideration numerous and often conflicting evaluations simultaneously;

- limited ability of the DM for assessing a lot of parameters concerning subjective evaluations of general validity and considering all available information together.

Of course, this approach requires a larger number of computations and preference information, but:

- it actually helps in understanding and it supports the entire decision making process itself;

- it allows DM to use in an appropriate way all own preference information, requiring weaker coherence conditions, and to obtain further information about partial comparisons;

- it compares actions with respect to two criteria a time and, then it is easier to set appropriate parameters reflecting the partial comparison at hand;
- it offers greater flexibility in the preference modeling, allowing explicitly the representation of specific preference framework and information DM wants to use each time in the considered comparison;
- it allows useful extensions of some well-known basic concepts, like weighting, compensation, dominance, indifference, incomparability, etc.
- it actually allows to model interaction between each couple of criteria, possibly the most important and really workable in an effective way.

Therefore, in our opinion the PCCA satisfies the following principles, relevant in any decision process, to build up realistic preference models and to obtain actual recommendations:

- *transparency,* making some light in any phase of the "black box" process (about the aggregation procedure in itself, the meaning of each parameter and index, their exploitation, etc.);
- *faithfulness,* respecting accurately the DM's preferences, without imposing too axiomatic constraints;
- *flexibility,* accepting and using any kind of information the DM wants and is able to give, neither more, nor less.

This means that DM will not be forced to "consistency" or "rationality". In other words, not too "external conditions" will be imposed to DM in expressing his/her preferences, but all actual information will be used. So, for example, not transitive trade-offs, $w_{i,k}$, (different from $w_{i,j} \cdot w_{j,k}$ where $w_{r,s}$ is the trade-off between criteria g_r and g_s), and or not complete importance weights (to some criterion no weight is associated) and also aggregated information (i.e., pooled importance weights, reflecting the interaction among criteria of each coalition) will be accepted as input.

Roughly speaking, the PCCA aggregation procedure can be applied to a lot of well-known compensatory or noncompensatory aggregation procedures resulting in binary preference indices. For each $j \in \mathcal{J}$, let $g_j \in F$ be an *interval scale* of measurement (i.e., unique up to a positive linear transformation) and $w_j, w_j \in \mathbb{R}^+$, be a suitable scale constant, called *trade-off* weight or *constant substitution rate,* reflecting (in a compensatory aggregation procedure) the increase on criterion value g_j necessary to compensate a unitary decrease on other criterion from F in terms of global preference. In other words, w_j is used to transform the scale g_j for normalizing and weighting the criteria values in order to compare units on different criterion scales, for each $g_j \in F$. Often this normalization is made introducing two parameters g_j^* and g_{*j}, $j \in \mathcal{J}$, $(g_{*j} < g_j^*)$, usually fixed *apriori* by DM according to the specific decision problem at hand and related with the discrimination power of the criterion scales. These parameters represent, in the DM's view, respectively two suitable "levels" on criterion g_j to normalize its evaluations of feasible actions. For example, g_{*j} and g_j^* can be respectively the "neutral" and the "excellent" level or the minimum and maximum value that can be assumed on criterion g_j; currently, $g_{*j} \leq \min\{g_j(x)\}$ and $g_j^* \geq \max\{g_j(x)\}$. Therefore we can write $w_j = \frac{t_j}{g_j^* - g_{*j}}$, where t_j represent the marginal weight ("importance") of criterion g_j after normalization of its scale.

Let consider the following subsets of $\mathcal{J}$:

$$\begin{aligned}
\mathcal{J}_{a>b} &= \{j \in \mathcal{J} : g_j(a) > g_j(b)\}, \\
\mathcal{J}_{a=b} &= \{j \in \mathcal{J} : g_j(a) = g_j(b)\}, \\
\mathcal{J}_{a<b} &= \{j \in \mathcal{J} : g_j(a) < g_j(b)\};
\end{aligned}$$

In this way, each doubleton $\{a, b\} \subseteq A$ determines a partition of $\mathcal{J}$, (possible an improper one, since some of the three subsets may be empty), whose elements are the subsets of criteria for which there is preference of a over b, indifference of a and b, preference of b over a, respectively.

Moreover, let be

$$\mathcal{J}_{a \geq b} = \{j \in \mathcal{J} : g_j(a) \geq g_j(b)\},$$

i.e. the subset of criteria for which there is a weak preference of a over b.

Let us remember, for example, the following elementary indices:

$$\begin{aligned}
m(a, b) &= |\mathcal{J}_{a>b}| \text{ (majority index)}, \\
\lambda(a, b) &= \sum_{(j \in \mathcal{J}_{a>b})} \lambda_j \text{ (Condorcet index)},
\end{aligned}$$

where $\lambda_j \in \mathbb{R}^+$ is the importance weight associated with criterion $g_j \in F$ and

$$w(a, b) = \sum_{(j \in \mathcal{J}_{a \geq b})} w_j \Delta_j(a, b) \text{ (weighted difference)},$$

where $\Delta_j(a, b) = g_j(a) - g_j(b)$ and all criteria are interval scales.

If we consider the subset of criteria $G = \{g_i, g_j\} \subseteq F$, indicating by f_{ij} any one of the above indices, computed with respect to G, it is possible to derive thence a new binary preference index $\delta_{ij}(a, b)$, defined as follows:

$$\delta_{ij}(a, b) = \begin{cases} \frac{f_{ij}(a,b)}{f_{ij}(a,b) + f_{ij}(b,a)} & \text{if } f_{ij}(a, b) + f_{ij}(b, a) > 0 \\ \frac{1}{2} & \text{if } f_{ij}(a, b) + f_{ij}(b, a) = 0 \end{cases} \quad (6.3)$$

The following properties hold, $\forall(a,b) \in A^2$:

$$0 \leq \delta_{ij} \leq 1, \qquad \delta_{ij}(a,b) + \delta_{ij}(b,a) = 1,$$
$$\delta_{ij}(a,b) \quad = \quad 1 \Leftrightarrow a \text{ partially dominates } b,$$
$$\delta_{ij}(a,b) \quad = \quad 0 \Leftrightarrow b \text{ partially dominates } a,$$

both being partial dominance relations defined with respect to the considered couple of criteria $\{g_i, g_j\} \subseteq F$.

Therefore, the general index $\delta_{ij}(a,b)$, obtained by the PCCA partial aggregation procedure, indicates the credibility of the dominance of a over b with respect to criteria g_i and g_j.

Let now λ_j, $\lambda_j \in \mathbb{R}^+$ be the normalized weight used in a noncompensatory aggregation procedure, called *importance weight,* associated with criterion $g_j \in F$, indicating the intrinsic importance of each criterion, independently by its evaluation scale. Then, we can aggregate the partial indices $\delta_{ij}(a,b)$ computed with respect to all the pairs of different criteria g_i and g_j from F according to the PCCA logic, considering also the normalized *importance weight* λ_{ij} (i.e. $\sum_{i<j} \lambda_{ij} = 1$) *of the coalition* (couple) of criteria g_i and g_j, $i, j \in \mathcal{J}$.

We obtain the following aggregated index:

$$\delta(a,b) = \frac{1}{n-1} \sum_{ij(i<j)} \lambda_{ij} \delta_{ij}(a,b). \tag{6.4}$$

If there is no interaction between these criteria, additive weights can be used in equation (6.4), i.e. $\lambda_{ij} = \lambda_i + \lambda_j$. The following properties hold, $\forall(a,b) \in A^2$ (see Section 3.1):

$$0 \leq \delta(a,b) \leq 1, \qquad \delta(a,b) + \delta(b,a) = 1,$$
$$\delta(a,b) \quad = \quad 1 \text{ if and only if } a \text{ strictly dominates } b,$$
$$\delta(a,b) \quad = \quad 0 \text{ if and only if } b \text{ strictly dominates } a.$$

Therefore, the particular meanings (credibility of dominance) of the partial and global indices $\delta_{ij}(a,b)$ and $\delta(a,b)$ respectively are results essentially linked to the *peculiar aggregation procedure* of PCCA and not to the specific bicriteria index considered each time.

In the framework of the PCCA methodology, different methods have been proposed: MAPPAC, PRAGMA, IDRA, PACMAN, each one with its own features to build up the correspondent outranking relations and indices.

3.1 MAPPAC

We recall that a *dominates* b (aDb), $a, b \in A$, with respect criteria from F if a is at least as good as b for the considered criteria and is strictly preferred to b

for at least one criterion:

$$aDb \Leftrightarrow g_i(a) \geq g_i(b), \forall g_i \in F \text{ and } \exists\, j \in \mathcal{J} : g_j(a) > g_j(b)\,.$$

We say that a *weakly dominates* b (aD_wb) if a is at least as good as b for all the criteria from *F:*

$$aD_wb \Leftrightarrow g_i(a) \geq g_i(b), \forall\, g_i \in F.$$

We say that a *strictly dominates* b (aD_sb) iff $g_i(a) \geq g_i(a)$, $\forall i \in F$, where at most only one equality is valid. The binary relation D_w is a partial preorder (reflexive and transitive), while *D* (and D_s) is a partial order (irreflexive, asymmetric and transitive); the correspondent preference structures are partial order and strict partial order respectively. Of course, $D_s \subset D \subset D_w$, aDb, $bD_wc \Rightarrow aDc$ and $aD_wb, bDc \Rightarrow aDc, \forall a, b, c \in A$.

In PCCA, where a subset of criteria $G = \{g_i, g_j\} \subset F$, is considered at the first level of aggregation, we say that a partially dominates b $(aD_{ij}b)$, if the relation of dominance is defined on *G.* We say that a is partially preferred or is partially indifferent to b $(aP_{ij}b_j$ and $aI_{ij}b$ respectively) if these relations hold with respect to the set of criteria $\{g_i, g_j\}$.

We observe that

$$\begin{aligned} aD_{ij}b &\Rightarrow aP_{ij}b, \\ \text{and } aD_{ij}b, \forall i, j \in \mathcal{J} &\Leftrightarrow aD_sb \Rightarrow aDb \Rightarrow aPb, \end{aligned}$$

if all criteria from *F* are true criteria.

In the MAPPAC method [25] the basic (or partial) indices $\pi_{ij}(a, b)$ can be interpreted as credibility indices of the partial dominance $aD_{ij}b$, indicating also the fuzzy degree of preference of a over b; the global index $\pi(a, b)$ can be interpreted as the credibility index of strict dominance aD_sb, i.e. as the fuzzy degree of comprehensive preference of a over b.

If all criteria from *F* are interval scales, recalling that $\Delta_j(a, b) = g_j(a) - g_j(b)$, for each $j \in \mathcal{J}$ and $a, b \in A$, w_j, is the trade-off weight and λ_j the (normalized) importance weight of criterion g_j, $j \in \mathcal{J}$, the axiomatic system of MAPPAC partial indices can be summarized as follows (see Table 6.18) for each $a, b \in A$:

- The basic indices $\pi_{ij}(a, b)$ are functions only of the signs of the differences in evaluations of a and b with respect to criteria g_i and g_j in case of concordant evaluations, i.e. iff $\Delta_i(a, b)\Delta_j(a, b) \geq 0$. In this case,

  $$aD_{ij}b \Leftrightarrow \Delta_i(a, b) + \Delta_j(a, b) > 0$$

  and

  $$bD_{ij}a \Leftrightarrow \Delta_i(a, b) + \Delta_j(a, b) < 0$$

Table 6.18. Axiomatic system of MAPPAC basic indices.

$\pi_{ij}(a,b)$	Binary Relations	Signs of $\Delta_i(a,b)\cdot\Delta_j(a,b)$	Signs of $\Delta_i(a,b)+\Delta_j(a,b)$	Pair of Signs of $\Delta_i(a,b),\ \Delta_j(a,b)$
$]0,1[$	$aP_{ij}b, bP_{ij}a,$ $aI_{ij}b$	<0	$\gtreqless$	$(+,-),(-,+)$
$\frac{1}{2}$	$aI_{ij}b$	$=0$	$=0$	$(0,0)$
1	$aD_{ij}b$	≥ 0	>0	$(+,+),(+,0),$ (0,+)
0	$bD_{ij}a$	≥ 0	<0	$(-,-),(-,0),$ (0,-)

and then $\pi_{ij}(a,b) = 1$ and $\pi_{ij}(b,a) = 0$ in the first case, and $\pi_{ij}(a,b) = 0$ and $\pi_{ij}(b,a) = 1$ in the second case.

- The basic indices $\pi_{ij}(a,b)$ are functions of the values of the differences in evaluations of a and b with respect to criteria g_i and g_j and of trade-off weights w_i and w_j in case of discordant evaluations, i.e. iff $\Delta_i(a,b)\,\Delta_j(a,b) < 0$. In this case, the indices $\pi_{ij}(a,b)$ and $\pi_{ij}(b,a)$ will be of a compensatory type, lying in the interval]0,1[, and they will indicate the fuzzy degree of preference of a over b and of b over a respectively; if $w_i\Delta_i(a,b) + w_j\Delta_j(a,b) = 0$, $\pi_{ij}(a,b) = \pi_{ij}(b,a) = \frac{1}{2}$.

- The global indices $\pi(a,b)$ are functions of all the basic indices $\pi_{ij}(a,b)$ and of the importance weights λ_{ij} of all coalitions $\{g_i, g_j\}$ of criteria. If there is no interaction between criteria g_i and g_j, we have $\lambda_{ij} = \lambda_i + \lambda_j$. In case of strict dominance aD_sb or bD_sa, $\pi(a,b) = 1$ and $\pi(b,a) = 0$, or $\pi(a,b) = 0$ and $\pi(b,a) = 1$, respectively. Otherwise, $\pi(a,b)$ and $\pi(b,a)$ will lie in the interval]0, 1[and they will indicate the fuzzy degree of comprehensive preference of a over b and of b over a respectively.

Preference Indices. We recall that $w_j\Delta_j(a,b) = w_j(g_j(a)-g_j(b))$, $j \in \mathcal{J}$, $a,b \in A$, is the normalized weighted difference of evaluations of actions a and b with respect to criterion g_j.

If we assume $f_{ij}(a,b) = \sum_{h\in\{i,j\}\cap\mathcal{J}_{a\geq b}} w_h\Delta_h(a,b)$ in the equation (6.3) we obtain the partial index $\pi_{ij}(a,b)$ of MAPPAC, $a,b \in A$, $\{g_i, g_j\} \subset F$ ($|F| \geq 3$). This index can also be explicitly written as shown in Table 6.19.

It is invariant to the admissible transformation of any $g_j \in F$, i.e. all the affine transformations of the type $g'_j(\bullet) = \alpha g_j + \beta$, with $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}$, being the criteria interval scales. It is the image of a valued binary relation, strictly complete, transitive and ipsodual (i.e. $\pi_{ij}(a,b) = 1-\pi_{ij}(b,a)$), that constitutes a complete preorder on *A,* and it indicates the fuzzy partial preference intensity of a over b.

Table 6.19. Preference indices.

$\pi_{ij}(a,b)$	$\pi_{ij}(b,a)$	
1	0	if $g_i(a) > g_i(b)$ and $g_j(a) > g_j(b)$
0	1	if $g_i(a) < g_i(b)$ and $g_j(a) < g_j(b)$
0.5	0.5	if $g_i(a) = g_i(b)$ and $g_j(a) = g_j(b)$
$\frac{w_i(g_i(a)-g_i(b))}{w_i(g_i(a)-g_i(b))+w_j(g_j(b)-g_j(a))}$	$\frac{w_j(g_j(b)-g_j(a))}{w_i(g_j(a)-g_i(b))+w_j(g_j(b)-g_j(a))}$	if $g_i(a) > g_i(b)$ and $g_j(a) \leq g_j(b)$ $g_i(a) = g_i(b)$ and $g_j(a) < g_j(b)$
$\frac{w_j(g_j(a)-g_j(b))}{w_i(g_i(b)-g_i(a))+w_j(g_j(a)-g_j(b))}$	$\frac{w_i(g_i(b)-g_i(a))}{w_i(g_i(b)-g_i(a))+w_j(g_j(a)-g_j(b))}$	if $g_i(a) \leq g_i(b)$ and $g_j(a) > g_j(b)$ $g_i(a) < g_i(b)$ and $g_j(a) = g_j(b)$

The basic preference index $\pi_{ij}(a,b)$ may be immediately interpreted geometrically by considering the partial profiles of the actions a and b with respect to criteria g_i and g_j (see Fig. 6.4).

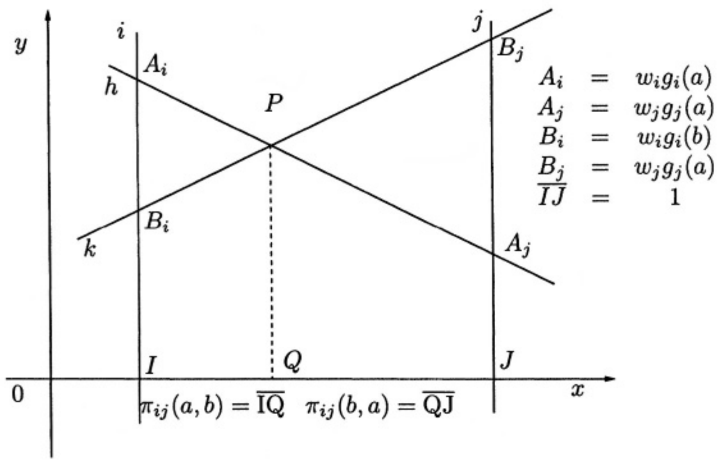


Figure 6.4. Geometrical interpretation of preferences indices.

Let us consider the following subsets of *F:*

$$\begin{aligned}
G^+(a,b) &= \{g_h \in F : \Delta_h(a,b) > 0\}; \quad |G^+(a,b)| = p, \\
G^=(a,b) &= \{g_h \in F : \Delta_h(a,b) = 0\}; \quad |G^=(a,b)| = o, \\
G^-(a,b) &= \{g_h \in F : \Delta_h(a,b) < 0\}; \quad |G^-(a,b)| = n, \\
D_1(a,b) &= \{(g_i,g_j) \in F^2, g_i \neq g_j : aD_{ij}b\}, \\
D_0(a,b) &= \{(g_i,g_j) \in F^2, g_i \neq g_j : bD_{ij}a\}.
\end{aligned}$$

Of course, $G^+(a,b) \cup G^=(a,b) \cup G^-(a,b) = F$ and $|F| = m = p+o+n$, $(p,o,n \geq 0)$. Since (see [26])

$$\begin{aligned}
&\binom{m}{2} = \binom{p+o+n}{2} = \\
&\binom{p+o}{2} + \binom{p+n}{2} + \binom{o+n}{2} - \binom{p}{2} - \binom{o}{2} - \binom{n}{2} = \\
&\binom{p}{2} + \binom{o}{2} + \binom{n}{2} + po + pn + on,
\end{aligned}$$

we can split all the $\binom{m}{2}$ basic preference indices $\pi_{ij}(a,b)$ as follows:

$$\binom{m}{2} = |D_1(a,b)| + |D_o(a,b)| + \binom{o}{2+pn}.$$

Thus, $|D_1(a,b)| = \binom{m}{2}$ if and only if $p \geq m-1$ and $n = 0$ (i.e., $\Delta_h(a,b) \geq 0$ for each $h \in \mathcal{J}$, with at most only one equality), $|D_0(a,b)| = \binom{m}{2}$ if and only if $n \geq m-1$ and $p = 0$, $|D_1(a,b)| = |D_0(a,b)| = 0$ and $\pi_{ij}(a,b) = \frac{1}{2}$ for each $i,j \in \mathcal{J}$ if and only if $o = \frac{m}{2}$ (i.e., $\Delta_h(a,b) = 0$ for each $h \in \mathcal{J}$).

The *global preference index* $\pi(a,b)$ is the sum of all the $\binom{m}{2}$, $m > 2$, basic preference indices $\pi_{ij}(a,b)$, weighted each time by the normalized importance weights λ_{ij} of the considered couple of criteria g_i, g_j:

$$\pi(a,b) = \sum_{ij(i<j)} \pi_{ij}(a,b)\frac{\lambda_{ij}}{\Lambda},$$

where $\Lambda = \sum_{ij(i<j)} \lambda_{ij}$.

If there is no interaction between each couple of criteria, we have $\lambda_{ij} = \lambda_i + \lambda_j$, where λ_h is the normalized importance weight of criterion g_h, $h = 1, 2 \cdots m$, and therefore:

$$\pi(a,b) = \sum_{ij(i<j)} \pi_{ij}(a,b)\frac{\lambda_i + \lambda_j}{m-1}, \quad i,j \in \mathcal{J}, \left(\sum_{ij(i<j)} (\lambda_i + \lambda_j) = m-1\right) \tag{6.5}$$

Therefore, in this case we can write $\pi(a,b)$ as:

$$\pi(a,b) = \pi_{PP}(a,b) + \pi_{PO}(a,b) + \pi_{NN}(a,b) + \pi_{NO}(a,b) + \pi_{OO}(a,b) + \pi_{PN}(a,b), \quad (6.6)$$

where:

$$\begin{aligned}
\pi_{PP}(a,b) &= \frac{p-1}{m-1} \sum_{i \in G^+(a,b)} \lambda_i; \\
\pi_{PO}(a,b) &= \frac{1}{m-1}\left[p \sum_{i \in G^=(a,b)} \lambda_i + o \sum_{i \in G^+(a,b)} \lambda_i\right]; \\
\pi_{NN}(a,b) &= \pi_{NO}(a,b) = 0; \\
\pi_{OO}(a,b) &= \frac{1}{2}\frac{o-1}{m-1} \sum_{i \in G^=(a,b)} \lambda_i; \\
\pi_{PN}(a,b) &= \sum_{rs} \pi_{rs}(a,b) \frac{\lambda_r + \lambda_s}{m-1}, (g_r, g_s) \in G^+(a,b) \times G^-(a,b).
\end{aligned}$$

Let $S(a,b) = G^+(a,b) \cup G^=(a,b)$. We can write:

$$\pi_{D_1} = \pi_{PP}(a,b) + \pi_{PO}(a,b)$$

and, recalling equation (6.6),

$$\pi_S = \pi_{D_1}(a,b) + \pi_{OO}(a,b) = \frac{1}{m-1}\left[(p+o-1) \sum_{i \in S(a,b)} \lambda_i - \frac{1}{2}(o-1) \sum_{i \in G^=(a,b)} \lambda_i\right].$$

We observe that:

a) if $G^=(a,b) = \emptyset$ or $|G^=(a,b)| = 1, \pi_{D_1}(a,b) = \pi_S(a,b)$;

b) if $G^-(a,b) = \emptyset$, $\pi(a,b) = \pi_S(a,b)$;

c) the index $\pi_S(a,b)$ is a linear combination of the *crisp* concordance index $c(a,b)$ of the ELECTRE methods (see Chapter 4 in this book) and the opposite of semi-sum of the importance weights of criteria from set $G^=(a,b)$; their coefficients are respectively given by the ratios between the number of criteria belonging to the corresponding classes and the total number of criteria up to one unit (i.e., the number of *significant* criteria for a comparisons by means of pairs of criteria);

d) if $|S(a,b)| \geq 2$, $\frac{1}{2} \leq \pi_S(a,b) \leq c(a,b) \leq 1$, and $\pi_S(a,b) = c(a,b) = 1$ if and only if aD_Sb (but $c(a,b) = 1$ does not imply $\pi_S(a,b) = 1$);

e) $\pi_S(a,b) = 0$ if and only if $|S(a,b)| < 2$, and $\pi_S(a,b) = c(a,b) = 0$ if and only if $S(a,b) = \emptyset$ (but $\pi_S(a,b) = 0$ does not imply $c(a,b) = 0$);

f) the compensatory component $\pi_{PN}(a,b)$ of $\pi(a,b)$ (see equation (6.6)) may be methodologically linked to the MAUT approach, in particular to the weighted sum with constant marginal substitution rates (trade-off weights);

g) if the number o of the criteria g_h from F for which $g_h(a) = g_h(b)$ changes without modification in the sum of the relative importance weights of coalitions $G^+(a,b)$, $G^+(b,a)$ and $G^=(a,b)$, the value of the aggregate preference index $\pi(a,b)$ may vary, as a consequence of changing of its component $\pi_{PN}(a,b)$ value. More precisely:

- $G^+(a,b) = \emptyset$ and $G^-(a,b) \neq \emptyset \Rightarrow \pi(a,b)$ increases with *o*, i.e. $\Delta_o\pi(a.b) > 0$,
- $G^+(a,b) \neq \emptyset$ and $G^-(a,b) = \emptyset \Rightarrow \pi(a,b)$ decreases with *o*, i.e. $\Delta_o\pi(a.b) < 0$,
- $\lim_{o\to+\infty} \Delta_o\pi(a,b) = 0$,
- $\lim_{o\to+\infty} \pi(a,b) = \sum_{i\in G^+(a,b)} \lambda_i + \frac{1}{2}\sum_{i\in G^=(a,b)} \lambda_i$ (< 1 since $G^+(a,b) \subset F$),
- if the relative importance of $G^+(a,b)$ and $G^-(a,b)$ are equal, the relation aIb is stable with respect to *o;*
- $\forall o \geq 1, \frac{p}{p+n}\sum_{i\in S(a,b)} \lambda_i \geq \frac{1}{2} \Rightarrow aPb$ stable with respect to *o,*
- if there is a perfect compensation between the normalized weighted differences in evaluations of opposite signs (i.e. neutral behavior of $\pi_{PN}(a,b)$), $\Delta_o\pi(a,b) > 0$ $[< 0] \Leftrightarrow n\sum_{i\in S(a,b)} \lambda_i - p\sum_{i\in S(a,b)} \lambda_i > 0$ $[< 0]$, i.e. if and only if $n > [<]p$,
- the aggregate preference index $\pi(a,b)$ is an increasing function of p (i.e. $\Delta_p\pi(a,b) > 0$) if $\pi(a,b) < 1$,
- $\lim_{p\to+\infty} \Delta_p\pi(a,b) = 0$,
- $\lim_{p\to+\infty} \pi(a,b) = 1 - \frac{1}{2}\sum_{i\in G^-(a,b)} \lambda_i$.

Following the same principle of PCCA, it is possible to build up other partial and global preference indices, based on a logic of noncompensatory aggregation [24]. The common feature of all these indices is that they are based on bicriteria and global indices, measuring respectively the credibility of partial dominance and of strict dominance of a over b, $a, b \in A$. So, for example, if no 2-level interaction occurs among considered criteria, let us consider the following two aggregated indices:

$$\pi'(a,b) = \frac{1}{m-1}\left[(m-1)\sum_{i\in G^+(a,b)} \lambda_i - (p + \frac{o-1}{2})\sum_{i\in G^=(a,b)} \lambda_i\right],$$

$$\pi^*(a,b) = \frac{1}{m-1}\sum_{i,j;(i<j)}\left[\sum_{(i,j):\pi_{ij}(a,b)>0.5} (\lambda_i + \lambda_j) + \frac{1}{2}\sum_{(i,j):\pi_{ij}(a,b)=0.5} (\lambda_i + \lambda_j)\right].$$

We can observe that index $\pi'(a,b)$ is totally noncompensatory and it is analogous to the concordance indices of ELECTRE I and II methods. On the other hand, index $\pi^*(a,b)$ is PCCA-totally noncompensatory (see [24]), depending on the *"coalition strength"* of the subsets (couples of criteria) of G^2 such that $aP_{ij}b$ or $aI_{ij}b$. Both these indices, like index $\pi(a,b)$, are also functions of p, n, o.

Taking into account the above properties and the peculiar features of the basic preference indices with respect to the dominance and compensation, MAPPAC and – more generally – PCCA may be considered as an *"intermediate"* MCDA methodology between the outranking (particularly ELECTRE) and MAUT methods.

Indifference Modelling. Since the evaluations of actions a and b with respect to the couple of criteria g_i, g_j from F are compared each time to build up index $\pi_{ij}(a,b)$, and recalling that $\Delta_i(a,b)\ \Delta_j(a,b) > 0$ means by definition active or passive partial dominance of a over b (and then $\pi_{ij}(a,b) = 1$ or 0 respectively), it is useful to confine the dominance relation only if well founded situations will occur. Therefore, in order to take into account the inevitable inaccuracies and approximations in the actions evaluations, and in order to prevent small differences between these evaluations from creating partial dominance relations or preference intensities close to the maximum or minimum values, it is advisable to introduce suitable indifference areas on the plane $Og_i(a)g_j(a)$ in the neighborhood of point $I = (g_i(a) = g_i(b), g_j(a) = g_j(b))$, see Fig. 6.5.

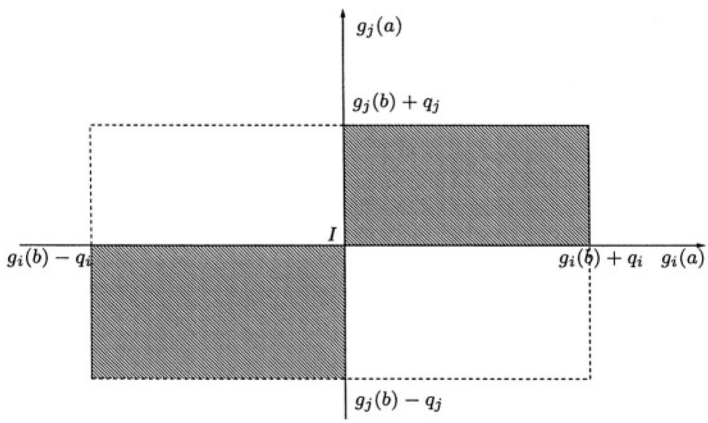


Figure 6.5. Indifference areas.

These areas may be defined in various way, as functions of correspondent indifference thresholds, one for each criterion considered (see [22]). The marginal indifference threshold for criterion g_j, denoted by q_j, is not negative and unique for every couple of distinct actions $a, b \in A$ ($q_j(a,b) = q_j(b,a) \geq 0, \forall a, b \in A$) and it is a function of the evaluations of these actions according to the criterion considered:

$$q_j(a,b) = \alpha_j + \beta_j \left| \frac{g_j(a) + g_j(b)}{2} \right|, g_j \in F, \alpha_j, \beta_j \geq 0. \qquad (6.7)$$

The first parameter α_j is expressed in the same scale of values as the criterion g_j, and q_j is a linear function of the arithmetical mean of the evaluations of the considered actions, being β_j the constant of proportionality. Then, if $\beta_j = 0$ or $\alpha_j = 0$, equation (6.7) supplies constant indifference thresholds, in absolute or relative value respectively. It is therefore possible to define an indifference area IA_{ij} for each pair of actions $a, b \in A$ and criteria $g_i, g_j \in F$ as a function of the marginal indifference thresholds (6.7). This area may assume various shapes, for example:

- rectangular, if $\pi_{ij}(a,b) = \frac{1}{2}$ for $|g_i(a) - g_i(b)| \leq q_i(a,b)$ and $|g_j(a) - g_j(b)| \leq q_j(a,b)$ (see Fig. 6.6);

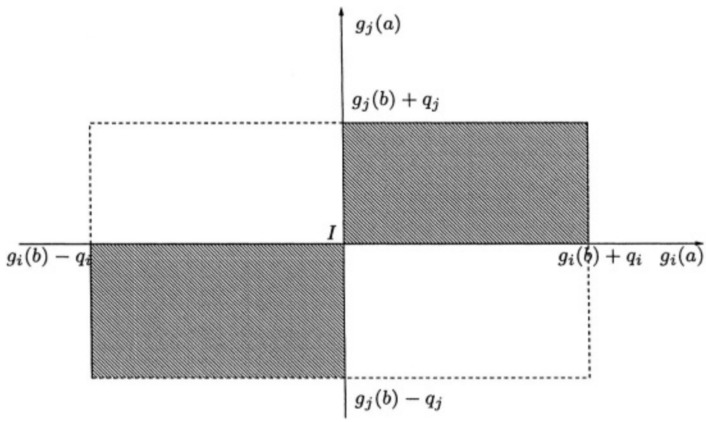


Figure 6.6. Indifference areas: rectangular.

- rhomboidal, if $\pi_{ij}(a,b) = \frac{1}{2}$ for $\frac{|g_i(a)-g_i(b)|}{q_i(a,b)} + \frac{|g_j(a)-g_j(b)|}{q_j(a,b)} \leq 1$ (see Fig. 6.7);
- elliptical, if $\pi_{ij}(a,b) = \frac{1}{2}$ for $\frac{(g_i(a)-g_i(b))^2}{q_i^2(a,b)} + \frac{(g_j(a)-g_j(b))^2}{q_j^2(a,b)} \leq 1$ (see Fig. 6.8).

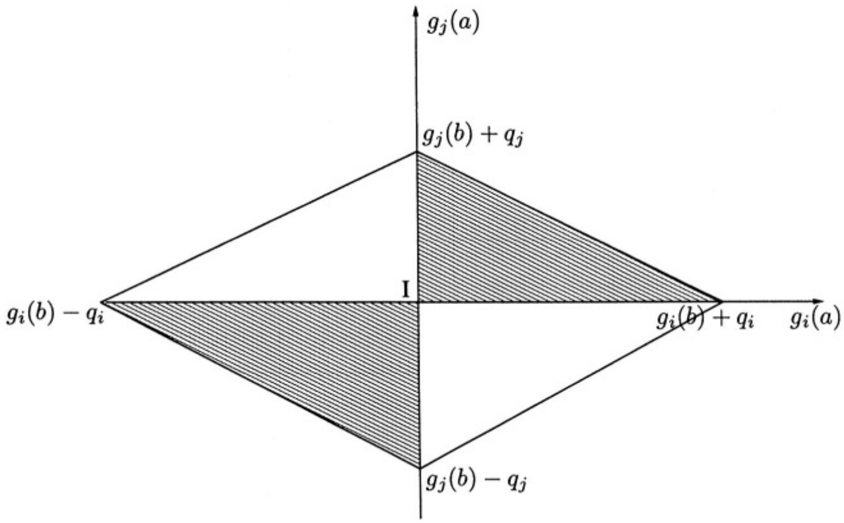


Figure 6.7. Indifference areas: rhomboidal.

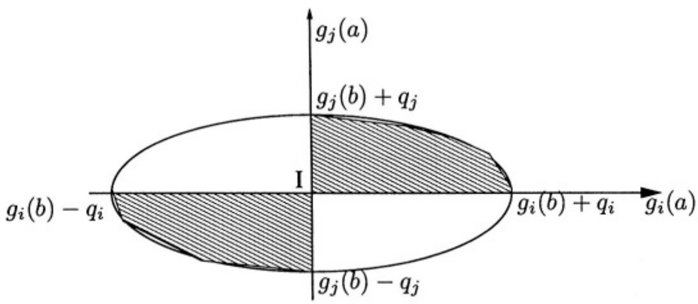


Figure 6.8. Indifference areas: elliptical.

It also possible to introduce semi-rectangular, semi-rhomboidal and semi-elliptical indifference areas, corresponding to the shadowed areas in Figures 6.6, 6.7, 6.8 respectively, with the specific aim of eliminating the effect of partial dominance only, adding each time the further conditions:

$$\begin{cases} g_i(b) & \leq & g_i(a) \\ g_j(b) & \leq & g_j(a) \end{cases} \text{ or } \begin{cases} g_i(a) & \leq & g_i(b) \\ g_j(a) & \leq & g_j(b). \end{cases}$$

Finally, it is also possible to consider mixed indifference areas, as a suitable combination of two or more of the cases considered above for each quadrant centered in point I. We can then modeling indifference in a flexible way, by setting different thresholds and/or shapes for each couple of criteria, according to the DM's preferential information.

Therefore, two separate indifference relations are obtained: strict indifference, denoted by $aI_{ij}b$, iff $\pi_{ij}(a,b) = \frac{1}{2}$ as a result of Table 6.19; large indifference, denoted by $aI^*_{ij}b$, iff a vector $\mathbf{q} \geq \mathbf{0}$ is introduced, $\mathbf{q} = [q_j(a,b)], j \in \mathcal{J}$, and some of the corresponding above indifference area conditions are satisfied, and thus $\pi_{ij}(a,b) = \frac{1}{2}$ is assumed.

Note that I_{ij} is an equivalence relation, whereas the relation $aI^*_{ij}b$ is not necessarily transitive.

Preference Structures. Using the basic and global preference indices $\pi_{ij}(a,b)$ and $\pi(a,b)$, it is possible to immediately define the following correspondent binary relations of partial and comprehensive indifference and preference relations respectively, with the particular cases of dominance recalled above:

- Partial relations

$$\begin{aligned}
\pi_{ij}(a,b) = 0.5 \quad &\Leftrightarrow \quad aI_{ij}b,\\
0.5 < \pi_{ij}(a,b) \leq 1 \quad &\Leftrightarrow \quad aP_{ij}b,\\
(\pi_{ij}(a,b) = 1 \quad &\Leftrightarrow \quad aD_{ij}b),\\
0 \leq \pi_{ij}(a,b) < 0.5 \quad &\Leftrightarrow \quad bP_{ij}a\\
(\pi_{ij}(a,b) = 0 \quad &\Leftrightarrow \quad bD_{ij}a).
\end{aligned}$$

- Comprehensive relations

$$\begin{aligned}
\pi(a,b) = 0.5 \quad &\Leftrightarrow \quad aIb,\\
0.5 < \pi(a,b) \leq 1 \quad &\Leftrightarrow \quad aPb \quad (\pi(a,b) = 1 \Leftrightarrow aDb),\\
0 \leq \pi(a,b) < 0.5 \quad &\Leftrightarrow \quad bPa \quad (\pi(a,b) = 0 \Leftrightarrow bDa).
\end{aligned}$$

Both these structures constitute a complete preorder on A. We observe that, if no indifference areas are introduced, will be $\pi_{ij}(a,b) + \pi_{ij}(b,a) = 1$ for each $i,j \in \mathcal{J}$ and $(a,b) \in A^2$ and therefore also $\pi(a,b) + \pi(b,a) = 1$.

Of course, by means of the same indices, we can also build up some other particular complete valued preference structures. For example, we may consider the structure of semiorder, obtained by introducing a real parameter $\delta \in [1/2, 1]$, which emphasizes the partial or global indifference relations (see Figure 6.9).

In this case, the indifference relations are reflexive, symmetric and not transitive, while the preference relations are transitive, non reflexive and asymmetric. We note that if $\delta = \frac{1}{2}$ we obtain again a complete preorder with "punctual" indifference, i.e. only for $\pi(a,b) = \pi(b,a) = \frac{1}{2}$, while if $\delta = 1$, the binary preference relation is empty. Alternatively, by introducing two real parameters

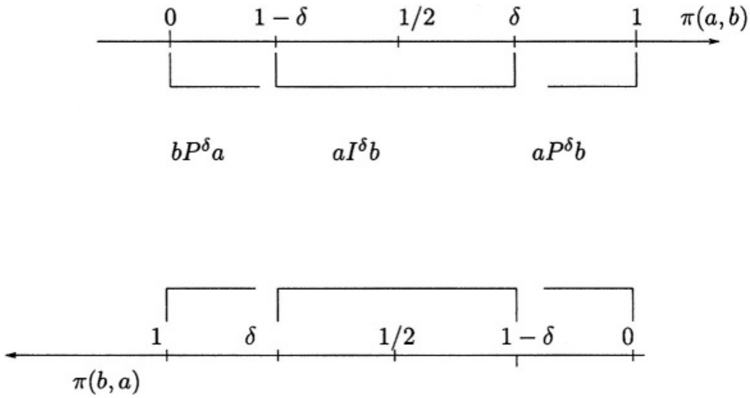


Figure 6.9. Aggregated semiorder structure.

δ and ϵ, $\frac{1}{2} \leq \delta < \epsilon \leq 1$, it is possible to build a complete two-valued preference structure, assuming that there are two preference intensity levels, represented by the preference relations P^{τ} (strict preference) and Q^{τ} (weak preference) (see Figure 6.10).

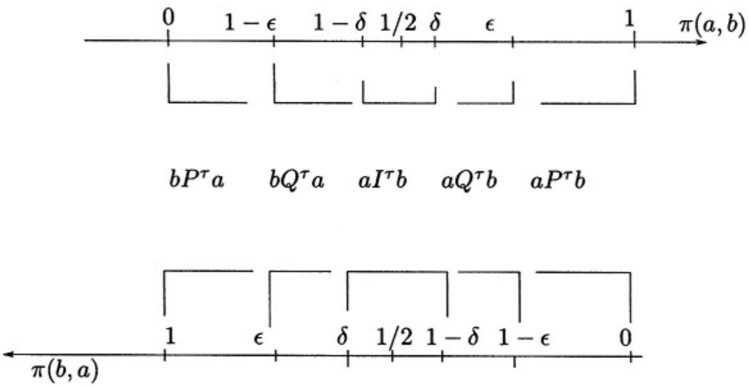


Figure 6.10. Aggregated pseudo-order structure.

In this case the relations of indifference and of weak preference are not transitive and the preference model presents the properties of the well-known pseudo-order structure (see [38]).

Conflict Analysis. Besides the concept of discordant criterion and veto threshold often used for building outranking relations, another interesting feature of PCCA approach is the possibility to consider a peculiar conflict analysis, taking into consideration the differences in evaluations of two actions with respect to each couple of criteria. The main aims of this analysis are the following:

- to explicitly define binary *incomparability relations* in presence of evaluations of two actions a and b in strong contrast on two criteria g_i and g_j, in the preference modeling phase (refusal to make a decision)
- to allow compensation only if differences in the conflicting evaluations are not too large; otherwise, to use non compensatory basic indices (functions only of importance weights), obtaining a *partially compensatory approach* (reduction of compensation) (see [24]).

These aims can be reached by defining a suitable *partial discordance index* $d_{ij}(a,b)$, $i,j \in \mathcal{J}$, $a,b \in A$, as a function of conflicting evaluations and entropy of information, and comparing this one with correspondent incomparability threshold r_{ij}, given by DM (see [22]). If we note by R_{ij} the partial incomparability relation with respect the couple of criteria g_i and g_j, we have:

$$d_{ij}(a,b) \geq r_{ij} \Leftrightarrow aR_{ij}b,\ (a,b) \in A^2, g_i, g_j \in F.$$

Then, considering all the possible couples of distinct criteria g_i, g_j from F, we have:

$$aRb \Leftrightarrow [aR_{ij}b \text{ for at least one couple } i,j \in \mathcal{J}].$$

This global incomparability relation R, symmetric but neither reflexive nor transitive, arise if at least one partial incomparability relation holds with respect to actions a and b.

The symmetric discordance index $d_{ij}(a,b)$, $i,j \in \mathcal{J}$, is defined as follows [21].

$$d_{ij} = |w_i\Delta_i(a,b) + w_j\Delta_j(b,a)|(1 - 2|\pi_{ij}(a,b) - 0.5|).$$

It lies in $[0, t_i + t_j]$ and reaches its maximum value only in case of maximum effective discordance of evaluations of a and b with respect to g_i and g_j (i.e. $g_i(a) = g_i^*$, $g_j(a) = g_{j*}$ and $g_i(b) = g_{i*}$, $g_j(b) = g_j^*$ or viceversa) and $t_i = t_j$ (equal normalized trade-off weights). Moreover, $d_{ij}(a,b) = 0$ if $\Delta_i(a,b) = \Delta_j(b,a) = 0$ or in case of partial dominance (evaluation concordance). Therefore, it is possible to set the incomparability thresholds r_{ij} according to the real preferential information of DM about the different level of compensation for each couple of criteria g_i and g_j:

$$r_{ij} = \begin{cases} = 0 & \text{completely non compensatory approach} \\ \cong 0 & \text{low compensation is allowed} \\ \cong t_i + t_j & \text{high compensation is allowed} \\ = t_i + t_j & \text{totally compensatory approach.} \end{cases}$$

The concepts introduced above therefore permit also a modelling by means of the four binary relations *I,P,Q,R,* defined on *A,* which are exhaustive and mutually exclusive and constitute a fundamental relational preference system.

Exploitation Phase. The results of the relational model in the form of fuzzy binary relations obtained can be presented in the form of suitable $\binom{m}{2}$ bicriteria $n \times n$ (i.e. $|A| \times |A|$) square matrices: $\mathbf{\Pi_{ij}} = [\pi_{ij}(a,b)]$, one for each couple of criteria g_i, g_j from *F,* containing the partial preference indices, and one aggregated matrix $\mathbf{\Pi} = [\pi(a,b)]$, with the comprehensive preference indices, $(a,b) \in A^2$.

The peculiar preference modeling flexibility of PCCA allows to respect accurately the DM's preference, without imposing too strong axiomatic constraints, and accepting and using any kind of information the DM is able to give. Therefore, DM is not forced to be *"consistent", "rational"* or *"complete",* but all information given by DM is accepted and used, neither more, nor less. Consequently, with respect to two criteria trade-offs w_{ij}, $i,j \in I$, it is possible to use as input not transitive (i.e. $w_{ij}w_{jk} = \frac{w_i}{w_j}\frac{w_j}{w_k} \neq \frac{w_i}{w_k} = w_{ik}$) or not complete (some w_{ij} not given by DM) trade-offs for some pairs of criteria (and therefore the component $\pi_{ij}(a,b)$ of index $\pi(a,b)$ correspondent to these criteria will be absent); and, with reference to importance weights λ_j, $j \in \mathcal{J}$, the DM may assign non additive weights λ_{ij} to some couple of criteria, modelling thus their interaction (i.e. weighting some index $\pi_{ij}(a,b)$ with a weight different from $\lambda_i + \lambda_j$). In all these cases, the aggregate index $\pi(a,b)$ will be computed taking into account the peculiar information actually used as input.

The indices of preference intensity contained in the aggregated matrix $\mathbf{\Pi}$ may, among other things, permit in the *exploitation phase* the building of specific partial or complete rankings of feasible actions as final prescription.

A first possible technique to build rankings can be based on the concept of qualification of a feasible action, introduced by Roy (see [34]). But, in order to take into consideration the most complete preference information given by the fuzzy relations, we can sum the global preference indices referred to each feasible action in comparison with others, obtaining its comprehensive preference index, aiming to build up the partition of *A* into *S* equivalence classes $C_1, C_2, \cdots, C_S, S \leq n$ (complete preorder), by means of a descending procedure (from the best action to the worst) or by an ascending procedure (from the worst to the best).

In either case, the peculiar feature of these techniques is that at every step they select the action(s) assigned to a certain position in the ranking considered and then repeat the procedure with respect to the subset of the remaining actions, eliminating at each iteration the action, selected in the preceding one. Here is a brief example of one of the possible techniques.

Computation of the *comprehensive preference index,* $a \in A$:

$$\sigma_+^{(1)}(a) = \sum_{b \in A \setminus \{a\}} \pi(a, b).$$

This will be:

$$0 \leq \sigma_+^{(1)}(a) \leq n - 1, \ \ \forall a \in A.$$

In particular we obtain:

$$\sigma_+^{(1)}(a) = n - 1 \ \text{ or } \ \sigma_+^{(1)}(a) = 0,$$

if and only if a strictly dominates, or is strictly dominated by, respectively, all the remaining feasible actions. We then select the action(s) with the highest index $\sigma_+^{(1)}$. This action, or these actions, will occupy the first place in the decreasing ranking, forming class C_1. Then, given $A^{(1)} = A \setminus C_1$, we repeat the procedure with reference to the actions from this new subset, obtaining the indices:

$$\sigma_+^{(2)}(a) = \sum_{b \in A^{(1)} \setminus \{a\}} \pi(a, b), \ a \in A^{(1)}.$$

This iteration will make it possible to form class C_2, and so on *(descending procedure).*

The *increasing solution* may be obtained by calculating for each action a the comprehensive index

$$\sigma_-^{(1)}(a) = \sum_{b \in A \setminus \{a\}} \pi(b, a),$$

and placing in the last class C_s the action(s) which present the highest value for this index. We then proceed with the calculation of the indices $\sigma_-^{(2)}(a)$ related to the subset $A \setminus C_s$ and so on.

This way to build the rankings is suggested in order to reduce the risk that an action dominating or dominated by one or more feasible actions may assume a discriminatory role over these. A dominated action has a distorting effect during the descending procedure, while a dominating action produces the same effect during the ascending procedure.

A useful geometrical interpretation on omometric axes of the complete preorders related to the actions considered each time in the k-th iteration may efficaciously express the different rankings with the corresponding comprehensive intensities of preference (see[22]). If the broken lines connecting the points representing the comprehensive preferences of each action at all different iterations prove to be more or less parallel, the relative comprehensive preferences tend to remain constant. On the other hand, if these broken lines intersect one another, the ranking will present inversion in terms of comprehensive preferences at the considered iterations.

Of course, in the building of all complete preorders it is possible to introduce suitable indifferent thresholds, to prevent small differences in the comprehensive indices considered at every iteration from assuming a discriminating role (see [22]).

The building of preorders allows also to solve the choice problem. But it is also possible to directly use the information about strict dominance (given by the comprehensive preference) indices to support DM in choice problem.

Let $P(a,b) = \max[\pi(a,b) - \pi(b,a), 0]$, that is $P(a,b) = T_L[\pi(a,b), 1 - \pi(b,a)]$, where $T_L[.,.]$ means Lukasiewicz t-norm. Choice is usually based on the following scoring functions:

- non domination degree

$$\mu_{ND+}(a,\pi) = \min_{x\in A}[1 - P(x,a)] = \min_{x\in A} P^d(a,x),$$

  where $P^d(\bullet,\bullet)$ means "dual"of $P(\bullet,\bullet)$;

- non dominance degree

$$\mu_{ND-}(a,\pi) = \min_{x\in A}[1 - P(a,x)] = 1 - \max_{x\in A} P(a,x).$$

Let $A^{UND+} = \{a \in A : \mu_{ND+}(a,\pi) = 1\}$ (i.e. the subset of non-dominated actions from A) and $A^{UND-} = \{a \in A : \mu_{ND-}(\pi,a) = 1\}$(i.e. the subset of non-dominating actions from A). Clearly, best action(s) will belong to set A^{UND+} and worst action(s) to set A^{UND-}. We observe that, if relation $\pi(a,b)$ is transitive, A^{UND+} and A^{UND-} are non empty.

3.2 PRAGMA

The Preference RAnking Global frequencies in Multicriteria Analysis (PRAGMA) [23] method is based on the peculiar PCCA aggregation logic (that is firstly on pairwise comparisons by means of couples of distinct criteria, and then on the aggregation of these partial results), and use the same data input and preferential information of MAPPAC, of which it constitutes a useful complement and presents the same flexibility in preference modeling. Moreover, it instrumentally uses the MAPPAC basic preferences indices to compute its specific information to support DM in his/her decision problem at hand. From the methodological point of view, PRAGMA is neither a classical outranking neither a MAUT method. In fact, the output of this approach are not binary outranking relations or scores. But, following the aggregation procedure of PCCA, in the first and in the second phase partial and global ranking frequencies are respectively built, one for each feasible action, and these frequencies are then exploited to give DM a useful recommendation (partial or complete preorders are the final output).

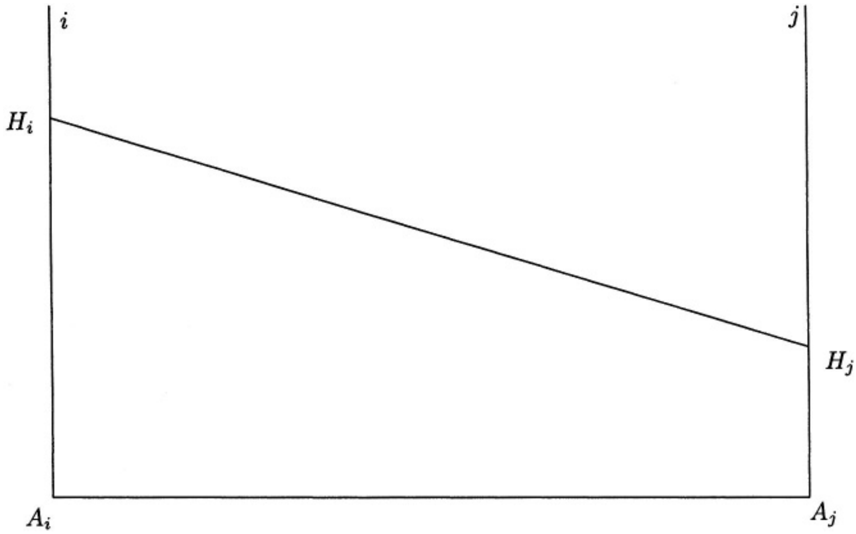


Figure 6.11. Partial profile of action a_h.

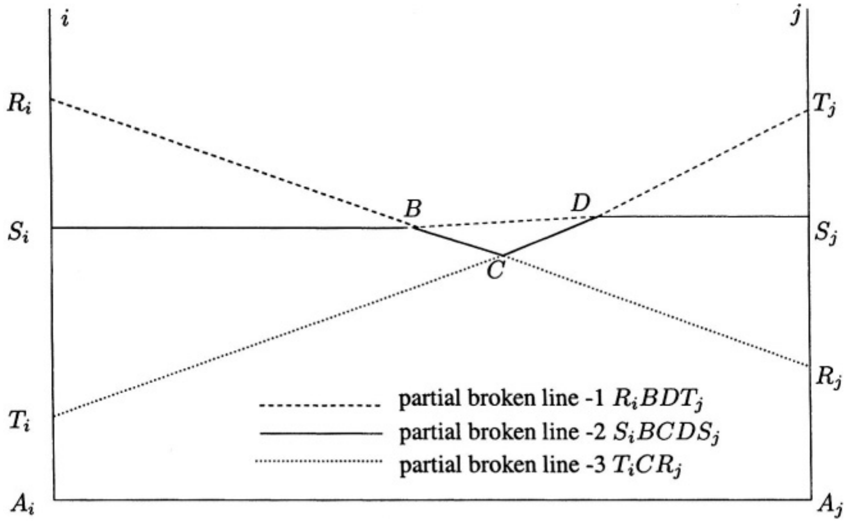
Partial and Global Frequencies. Let the segment H_iH_j (see Fig 6.11) be the *partial profile* of action $a_h \in A$, where the points H_i and H_j have as ordinates the weighted normalized evaluations of action a_h with respect to criteria g_i and g_j respectively, $g_i, g_j \in F$.

Considering all couples of criteria, it is possible to obtain $\binom{m}{2}$ distinct partial profiles of a_h and we call *global profile* of a_h the set of these $\binom{m}{2}$ partial profiles.

We define as *partial broken line-k*, or partial broken line of level k of a_h, $k = 1, 2 \cdots, n$ the set of consecutive segments of its partial profiles, to which correspond, for each point, $k - 1$ partial profiles (distinct or coinciding) of greater ordinate. If, for example, it is $A = \{a_r, a_s, a_t\}$, we obtain the partial profiles and partial broken lines represented in Figure 6.12.

We observe that the partial broken line-k, $k = 1, 2 \cdots, n$, coincides with the partial profiles of $a_h, a_h \in A$, if and only if a_h is partially dominated by d actions $(0 \leq d \leq k - 1)$ and dominates the remaining ones and/or if p couples of actions from A $(0 \leq p \leq k - 1, d + p = k - 1)$ exist such that, for each couple, their partial profiles come from opposite sides with respect to profile of a_h, and they intersect this profile at the same point.

Figure 6.12. Partial profiles and partial broken lines of a_r, a_s, a_t.



Further, we define as *global broken* ***line-k*** or global broken lines of level k $(k = 1, 2, \cdots, n)$ the set of $\binom{m}{2}$ partial broken lines-k obtained by considering all the couples of distinct criteria $g_i, g_j \in G$. The global broken line-k coincides with the global profiles of a_h if and only if all the partial broken lines of level k, obtained by considering each of the $\binom{m}{2}$ couples of criteria, coincide with the corresponding partial profiles of a_h.

We define as the *partial frequency* of level k $(k = 1, 2 \cdots, n)$ of a_h, with reference to the criteria g_i and g_j, the value of the orthogonal projection on the straight line A_iA_j (given $\overline{A_iA_j} = 1$) of the intersection of the partial profile of a_h with the corresponding partial broken line of level k. If we indicate this frequency as $f_{ij}^k(a_h)$, it will be $0 \leq f_{ij}^k(a_h) \leq 1$, for all $a_h \in A, k = 1, 2 \cdots, n$. Thus, for example, from the graphics in Figure 6.13.

$$
\begin{aligned}
\overline{A_iA_j} &= 1; & \overline{A_iB} &= 0.3; & \overline{BC} &= 0.1;\\
\overline{CD} &= 0.2; & \overline{DA_j} &= 0.4; & &\\
f_{ij}^{(1)}(a) &= 0.3; & f_{ij}^{(2)}(a) &= 0.3; & f_{ij}^{(3)}(a) &= 0.6;\\
f^{(1)}{}_{ij}(b) &= 0.3; & f_{ij}^{(2)}(b) &= 0.7; & f_{ij}^{(3)}(b) &= 0\\
f_{ij}^{(1)}(c) &= 0.4; & f_{ij}^{(2)}(c) &= 0.2; & f_{ij}^{(3)}(c) &= 0.4
\end{aligned}
$$

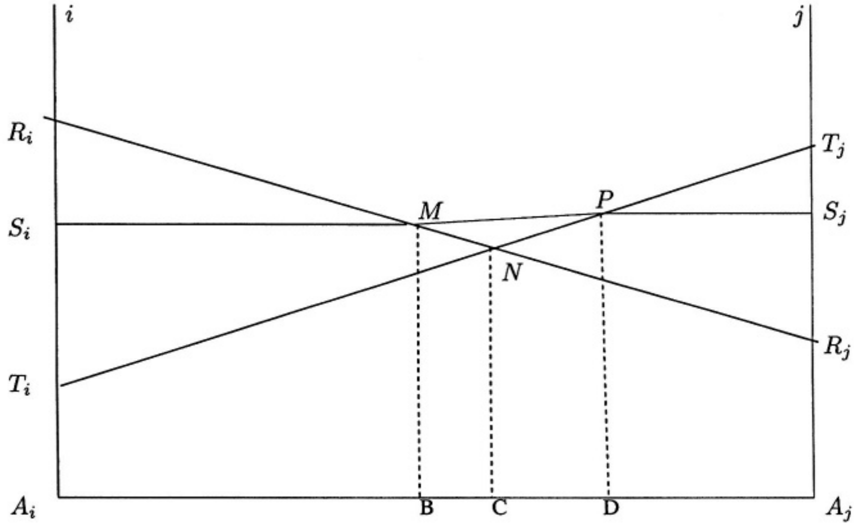


Figure 6.13. Partial frequencies of a_r, a_s, a_t.

The partial frequencies may be represented in matrix form, obtaining $\binom{m}{2}$ square $n \times n$ matrices $\mathbf{F_{ij}}$, which is the matrix of the partial ranking frequencies:

$$\mathbf{F_{ij}} = \left[f_{ij}^{(k)}(a_h)\right], a_h \in A;\ k = 1, 2 \cdots, n;\ i, j \in I. \qquad \textbf{(6.8)}$$

The elements of the h^{th} line of matrix (6.8) indicate in order the fractions of the interval unitary (A_i, A_j) for which the action a_h is in the k^{th} position $(k = 1, 2 \cdots, n)$, while the elements of the k^{th} column of the same matrix indicate those fractions for which the k^{th} position (in the partial preference ranking considered) is assigned to the actions $a_1, a_2, \cdots, a_n$, respectively. Obviously:

$$\sum_{k=1}^{n} f_{ij}^{(k)}(a_h) = 1,\ \forall a_h \in A \text{ and } \sum_{h=1}^{n} f_{ij}^{(k)}(a_h) = 1, k = 1, 2, \cdots n.$$

If $f_{ij}^{(k)}(a_h) \in \{0, 1\}$, for all $a_h \in A$ and $k = 1, 2 \cdots, n$, the partial profiles of all the actions will be non-coinciding, and there will be no inversions with respect to the preference relation in the two complete preference preorders with respect to the criteria g_i and g_j, i.e. all actions from *A* partially dominate one another.

If $v\,(v = 2, 3 \cdots, n)$ partial profiles are coinciding, the corresponding partial broken **lines-k** must be built taking distinctly into account the coinciding profiles v times (see [23]).

Let us then define *global frequency of level* k, $(k = 1, 2 \cdots, n)$ of a_h as the weighted arithmetic mean of all the $\binom{m}{2}$ partial frequencies of level k of a_h, obtained by considering all the couples of distinct criteria g_i and g_j. Therefore, designating this frequency by $f^{(k)}(a_h)$, we obtain, if no interaction between criteria is considered (see Section 3.1):

$$f^{(k)}(a_h) = \sum_{(i<j)i,j} f_{ij}^{(k)}(a_h) \frac{\lambda_i + \lambda_j}{m-1},\ a_h \in A,\ k = 1, 2 \cdots, n.$$

The linear combination of the matrices (6.8) with weights $\frac{\lambda_i+\lambda_j}{m-1}$ will therefore give the square $n \times n$ matrix $\mathbf{F} = [f^k(a_h)]$ $(h = 1, 2, \cdots, n;\ k = 1, 2, \cdots, n)$, called the global ranking frequency matrix. Its generic element $f^k(a_h)$ indicates the relative frequency with which $a_h \in A$ is present in the k^{th} position $(k = 1, 2, \cdots, n)$ in the particular ranking obtained by considering all the criteria $g_j \in F$ and the global profiles of all the feasible actions. It will therefore be:

$$\sum_{k=1}^{n} f^{(k)}(a_h) = 1, \forall a_h \in A \text{ and } \sum_{h=1}^{n} f^{(k)}(a_h) = 1, k = 1, 2, \cdots, n.$$

It is possible to calculate the partial frequencies $f_{ij}^{(k)}(a_h)$ by means of an algorithm which uses the indices $\pi_{ij}(a_h, a_k)$ of the MAPPAC method (see [23]). It is therefore possible to consider marginal indifference thresholds and suitable indifference areas also when the PRAGMA method is implemented. In other words, the indices $\pi_{ij}(a_h, a_k)$ here instrumentally introduced, may be calculated in advance by using all the techniques adopted with reference to the MAPPAC method (see Section 3.1).

Apart from these calculations, it is useful in any case to remember among others some particular features of the ranking frequencies obtained by the PRAGMA method:

1 The partial frequencies (and therefore also the global ones) of $a_h \in A$ are functions of the value of the normalized weighted differences between the evaluations of a_h and those of the remaining feasible actions with respect to the criteria considered. The values of these weighted differences may be overlooked only in the case of partial dominance (for partial frequencies) or strict dominance (for global frequencies), active or passive, of the action a_h.

2 If a_h partially dominates $n - k$ actions and it is partially dominated by the remaining $k - 1$ actions, $k = 1, 2 \cdots n$ the result is $f_{ij}^{(k)}(a_h) = 1$, whatever the values λ_i and λ_j.

3 If a_h strictly dominates $n - k$ actions and is strictly dominated by the remaining $k - 1$ actions, $k = 1, 2 \cdots, n$, the result is $f^{(k)}(a_h) = 1$, whatever the values of the weights $\lambda_j, j \in \mathcal{J}$.

4 If $f^{(k)}(a_h) = 1$, the action a_h occupies the k^{th} position, $k = 1, 2 \cdots, n$, in every monocriterion ranking and a_h is preceded and followed by the same subset of actions in these rankings.

Therefore, the information obtained by means of analysis of the global frequencies $f^{(k)}(a_h)$ is more complete and more accurate than that obtained from an examination of all the distinct monocriterion rankings of the feasible actions, or from a mixture of these.

Exploitation and Recommendation. In order to support DM in the decision problem at hand, it is often sufficient to analyze the elements of matrices $\mathbf{F_{ij}}$ and/or **F**. For example, a straightforward reading of the global frequencies of matrix **F** could indicate which action(s) will finally be chosen. But the concise and accurate information regarding the frequencies of ranks each action may occupy can be extremely useful to build up final rankings.

If we want to obtain complete or partial rankings of the feasible actions in order to build up comprehensive evaluations and recommendations, it is possible, for example, to proceed in this way. Calculate for each action $a_h \in A$, the *accumulated frequencies of order* k, $k = 1, 2 \cdots, n$, summing the first k elements of the h^{th} row of matrix **F**, that is:

$$F^{(1)}(a_h) = f^{(1)}(a_h) \text{ and } F^{(k)}(a_h) = \sum_{i=1}^{k} f^{(i)}(a_h), k = 2, 3 \cdots, n$$

Then establish the order q ($q = 1, 2, \cdots, n-1$) of the frequencies which are considered relevant to the building of the ranking, that is indicate to what order q we intend to take into consideration the accumulated frequencies $F^k(a_h)$ for this purpose. The following comprehensive index is then built:

$$S^q(a_h) = \sum_{k=1}^{q} \alpha_k F^{(k)}(a_h),\ a_h \in A; 1 \geq \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_q > 0. \quad (6.9)$$

This gives the measure of the "strength" with which a_h occupies the first q positions in the aggregated ranking. This in practice will be $1 \leq q \leq \frac{n}{2}$, which regards the first positions in the ranking; the coefficients α_k indicate the relative importance (not increasing with k) of accumulated frequence of order k. In the first class C_1 of the decreasing ranking will be placed the action(s) to which the maximum value of $S^{(q)}(a_h)$ corresponds. In order to avoid ex aequo rankings, we proceed by selecting whichever actions have obtained an equal value of $S^{(q)}$ on the basis of the values of the indices $S^{(q+1)}$ and, in the case of further equality, on those of the indices $S^{(q+2)}$ and so on. In this case ex aequo actions would be accepted only if their corresponding indices $S^{(i)}$ proved equal for $i = q, q+1, \cdots, n$. If, on the other hand, we desire to prevent small differences in the indices $S^{(q)}$ from having a discriminatory role in the building of the rankings, it is possible to consider global indifference thresholds (see [22]).

If we place $\alpha_k = 1$, for all k, in 6.9, we do not emphasize the greater importance of the global ranking frequencies of the first positions. On the other hand, if we accept $q = 1$, we take into account only the global frequencies of the first position for the purpose of building the rankings.

After building class C_1, with reference to the subset of the remaining actions $A^{(1)} = A - C_1$, we calculate again the partial, global and accumulated frequencies and the index 6.9, proceeding as above in order to build class C_2, and so on. We observe that at each iteration t the order q_t, on the basis of the which the index $S^{(q_t)}$ 6.9 is to be calculated, must be restated so that it is a non increasing whole number and, taking into account the number $|A^{(t)}|$ of actions of the evaluation set, so that at each iteration t the ratio $\frac{q_t}{|A^{(t)}|}$ is as near as possible to the ratio $\frac{q}{n}$ of the first iteration (see [23]). In general, the rankings obtained are a function of the value of the order q originally selected (see [22]).

If at each useful iteration $F^{(k)}(a_r) \geq F^{(k)}(a_s)$ for all $k = 1, 2, \ldots, n$ and $F^{(k)}(a_r) > F^{(k)}(a_s)$ for some k, or if $\sum_{k=1}^{s}[F^{(k)}(a_r) - F^{(k)}(a_s)] \geq 0$ for all $s = 1, 2, \cdots, n$ and $F^{(k)}(a_r) \neq F^{(k)}(a_s)$ for some k, it is possible to speak of first degree or second degree *frequency dominance,* respectively, of a_r over a_s. In both cases, if $\alpha_1 > \alpha_2 > \cdots > \alpha_q$, a_r will precede a_s in any of the rankings obtained, whatever value may be chosen for the other q.

Besides the partition of the actions of A into equivalence classes (complete preorder) obtained with the descending procedure (or procedure *from above*) described, it is also possible to build another complete preorder in the same way using the ascending procedure (or procedure *from below*), that is selecting the action(s) to be placed in the last, next to last, $\cdots$ and finally in the first equivalence class.

In conclusion, it is possible to build a final ranking (partial preorder) of the feasible actions, as the intersections of the two decreasing and increasing rankings obtained by means of two separate procedures described. Using the PRAGMA method for the building of rankings, it is possible not only to establish any implicit incomparability deriving from the inversion of preferences in the preorders obtained by means of the two separate procedures, but also in this case it is possible to consider an explicit incomparability, obtained if the relative tests give a positive result, during the preference modeling phase. Since, as we have said, the PRAGMA method makes instrumental use of the basic preference indices, it is possible to use once again the same discordance indices already introduced in the MAPPAC method (see Section 3.1).

Besides these, moreover, it is also possible to consider other analogous discordance indices peculiar to the PRAGMA method, that is using the partial and global ranking frequencies. Thus, for example, with respect to a couple or all criteria simultaneously, a strengthening of the ranking frequencies of an action a_h, respectively partial or global, corresponding to the first and last positions in the ranking, can reveal strongly discordant evaluations of a_h by means of those criteria. Therefore this kind of situation, suitably analyzed, could lead the DM to reconsider the nature of a_h, therefore, in the building phase of the rankings, this situation may lead to a rapid choice of a_h both in the descending and in the ascending procedure, resulting in situations of conflictuality and implicit incomparability.

Software. M&P (MAPPAC and PRAGMA) is a software to rank alternatives using the methods previously described. It presents a lot of options in order to be very flexible in the preference modeling, according to the PCCA philosophy. After loading or writing a file concerning the decisional problem at hand, in the Edit menu it is possible to set all the parameters required to compute the basic and global preference indices or ranking frequencies, i.e. trade-off and importance weights etc.. Some classical statistical analyses on the alternatives evaluations are also allowed (average values, standard deviations, correlations between criteria). The indifference areas can be performed in the Calculation menu. For each couple of criteria, suitable indifference thresholds and shapes can be defined. This option results in some non punctual indifference relations, that can also be seen on useful graphics, showing the indifference area and each pair of alternatives in the chosen plane Og_ig_j, g_i,$g_j \in F$. It is also possible to graphically represent the partial and global profiles and levels of the considered alternatives. Going to Solutions menu, after setting other optional parameters, we can firstly obtaining the (partial and global) preference matrices (MAPPAC) and frequencies matrices (PRAGMA); then, exploiting these data, the descending and ascending complete preorders and the final (partial) preorder (as their intersection) can be built up, respectively for MAPPAC and PRAGMA methods. On interesting geometrical interpretation on omometric axes of the complete preorders computation procedure, expresses with respect to each iteration the different rankings with the corresponding global preference intensities of the alternatives considered each time. This representation shows eventual inversion of preferences (as intersection of the corresponding straight lines) due to the presence of some strong dominance effect. Finally, it is possible to perform a suitable Conflict analysis among the alternatives, by setting the parameters needed to compute the bicriteria discordance indices and the incomparability relations, each time according to the corresponding compensation level established by the DM. The indifference and incomparability relations are also suitably presented in a geometrical way in the bicriteria planes Og_ig_j, for each $g_i,g_j \in F$, where the pairs of action are represented using different colours for different binary relation.

3.3 IDRA

A new MCDA methodology in the framework of PCCA was presented by Greco [7] in IDRA (Intercriteria Decision Rule Approach). Its main (and original) features are: to use mixed utility function (i.e. in the decision process both trade-off and importance intercriteria information are considered) and to allow bounded consistency, i.e. no hard constraint is imposed to the satisfaction of some axiomatic assumptions concerning intercriteria information obtained by DM. With respect to the last point, in a MCDA perspective two different kinds of coherence should be considered: the judgemental and the methodological. The first one concerns the intercriteria information supplied by DM and there is no room for technical judgement with respect to its internal coherence. The second one is related to the exploitation of intercriteria information in order to obtain the final recommendation and a coherence judgment based on some MCDA principles and axioms is allowed. Therefore, according to the judgemental coherence principle, within the IDRA method DM is allowed to give both trade-off and importance intercriteria information, without checking its not requested coherence.

Let $g_j : A \rightarrow \mathbb{R}, \forall j \in I$, an interval scale of measurement; a normalized value c_{hj} of $g_j(a_h)$, $a_h \in I$, can be obtained by introducing two suitable parameters $a(j)$, a minimum aspiration level, and $b(j)$, a maximum aspiration level, for each criterion $g_j \in F$, with $a(j) \leq \min g_j(x)$ and $b(j) \geq \max g_j(x)$ by defining

$$c_{hj} = \begin{cases} \frac{(g_j(a_h)-(a_j))}{b(j)-a(j)} & \text{if } a(j) < b(j), \\ 0 & \text{if } a(j) = b(j). \end{cases}$$

In IDRA, as above emphasized, the compensatory approach and the non-compensatory approach are complementary, rather than alternative, aggregation procedures, following the line coming out from some well known experiments carried out by Slovic [36] and others. The basic idea within IDRA [7] is that matching (i.e. comparing two actions by making the action that is superior on one criterion to be so inferior in the other one that the previous advantage is canceled) is not a decision problem: it is rather a questioning procedure for obtaining the intercriteria information called trade-off. On the contrary, choosing among equated (by matching) packing of actions is a typical decision problems, as ranking and sorting. Therefore, if this assumption is accepted, in each decision problem, like choice, there are two different types of intercriteria information: trade-off, which can be derived from a matching, and importance weights, linked to the intrinsic importance of each subset (also a singleton) of criteria from F.

As a consequence, there is only one utilityfunction U^M, called mixed (see [7]), because both trade-off (α_j) and importance (λ_j) weights are considered, $j \in \mathcal{J}$; thus for each $a_h \in A$:

$$U^M(a_h) = \sum_{j=1}^{m} \lambda_j \alpha_j g_j(a_h).$$

The bounded consistency hypothesis

- for trade-off weights, $w_{ik}w_{kj} = w_{ij}, i, j, k \in \mathcal{J}$, where, in general, w_{pq} is the tradeoff between the criteria g_p and g_q; ·
- for importance-weights, given $G_1, G_2 \subset F$, if G_1 is more important than G_2, then $\sum_{g_i \in G_1} \alpha_j > \sum_{g_i \in G_2} \alpha_j$; if G_2 is more important than G_1, then $\sum_{g_i \in G_1} \alpha_j < \sum_{g_i \in G_2} \alpha_j$; if G_1 and G_2 are equally important, then $\sum_{g_i \in G_1} \alpha_j = \sum_{g_i \in G_2} \alpha_j$;

Very often these requirements are not satisfied by the answers given by the DM and the DM is said "incoherent". But, as remarked by Greco [7], most of these "inconsistencies" derive from the attempt to use information relative to partial comparisons (i.e. with respect to only some criteria from F) for global comparisons (i.e. where all the criteria from F are considered). In IDRA, the hypothesis of bounded consistency means that the information obtained from DM with respect to some criteria from F must be used only for comparisons with respect to the same criteria, according to the principle of judgemental coherence. Therefore, every above problem of intercriteria information consistency is "dissolved" in its origin. In IDRA the framework of PCCA is used to implement the bounded consistency hypothesis, considering therefore a couple of criteria at a time. We observe that, in particular, no requirement of completeness of the relations "more important than" and "equally important to" is assumed. As a consequence, for any couple of distinct criteria $g_i, g_j \in F$, one of the following intercriteria information can be obtained by the DM:

1 both the trade-off and the judgement about the relative importance of the criteria;

2 only the trade-off;

3 only the judgement about the relative importance of the criteria;

4 neither the trade-off nor the judgement about the relative importance of the criteria.

Using this information, a basic preference index $\pi^*_{ij}(a, b)$ can be suitably defined (see [7]). The index $\pi_{ij}: A \times A \rightarrow [0, 1]$ is the image of a valued binary relation, complete and ipsodual, and constitutes a complete valued preference structure (complete preorder) on set A. The index $\pi^*_{ij}(a, b)$ can be interpreted as the probability that a is preferred to b, with respect to a mixed utility function in which the trade-off and importance weights are randomly chosen in the set of intercriteria information furnished by the DM. In IDRA, each piece of intercriteria information concerning the trade-off or the relative importance of criteria can be considered a "decision rule" (tradeoff-rule or importance-rule respectively), since it constitutes a basis for an argumentation about the preference between the potential actions. The DM is asked to give a non negative credibility-weight to each decision rule, according to his/her judgment about the relevance of the corresponding pairwise criterion comparisons in order to establish a global preference [7]. Therefore, from the sum of the basic indices $\pi^*_{ij}(a, b)$, with respect all the considered couple of criteria, weighted by the correspondent credibility-weights for the tradeoff-rule or the importance-rule, the aggregated index $\pi(a, b)$ is obtained, for each $a, b \in A$. These indices can be then exploited using the same procedure proposed for MAPPAC in order to obtain two complete preorders (decreasing and increasing solutions); the intersection of these two rankings gives the final ranking (partial preorder). The aggregated index of IDRA mainly differs from the analogous index of MAPPAC in this point: in MAPPAC all (i.e. with respect to each couple of criteria from *F*) basic indices are aggregated, while in IDRA only the elementary indices corresponding to couples of criteria about which the DM has given decision rules are aggregated (faithfulness principle). In IDRA there is a peculiar characteristic: distinction between:

1 intercriteria information which is not supplied by the DM (i.e. the DM does not says anything about the relative importance between g_i and g_j);

2 intercriteria information by which the DM expresses his/her incapacity to say what is the trade-off or the relative importance between g_i and g_j (i.e. the DM says that he/she is not able to give this information).

In IDRA, in case 1. the comparison with respect to criteria g_i and g_j plays no part; in case 2. the same comparison contributes to the aggregated index by means of considering the corresponding basic index calculated taking into account all the possible importance-weights as equally probable, according to the "principle of insufficient reason" (so called Laplace criterion in the case of decision making under uncertainty).

3.4 PACMAN

A new DM-oriented approach to the concept of compensation in multicriteria analysis was presented by Giarlotta [5, 6] in PACMAN (Passive and Active Compensability Multicriteria ANalysis). The main feature of this approach is that the notion of compensability is analyzed by taking into consideration two criteria at a time and distinguishing the compensating (or active) criterion from the compensated (or passive) one. Separating active and passive effects of compensation allows one to point out a possible asymmetry of the notion of compensability and to introduce a suitable valued binary relation of compensated preference.

The concept of compensation has been analyzed in many papers [35, 37, 38]. The literature on this topic is mainly concentrated on the study of decision methodologies, aggregation procedures and preference structures on the basis of this concept. Therefore definition and usage of compensation have essentiality been *method-oriented,* since this concept has been regarded as a theoretical device of classification.

On the contrary, the notion of compensation examined in PACMAN, namely *compensability,* is aimed at capturing the behavior of a decision maker towards the possibility to compensate among criteria. In our approach, intercriteria compensability remains somehow "the possibility that an advantage on one criterion can offset a disadvantage on another one", but as it is determined by a DM and not by a method. Therefore, being more or less compensatory is not regarded here as the characteristic of a multicriteria methodology or of an aggregation procedure. Instead, it is an intrinsic feature of a DM. In this sense, we speak of a *DM-oriented* usage of the concept of compensation.

There are three steps in PACMAN:

- *compensability analysis,* the procedure aimed at modeling intercriteria relations by means of compensability;
- evaluation of the degree of active and passive preference of an alternative over another one by the construction (at several levels of aggregation) of *binary indices;*
- determination of a binary relation of strict preference, weak preference, indifference or incomparability for each couple of alternatives, on the basis of two valued relations of *compensated preference.*

At each step of the procedure PACMAN requires a strict interaction between the actors of the decision process. Therefore, also this approach allows application of the principles of faithfulness (to the information provided by DM), transparency (at each stage of the procedure) and flexibility (in preference modelization).

Compensability Analysis. Let $g_j: A \rightarrow \mathbb{R}$ be an interval scale of measurement, representing the j-**th** criterion according to a non decreasing preference. For each $j \in \mathcal{J}$, let $\Delta_j: A \times A \rightarrow \mathbb{R}$ be the normalized difference function, defined by $\Delta_j(a,b) = (g_j(a) - g_j(b))/(\beta_j - \alpha_j)$, where α_j and β_j ($\alpha_j < \beta_j$) are respectively the minimum and the maximum value that can be assumed on $j \in \mathcal{J}$.

The aim of compensability analysis is to translate into numerical form the definition of bicriteria compensability for each pair of criteria.This is done by constructing, for each pair (i,j) of criteria, the compensatoryfunction $CF_{i \rhd j}$ of i over j, which evaluates the compensating effect of a positive normalized difference on the passive criterion j.

Since a proper and complete estimation of the compensatory effect for every possible active and passive difference is too demanding in terms of amount and preciseness of the related information provided by the DM , we build $CF_{i \rhd j}$ as a *fuzzy* function. This function associates to any pair of normalized differences $(\Delta_i, \Delta_j) \in]0,1] \times [-1,0[$ a number belonging to [0, 1] the degree of confidence that the positive difference Δ_i totally compensates the negative differences Δ_j. Extending the function in frontier by continuity, we obtain a fuzzy compensatory function $CF_{i \rhd j}$: $[0,1] \times [-1,0] \rightarrow [0,1]$, which satisfies the following conditions:

Weak monotonicities

$$0 \leq \Delta_{i_1} \leq \Delta_{i_2} \leq 1 \text{ and } -1 \leq \Delta_j \leq 0 \quad \Rightarrow \quad CF_{i \rhd j}(\Delta_{i_1}, \Delta_j) \leq CF_{i \rhd j}(\Delta_{i_2}, \Delta_j),$$
$$0 \leq \Delta_i \leq 1 \text{ and } -1 \leq \Delta_{j_1} \leq \Delta_{j_2} \leq 0 \quad \Rightarrow \quad CF_{i \rhd j}(\Delta_i, \Delta_{j_1},) \leq CF_{i \rhd j}(\Delta_i, \Delta_{j_2})$$

Continuity $CF_{i \rhd j}$ is continuous everywhere on [0, 1] × [−1, 0].

The reason for a fuzzy modelling is to minimize the amount of information required from the DM, without losing too much in content. The two conditions stated above are very helpful in this sense. In fact, in order to assess a compensatory function, the DM is asked to determine just the zones where the *degree ofconfidence* expressed by $CF_{i \rhd j}$ is *maximmum* (usually equal to one) or *minimum* (usually equal to zero). Using monotonicity and continuity, it is possible to extend by linearization its definition to the whole domain [0, 1] × [−1, 0], without any further information. Bydefinition, $CF_{i \rhd i} \equiv 0$ for each $i \in \mathcal{J}$.

The procedure for the construction of compensatory functions aims at simplifying the task for the DM in providing meaningful information. On the other hand, this procedure requires the DM to provide a large amount of information. In fact, according to the PCCA philosophy, we estimate intercriteria compensability for each couple of criteria. Moreover, we still distinguish their compensatory reaction within the couple, according to whether they effect or endure compensation. This results in the necessity of assessing a compensatory function for each *ordered* pair of distinct criteria.

However, the large amount of information required by PACMAN allows one to model the relationships between each couple of criteria in a rather faithful and flexible way, according to the PCCA philosophy. Usually, an important criterion is relevant both actively (i.e., contributing to preference) and passively (i.e., opposing to preference). Therefore for each criterion we can treat separately passive resistance and active contribution, concepts related to the notion of veto thresholds and preference thresholds respectively in the outranking approach [35]. For a detailed description of the procedure used to construct compensatory functions see [6].

Preference Modeling. In PACMAN preferences are modelled on the basis of compensability analysis. This is accomplished in steps (2) and (3) of the procedure.

(2): Let $g_j \in G^+(a,b)$, i.e., $\Delta_j(a,b) > 0$. The positive difference $\Delta_j(a,b)$ has a double effect.

- active, because it gives some contribution to the (possible) overall preference of a over b (accept this global preference);
- passive, because it states a resistance to the (possible) overall preference of b over a (reject this global preference).

Active contribution and passive resistance of a over b are evaluated for each $g_j \in G^+(a,b)$, computing the *partial indices* $\Pi_j^+(a,b)$ and $\Pi_j^-(a,b)$, respectively. Successively, active and passive effects are separately aggregated, thus obtaining an evaluation of the total strength of the arguments in favour of a preference of a over b, and of those against a preference of b over a, respectively. Numerically, this is done by computing the two binary *global indices* $\Pi^+(a,b)$ and $\Pi^-(a,b)$. Clearly, the same evaluations are done for the pair (b,a), first computing the partial indices $\Pi_j^+(b,a)$ and $\Pi_j^-(b,a)$, and then the global indices $\Pi^+(b,a)$ and $\Pi^-(b,a)$.

The final output of this stage is a pair of *global net indices* $\Pi(a,b)$ and $\Pi(b,a)$ for each couple of alternatives $a,b \in A$. These indices express the degree of *compensated preference* of a over b and b over a, respectively. The index $\Pi(a,b)$ is obtained from the values of the indices $\Pi^+(a,b)$ and $\Pi^-(b,a)$; similarly, the index $\Pi(a,b)$ is obtained from the values of the indices $\Pi^+(b,a)$ and $\Pi^-(a,b)$. A formalization of the whole procedure can be found in [5].

(3) The last step of PACMAN is the construction of a fundamental system of preferences (*P, Q, I, R*). The relation between the alternatives a and b is determined from the values of the two global net indices $\Pi(a,b)$ and $\Pi(b,a)$.

One of the main interesting features of PACMAN is that intercriteria compensability can be modelled with respect to the real scenarios, treating each pair of criteria in a peculiar way. Complexity and length of the related decision

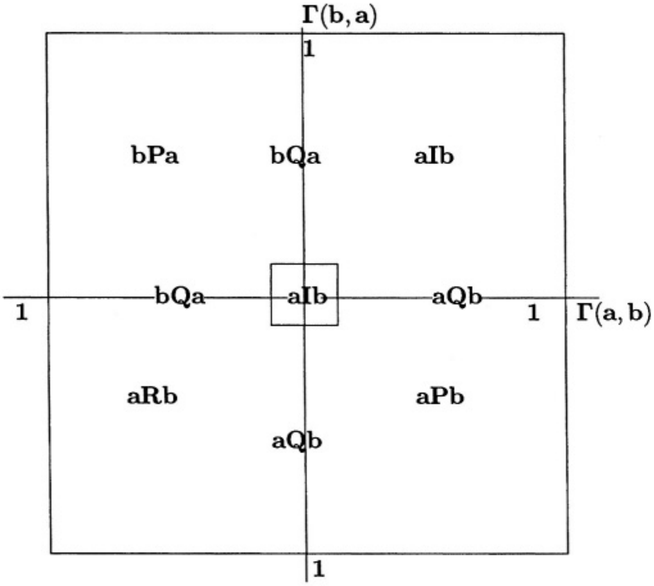


Figure 6.14. Determination of a relation between the two alternatives $a,b \in A$ on the basis of the values of global indices.

process is the price to pay for the attempt to satisfy the principles of faithfulness, transparency and flexibility.

4. One Outranking Method for Stochastic Data

It frequently happens that we have to treat a decision context in which the performance of the alternatives according to each criterion/attribute is subject to various forms of imperfection of the available data. The form of imperfection that interests us here concerns the uncertainty, in the sense of probability (statistic or stochastic data). For example, frequently the decision maker calls upon several experts in order to obtain judgements which then forms the basic data. Since each alternative is not necessarily evaluated at the same level of anticipated performance by all experts, each combination of 'alternative-criterion' leads to a distribution of expert's evaluation. This type of distributional evaluation is considered as stochastic data.

Even if the multi-criteria analysis with stochastic data has so far been treated nearly exclusively in the theory of the multi-attributes utility framework, the outranking synthesis approach can be constituted an appropriate alternative. Some multi-criteria aggregation procedures belonging to this second approach have been developed specially to treat stochastic data. For example, we can mention the works by [4, 19, 17, 18, 42, 20]. The majority of these methods construct outranking relations as in ELECTRE or PROMETHEE. In this chapter we have choose to present the Martel and Zaras' method that makes a link between the multi-attributes utility framework and the outranking approach.

4.1 Martel and Zaras' Method

We consider a multi-criteria problem which can be represented by the (A. A. E.) model (Alternatives, Attributes/Criteria, Evaluators). The elements of this model are as follows:

- $A = \{a_1, a_2, \ldots, a_m\}$ representing the set of all potential alternatives;
- $F = \{X_1, X_2, \ldots, X_n\}$ representing the set of attributes/criteria, an attribute X_j defined in the interval $[x_j^0, x_j^1]$ where x_j^0 is the worst value obtained with the attribute X_j and x_j^1 is the best value; $E = \{f_1, f_2, \cdots, f_n\}$ the set of evaluators, an evaluator $f_j(x_{ij})$ being a probability function associating to each alternative a_i a non-empty set of x_{ij} (a random variable) representing the evaluation of a_i relative to the attribute X_j.

In this method, it is assume known the distributional evaluation of the alternatives according to each attribute and the weight of the attributes.

These attributes (criteria) are defined such that a larger value is preferred to a small value and that the probability functions are known. It is also assume that the attribute set F obeys the additive independence condition. Huang, Kira and Vertinsky (see [11]) showed in the case of the probability independence and the additive multi-attributes utility function, that the necessary condition for the multi-attributes stochastic dominance is to verify stochastic dominance on the level of each attribute. In practice, the essential characteristic of a multi-attributes problem is that the attributes are conflicting. Consequently, the Multi-attributes Stochastic Dominance relation results poor and useless to the DM. It seems to be reasonable to weaken this unanimity condition and accept a majority attribute condition.

Thus, Martel and Zaras' method [20] use the stochastic dominance to compare the alternatives two by two, on each attribute. These comparisons are interpreted in terms of partial preferences. Next, the outranking approach is used for constructing outranking relations based on a concordance index and eventually on a discordance index. With this approach, a majority attribute condition (concordance test) replaces the unanimity condition of the classic dominance. Finally, these outranking relations are used in order to construct the prescription according to a specific problem statement.

Often, in order to conclude that alternative a_i is preferred or is at least as good as $a_{i'}$, with respect to the attribute X_j, it is unnecessary to make completely explicit all the decision-maker's partial preferences. In fact, it can be possible to conclude on the basis of stochastic dominance conditions of first, second and third order (i.e. FSD, SSD and TSD relations), for a class of concave utility functions with decreasing absolute risk aversion (i.e. DARA utility functions class). If the decision-maker's (partial) preference for each attribute X_j can be related by the utility function $U_j \in$ **DARA,** then his preference for the $F_j(x_{ij})$ distribution associated with alternative a_i for each attribute X_j will be:

$$g_j(F_j(x_{ij})) = \int_{x_j^0}^{x_j^1} U_j(x_{ij})\, dF_j(x_{ij}).$$

THEOREM 1 (HADAR AND RUSSEL, 1969) *: If $F_j(x_{ij})$ FSD $F_j(x_{i'j})$ or $F_j(x_{ij})$ SSD $F_j(x_{i'j})$ or $F_j(x_{ij})$ TSD $F_j(x_{i'j})$ and $F_j(x_{ij}) \geq F_j(x_{i'j})$, then $g_j(F_j(x_{ij})) \leq g_j(F_j(x_{i'j}))$for all $U_j \in$ **DARA**, where $F_j(x_{ij})$ and $F_j(x_{i'j})$ represent cumulative distributionfunctions associated with a_i and $a_{i'}$ respectively.*

This theorem allows to conclude clearly that a_i is preferred to $a_{i'}$, with respect to the attribute X_j. We refer the reader to Zaras (see [41]) to review the concept of stochastic dominance.

In the MZ's model, two situations are identified; **clear situation** where the conditions imposed by the theorem are verified (**SD=FSD $\cup$ SSDU $\cup$ TSD** situations), and **unclear situation** where none of the three stochastic dominance is verified. The value of the concordance index can be decomposed into two parts:

Explicable concordance, that corresponds to cases in which the expression of the decision-maker's preferences is trivial or clear.

$$C_E(a_i, a_{i'}) = \sum_{j=1}^{n} \pi_j \delta_j^E(a_i, a_{i'}),$$

where

$$\delta_j^E(a_i, a_{i'}) = \begin{cases} 1 & \text{if } F_j(x_{ij})\ SD\ F_j(x_{i'j}) \\ 0 & \text{otherwise} \end{cases}$$

and π_j is the weight of attribute X_j, with $\pi_j \geq 0$ and $\sum_j^n \pi_j = 1$.

Non-Explicable concordance that corresponds to the potential value of the cases in which the expression of the decision-maker's preferences is unclear.

$$C_{NE}(a_i, a_{i'}) = \sum_{j=1}^{n} \pi_j \delta_j^{NE}(a_i, a_{i'}),$$

where

$$\delta_j^{NE}(a_i, a_{i'}) = \begin{cases} 1 & \text{if no } F_j(x_{ij})\ SD\ F_j(x_{i'j}) \text{ and} \\ & \text{no } F_j(x_{i'j})\ SD\ F_j(x_{ij}) \\ 0 & \text{otherwise.} \end{cases}$$

This second part of the concordance is only a potential value, as it is not certain that for each of these attribute $F_j(x_{ij})$ will be preferred to $F_j(x_{i'j})$.

In these cases, it may be useful to state a condition which tries to make explicit the decision-maker's value functions $U_j(x_{ij})$. If the condition

$$0 \leq p - C_E(a_i, a_{i'}) \leq C_{NE}(a_i, a_{i'}),$$

where $p \in [0.5, 1]$ is the concordance threshold, is fulfilled, then the explication of the unclear cases leads to a value of the concordance index such that the concordance test is satisfied for the proposition that "a_i globally outranks $a_{i'}$". The objective is to reduce as far as possible, without increasing the risk of erroneous conclusions, the number of time where the $U_j(x_{ij})$ functions must be to make explicit. It is notably in the case of unclear situation that [20] used the probabilistic dominance, as a complementary tool to the stochastic dominance, to build preference relationships.

A discordance index $D_j(a_i, a_i')$ for each attribute X_j may be eventually defined as the ratio between of the difference of the means of the distributions of a_i and a_i' to the range of the scale (if it is justified by the scale level of distributional evaluation):

$$D_j(a_i, a_{i'}) = \begin{cases} \frac{\mu(F_j(x_{i'j})) - \mu(F_j(x_{ij}))}{(x_i^1 - x_i^0)} & \text{if } F_j(x_{i'j}) \; FSD_j \; F_j(x_{ij}) \\ 0 & \text{if } F_j(x_{i'j}) \text{ not } FSD_j F_j(x_{ij}). \end{cases}$$

The difference between the average values of two distributions gives a good indication of the difference in performance of the two compared alternatives. If this difference is large enough in relation to the range of the scale, and FSD is fulfilled on attribute X_j, then the chances are large that a_i is 'dominated' by $a_{i'}$. In this case, MZ assume a minimum level ν_j, called a veto threshold, of the discordance index $D_j(a_i, a_{i'}$ giving to a discordant attribute X_j the power of withdrawing all credibility that a_i globally outranks $a_{i'}$).

The discordance test is related to veto threshold ν_j for each attribute. The concordance and discordance relations for the potential alternatives from A are formulated in a classical manner:

$$\text{For all } (a_i, a_{i'}) \in A \times A, (a_i, a_{i'}) \in C_p \quad \longleftrightarrow \quad C(a_i, a_{i'}) \geq p$$
$$\text{For all } (a_i, a_{i'}) \in A \times A, (a_i, a_{i'}) \in D_\nu \quad \longleftrightarrow \quad \exists j / D_j(a_i, a_{i'}) \geq \nu_j.$$

The outranking relations result from the intersection between the concordance set and the complementary set of discordance set:

$$S(p, \nu_j) = C_p \cap \bar{D}_\nu = C_p \setminus D_\nu.$$

Therefore, like in ELECTRE I, we can conclude that a_i globally outranks $a_{i'}$ $(a_i S a_{i'})$ if and only if $C(a_i, a_{i'}) \geq p$ and $D_j(a_i, a_{i'}) < \nu_j$ for all j. If we have no $a_i S a_{i'}$ and no $a_{i'} S a_i$, then a_i and $a_{i'}$ are incomparable, where S is a crisp outranking relation. On the basis on the level of overlapping of the compared distributions, Martel *et al.* [18] developed preference indices associated to the three types of stochastic dominance and constructed the valued outranking relations.

Depending on whether one is dealing with a choice or a ranking problematic, either the core of the graph of outranking relations is determined or the outranking relations are exploited as in ELECTRE II, for example.

EXAMPLE 10 *Given 6 alternatives a_1, a_2, a_3, a_4, a_5 and a_6, 4 attributes X_1, X_2, X_3 and X_4 and the stochastic dominance relation observed between each pair of alternatives according to each attribute (Table 6.20).*

Table 6.20. **Table of observed stochastic dominances.**

	X_1						X_2					
	a_1	a_2	a_3	a_4	a_5	a_6	a_1	a_2	a_3	a_4	a_5	a_6
a_1	×	—	—	—	TSD	—	×	FSD	FSD	FSD	FSD	FSD
a_2	FSD	×	—	—	FSD	—	—	×	FSD	FSD	FSD	FSD
a_3	FSD	FSD	×	SSD	FSD	FSD	—	—	×	FSD	FSD	FSD
a_4	FSD	FSD	—	×	SSD	FSD	—	—	—	×	FSD	—
a_5	—	—	—	—	×	—	—	—	—	—	×	—
a_6	FSD	FSD	—	—	FSD	×	—	—	—	—	FSD	×
	X_3						X_4					
	a_1	a_2	a_3	a_4	a_5	a_6	a_1	a_2	a_3	a_4	a_5	a_6
a_1	×	—	SSD	SSD	FSD	FSD	×	FSD	—*	FSD	FSD	SSD
a_2	FSD	×	SSD	SSD	FSD	FSD	—	×	FSD	FSD	FSD	FSD
a_3	—	—	×	—	FSD	FSD	—*	—	×	FSD	FSD	FSD
a_4	—	—	SSD	×	FSD	FSD	—	—	—	×	FSD	FSD
a_5	—	—	—	—	×	—	—	—	—	—	×	—
a_6	—	—	—	—	FSD	×	—	—	—	—	FSD	×

* no $a_1 SD a_3$ and no $a_3 SD a_1$ according to X_4.

It is assumed that the weights of the attributes are respectively .09, .55, .27 and .09. The explicable concordance indices was calculated and are presented in Table 6.21. The discordance indices are not considered in this example.

On the basis of the explicable concordance indices, we can build up the following outranking relations for a concordance threshold $p = .90$: $a_1 S a_4$, $a_1 S a_5$, $a_1 S a_6$; $a_2 S a_3$, $a_2 S a_4$, $a_2 S a_5$, $a_2 S a_6$; $a_3 S a_5$, $a_3 S a_6$; $a_4 S a_5$ and $a_6 S a_5$. It is possible to construct the following partial pre-order graph (Figure 6.15); within this graph, the transitivity is respected.

In the Table 6.20 we observe that the relation between a_1 and a_3 according to attribute X_4 is ***unclear*** *since no $F_4(x_{14})$ SD $F_4(x_{34})$ and no $F_4(x_{34})$ SD $F_4(x_{14})$. If the decision-maker can explicit $U_4(x)$ and if a_1 is preferred to a_3*

Table 6.21. **Explicable concordances indices.**

	a_1	a_2	a_3	a_4	a_5	a_6
a_1	×	0.64	0.82*	0.91	1	0.91
a_2	0.36	×	0.91	0.91	1	0.91
a_3	0.09*	0.09	×	0.73	1	1
a_4	0.09	0.09	0.27	×	1	0.45
a_5	0	0	0	0	×	0
a_6	0.09	0.09	0	0.55	1	×

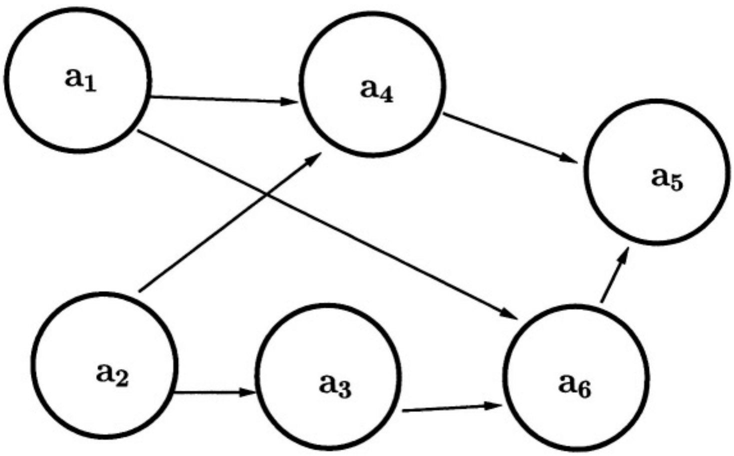


Figure 6.15. Partial preorder.

according to this attribute, then globally a_1Sa_3 *with a concordance thresholds* $p = .90$ *since* $C(a_1, a_3) = .91$ *(.82 in Table 6.21 +.09 (the weight of* X_4*)).*

5. Conclusions

In this chapter some outranking methods different from ELECTRE and PROMETHEE family have been presented, able to manage different type of data (ordinal, cardinal and stochastic). Their description proved again the richness and flexibility of the outranking approach in preference modelling and in supporting DM in a lot of decisional problem at hand. Some properties of this approach are common to all the outranking methods, others are peculiar features of some of them. In the following we recall the main characteristics of the considered methods.

a) The input of these methods are alternative evaluations that can be given in the form of qualitative (ordinal scale), numerical non-quantitative (with the particular case of interval scales) or stochastic (probability distribution) data with respect to all considered criteria. Sometimes also some technical parameters should be supplied by DM as infracriterion information (indifference, preference, veto thresholds).

b) All these methods need as infracriterion information the importance weights in numerical terms. In some of them, just a particular order of criteria is explicitly requested, otherwise a random weight approach.

d) The outranking methods within the PCCA approach need the elicitation of both importance and trade-off weights, but the information concerning weights does not need to respect completeness (i.e. all pairwise trade-off and/or importance weights given) and transitivity with respect to trade off weights.

e) In their first step, all these methods (apart from PRAGMA) give as results some preference or outranking relations, crisp or fuzzy (preference relations and/or indices).

f) The preference structures associated with these methods is usually *P, I, R,* obtained at global level (comprehensive evaluation). In the PCCA approach is also possible to obtain the same binary relations with respect to each couple or pair (g_i, g_j) of considered criteria (P_{ij}, I_{ij}, R_{ij}).

g) Usually the final recommendation (complete or partial preorder) is obtained by the exploitation of the binary relations previously obtained. But in some ordinal method the complete final preorder is directly obtained as a result of the concordance-discordance analysis between different rankings.

Acknowledgments

I am grateful to Dr. Silvia Angilella for her kind cooperation and great help in the editing of this chapter.

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IV

MULTIATTRIBUTE UTILITY AND VALUE THEORIES