Chapter 15

VERBAL DECISION ANALYSIS

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Abstract Verbal Decision Analysis is a new methodological approach for the construction of decisions methods with multiple criteria. The approach is based on cognitive psychology, applied mathematics, and computer science. Problems of eliciting exact quantitative estimations from the decision makers may be overcome by using preferential information from the decision makers in the ordinal form (e.g., "more preferable", "less preferable",...). This type of judgments is known to be much more stable and consistent. Ways of how to obtain and use ordinal judgments for multicriteria alternatives' evaluation are discussed. Decision methods ZAPROS, and ORCLASS based on the approach are briefly described.

Keywords: Decision analysis, multiple criteria, ordinal judgments, preference elicitation, ZAPROS, ORCLASS.

1. Features of Unstructured Decision Problems

According to Simon [48] decision problems may be divided into three main groups: 1) well-structured problems, 2) ill-structured problems, and 3) unstructured problems.

Well-structured problems are problems where the essential dependencies between parameters are known and may be expressed in a formal way. Problems of this class are being rather successfully solved by operations management methods.

Ill-structured or mixed problems have both qualitative and quantitative elements, but unknown and undefined problem elements tend to dominate these tasks. Problems in this class are rather diversified and methods from different areas may be used to work with them including "cost-benefit" analysis, as well as multicriteria decision making and multicriteria decision aids.

Unstructured problems are the problems with mostly qualitative parameters with no objective model for their aggregation. We can see examples of such tasks in policy making and strategic planning in different fields, as well as in personal decisions. These problems are in the area of multicriteria decision aids but require some special considerations in the methods used.

Larichev and Moshkovich [33, 34] proposed the following list of general features for the unstructured problems:

- the problems in this class are unique in the sense that each problem is new to the decision maker and has characteristics not previously experienced;
- parameters (criteria) in these problems are mostly qualitative in nature, most often formulated in a natural language;
- in many cases evaluations of alternatives against these parameters may be obtained only from experts (or the decision maker him/her self);
- an overall evaluation of alternatives' quality may be obtained only through subjective preferences of the decision maker.

Human judgment is the basic source of information in unstructured problems. Being interested in the result, the decision maker would like to control the whole process, including selection of experts and formation of the decision rule(s). Verbal Decision Analysis (VDA) was proposed as a framework for the unstructured problems [34].

2. Main Principles of Verbal Decision Analysis

The role of decision making methods applied to unstructured problems should be to help the decision maker to structure the problem (form a set of alternatives and elaborate a set of relevant criteria) and work out a consistent policy for evaluating/comparing multicriteria alternatives.

As human judgment is the central source of information in unstructured problems, the proposed methods should consider the constraints of the human information processing system as well as the psychological validity of input data in decision analysis. This requires that the methods should: 1) use language for problem description that is natural to the decision maker; 2) implement psychologically valid measurement of criteria and psychologically valid preference elicitation procedures; 3) incorporate means for consistency check of the decision maker's information; 4) be "transparent" to the decision maker and provide explanations of the result.

Verbal Decision Analysis is oriented on construction of a set of methods for different types of decision tasks within the stated framework.

2.1 Natural Language of a Problem Description

Verbal Decision Analysis tries to structure a decision problem by using the natural language commonly used by a decision maker and other parties participating in the decision process [26]. The goal of problem structuring is to define alternatives and the primary criteria to be used for evaluation.

In unstructured practical decision tasks most decisions involve qualitative criteria with no natural numerical equivalents [28, 34].

People are known to be poor at estimating and comparing objects that are close in value. It is reasonable for qualitative as well as for originally quantitatively measured criteria to have scales with several distinct levels, possibly differentiated in words and examples [17, 20, 52]. For example, experts were found to have much closer estimates of applicants over separate criteria using scales with a small number of verbal estimates than when using a 1 to 10 quality scale [40].

Verbal descriptions over criteria scale levels instead of numerical values, not only allow the decision maker to be more confident in his(her) own evaluations, but also should lead to information from experts that is more stable. Therefore, Verbal Decision Analysis uses scales with verbal descriptions of criteria levels for unstructured problems.

2.2 Psychological Basis for Decision Rules Elaboration

The measurements discussed in the previous section may be referred to as primary measurements. These primary measurements structure the problem to allow construction of a decision rule for overall evaluation and/or comparison of alternatives. Construction of the decision rule for unstructured problems includes elicitation of the decision maker's preferences as there are almost no objective dependencies between decision criteria.

The complexity involved in eliciting preference information from human subjects has been widely recognized. The process of eliciting necessary information for such decisions is one of the major challenges facing the field [19, 27, 28, 49].

The limitations in human ability to evaluate and to compare multiattribute options can lead to inconsistencies in human judgments [45, 51] or to application of simplified rules that do not consider essential aspects of the options under consideration [32, 38, 43].

It is important to understand what input information is reliable. Larichev [28] attempted to collect and classify all elementary operations in information processing used in normative decision-making. Twenty-three operations were defined and analyzed from the perspective of their complexity for human subjects. The study concluded that quantitative evaluation and comparison of different objects was much more difficult for subjects than conducting the same operations through qualitative ordinal expression of preference.

The following operations were found admissible on the basis of the known research results [34]:

- rank ordering of criteria importance;
- qualitative comparison of attribute values for one criterion or two criteria;
- qualitative evaluation of probabilities.

Some other operations are expected to be admissible although not enough research has been obtained to date to be sure of admissibility.

Qualitative judgments are preferable for the majority of operations. Therefore, Verbal Decision Analysis uses ordinal (cardinal) judgments as compared to interval data.

2.3 Theoretical Basis for Decision Rules Elaboration

Ordinal comparisons are always the first practical step in preference elicitation procedures in multicriteria analysis. Rather often, scaling procedures follow this step (resulting in quantitative values for all elements of the model). There are ways to analyze the decision on the basis of ordinal judgments, sometimes leading to the preferred decision without resort to numbers [4, 22, 23, 34]. Possible types of available ordinal preference information can be grouped as follows:

- rank ordering of separate levels upon criterion scales (ordinal scales);
- rank ordering of criteria upon their importance;
- pairwise comparison of real alternatives;

- ordinal tradeoffs: pairwise comparison of hypothetical alternatives differing in estimates of only two criteria.

Ordinal Scales are used in the rule of dominance (Pareto Principle). This rule states that one alternative is more preferable than another if it has criterion levels that are not less preferable on all attributes and is more preferable on at least one. This rule does not utilize criterion importance and is not necessarily connected with an additive form of a value function but it requires preferential independence of each separate criterion from all other criteria.

Rank Ordering of Criteria upon Importance does not provide any decision rule by itself. In combination with ordinal scales and lexicographical criterion ranking, the rule for selection of the best alternative may be as follows: first select alternatives with the best possible level upon the most important criterion. From the resulting subset select alternatives with the best possible level upon the next important criterion and so on. This rule is based on the assumption that in the criterion ranking one attribute is more important than all the other attributes, which follow it in the ranking. This preemptive rule does not necessarily imply the additive value function, but has the obvious drawback of its non-compensatory nature, and is theoretically unpopular.

Pairwise Comparison of real alternatives may be directly used in some methods (see, e.g. [24]). In general this information by itself will lead to the solution (if you compare all pairs of alternatives then you can construct a complete rank order of alternatives). But the whole area of multicriteria decision analysis has evolved from the notion that this task is too difficult for the decision maker. This approach is mostly used in multicriteria mathematical programming (in which there is not a finite number of alternatives for consideration). Still this information is considered to be highly unstable [28, 51].

Ordinal Tradeoffs [33] exploit the idea of tradeoffs widely used in decision analysis for deriving criterion weights, but is carried out in a verbal (ordinal) form for each pair of criteria and for all possible criterion levels. To find the tradeoff we have to ask the decision maker to consider two criteria and choose which he(she) prefers to sacrifice to some lower level of attainment. When levels are changed from the best to the worst attribute level, this corresponds to the questions in the "swing" procedure for criterion weights [11, 52], but does not require quantitative estimation of the preference.

The use of such tradeoffs is valid if there is preferential independence of pairs of criteria from all other criteria. Two of these preference elicitation methods provide the safest basis for preference identification: ordinal criterion scales and ordinal tradeoffs.

2.4 Consistency Check of Decision Maker's Information

Valid implementation of both ordinal criterion scales and ordinal tradeoffs requires preferential independence of one or two criteria (for all practical purposes if there is pairwise criterion independence, there exists an additive value function and it is reasonable to conclude that any group of criteria is independent from the rest – see [53]). In addition, in many practical cases the decision rule would require transitivity of preferences. It is necessary to check for these conditions for the method to be valid.

The use of preferential independence conditions stems from the desire to construct an efficient decision rule from relatively weak information about the decision maker's preferences. On the other hand complete checking for this condition will require an exhaustive number of comparisons. Therefore it is reasonable [33, 34] to carry out a partial check of the independence condition over pairs of alternatives. First all necessary tradeoff comparisons are carried out with all criterion levels except those being considered held at their most preferable level. Then, the same tradeoffs are carried out with all other criteria held at their least preferable level. If preferences are the same in both cases, those two criteria are considered to be preferentially independent from all other criteria.

This check is considered to be profound as the change in criterion levels is the most drastic (from the best to the worst) and stability of preferences under those conditions is good evidence of independence.

In case of dependency Verbal Decision Analysis recommends trying to reformulate the problem: group some criteria if they seem to be dependent, or decompose some criteria if their dependence seems to have a root in some essential characteristic combining several others that should be considered separately (see [34] for more details).

To be able to check for consistency of the information elicited (for ordinal information in the form of transitivity of preferences), Verbal Decision Analysis applies "closed procedures" where subsequent questions can be used to check information over all previous questions. For instance, if we ask the decision maker to compare A and B, then B and C, it's a good idea to ask the decision maker to compare A and C as well. If A is preferred to B, B is preferred to C, and A is preferred to C, then everything is consistent. If C is preferred to A, the preferences are intransitive. Within our approach, transitivity of preferences is assumed, so the decision maker is asked to reconsider comparisons from which intransitivity arises.

2.5 Explanation of the Analysis

The last but not the least requirement for Verbal Decision Analysis is to demonstrate the results of the analysis to the decision maker in a way that connects the problem structure and the elicited information with the resulting recommended alternative or alternatives.

It should be possible for the decision maker to see how information provided by him(her) lead to the result obtained. This is a necessary condition for the decision maker to rely on the result and to have the necessary information for re-analysis in case the result does not seem plausible. Methods based on Verbal Decision Analysis principles provide the ability to give explanations due to their logical and valid elicitation and their use of qualitative information.

In the next two sections methods based on these principles are presented for two important decision problems: rank ordering of multicriteria alternatives and ordinal classification/sorting [9] of multicriteria alternatives.

3. Decision Methods for Multicriteria Alternatives Ranking

The problems of ranking alternatives evaluated against a set of criteria are wide spread in real life. There are many decision aiding methods oriented on the solution of these problems [21, 34, 44, 46].

Within the Verbal Decision Analysis framework, we consider an unstructured problem where there is a large number of alternatives with mostly qualitative characteristics evaluated by human experts. The task is to elaborate a subjective decision rule able to establish at least a partial order on the set of alternatives.

Alternatives are evaluated against a set of criteria with *verbal* formulations of quality grades along their scales and as the number of alternatives is large enough the idea is to construct a decision rule in the criteria space and then use it on any set of real alternatives.

A good example of such a problem is selection of applicants for an interview for a faculty position [40]. A variant of a set of criteria with simple ordinal scales for evaluation of an applicant for a position in Management Information Systems is presented in Table 15.1

Method ZAPROS was proposed to deal with this type of problem and was based on the VDA principles. The ideas of ZAPROS started to be developed in 80s by a group of Russian scientists under the leadership of Larichev. The first publication in English presenting fully developed version of earlier ideas appeared in a 1995 issue of European Journal of Operational Research [33].

The method is based on the implementation of ordinal verbal scales and ordinal tradeoffs on the scales of criterion pairs near two reference situations. The goal is the construction of the Joint Ordinal Scale for all criteria. The name ZAPROS is the abbreviation of Russian words: Closed Procedures near Reference Situations.

Let us look more closely at the method and its enhancement during recent years.

Table 15.1. Criteria for applicant evaluation.

Criteria	Scale
A. Ability to teach our students	A1. Above average
	A2. Average
	A3. Below average
B. Ability to teach SA&D and DBMS	B1. Above average
	B2. Average
	B3. Below average
C. Evaluation of completed research and scholarship	C1. Above average
	C2. Average
	C3. Below average
D. Potential in publications	D1. Above average
	D2. Average
	D3. Below average
E. Potential leadership in research	E1. Above average
	E2. Average
	E3. Below average
F. Match of research interests	F1. Above average
	F2. Average
	F3. Below average

3.1 Problem Formulation

Formal presentation of the problem under consideration is as follows:

Given:

1 There is a set of n criteria for evaluation of alternatives.

2 X_i is a finite set of possible verbal values on the scale of criterion $i = 1, ..., n$, where $|X_i| = n_i$.

3 $X = \prod_{i=1}^{n} X_i$ is a set of all possible vectors in the space of n criteria.

4 $A = \{a_1, ..., a_i, ..., a_m\} \subseteq X$ is a subset of vectors from *X* describing real alternatives.

Required: to rank order alternatives from the set A on the basis of the decision-maker's preferences.

We will use the following notations for relationships between alternatives:

- $\succeq_i$ is the weak preference relationship with respect to criterion i: for $a, b \in A, a \succeq_i b$ means a is at least as good as b with respect to criterion i;

- $\succ_i$ is the strict preference relationship with respect to criterion i: $a, b \in A, a \succ_i b$ iff $a \succeq_i b$ and not $b \succeq_i a$;

- $\sim_i$ is the indifference relationship with respect to criterion i: $a, b \in A$, $a \sim_i b$ iff $a \succeq_i b$ and $b \succeq_i a$;
- $\succeq$ is the weak preference relationship: for $a, b \in A$, $a \succeq b$ means a is at least as good as b;
- $\succ$ is the strict preference relationship: $a, b \in A$, $a \succ b$ iff $a \succeq b$ and not $b \succeq a$;
- $\sim$ is the indifference relationship with respect to criterion i: $a, b \in A$, $a \sim b$ iff $a \succeq b$ and $b \succeq a$.

3.2 Formation and Implementation of the Joint Ordinal Scale

The first step in any decision analysis is to form the set of alternatives, form the set of criteria, and to evaluate alternatives against criteria. As we have decided to use only ordinal judgments for comparison of alternatives, the first step in this direction is to elaborate ordinal scales for attributes.

Formally, ordering criterion values along one criterion scale requires the decision maker to select the preferred alternative out of two hypothetical vectors from *X* differing in values with respect to one criterion (with all other values being at the same level).

This information allows formation of a strict preference relation $\succ_i$ for each criterion $i = 1, ..., n$.

Ordinal scales allow pairwise comparison of real alternatives according to the rule of *dominance.*

DEFINITION 52 *Alternative* a *is not less preferable than alternative* b*, if for each criterion* i *alternative* a *is not less preferable than alternative* b *(*$a \succeq_i b$ *for* $i = 1, ..., n$*).*

The next level of preference elicitation is based on comparison in an *ordinal form* of combinations of values with respect to two criteria.

To carry out such a task we need to ask a decision maker questions of the kind: "what do you prefer: to have this (better) level with respect to criterion i and that (inferior) level with respect to criterion j, or this (better) level for criterion j and that (inferior) level for criterion i if all other criteria are at the same level?"

Possible responses in this case are: more preferable, less preferable or equally preferable [33].

The decision-maker may be asked to make these "ordinal tradeoffs" for each pair of criteria and for each pair of possible values in their scales.

The same information may be obtained with far fewer questions by comparing two hypothetical vectors from *X* differing in values with respect to two criteria (with all other values being at the same level). Still the number of the comparisons for all possible combinations of criterion values may be quite large.

ZAPROS [33, 34] uses only part of this information for the construction of the Joint Ordinal Scale (JOS). The decision-maker is asked to compare pairs of hypothetical vectors from $Y \subset X$, each vector with the *best possible values* for all criteria but one. The number of these vectors is not large $|Y| = \sum_{i=1}^{n}(n_i - 1) + 1$.

The goal is to construct a complete rank ordering of all vectors from *Y* on the basis of the decision maker's preferences. An example of a possible preference elicitation question is presented in Table 15.2.

Table 15.2. **Comparison of hypothetical alternatives.**

Criteria	Alternative 1		Alternative 2	
A. Ability to teach our students	Above average	A1	Above Average	A1
B. Ability to teach SA&D and DBMS	Above average	B1	Above Average	B1
C. Evaluation of completed research and scholarship	Above average	C1	Above Average	C1
D. Potential in publications	*Average*	*D2*	*Above Average*	*D1*
E. Potential leadership in research	Above average	E1	Above Average	E1
F. Match of research interests	*Above average*	*F1*	*Below Average*	*F3*

Possible Answers:
1. Alt.1 is more preferable than Alt.2
2. Alt.1 and Alt.2 are equally preferable
3. Alt.1 is less preferable than Alt.2

DEFINITION 53 *Joint Ordinal Scale (JOS) is a complete rank order of vectors from Y, where Y is a subset of vectors from X with all the best values but one. Complete rank order means that for each* $x, y \subseteq Y$ $x \succ y$ *or* $y \succ x$ *or* $x \sim y$.

If the comparisons do not violate transitivity of preferences, we are able to construct a complete rank order of the vectors from *Y* on the basis of this information, forming the Joint Ordinal Scale. An example of the JOS for the applicants' problem is presented in Table 15.3 with the JOS rank for the most preferred vector marked as 1.

Construction of the Joint Ordinal Scale provides a simple rule for comparison of multiattribute alternatives. The correctness of rule 54 in case of pairwise preferential independence of criteria was proven in [33]. The crucial difference between the rule of dominance and this rule is that we are able now to compare criterion values with respect to *different* criteria.

Table 15.3. An Example of a joint ordinal scale.

Equal Criterion Values	Rank in JOS	Corresponding vector(s)
A1,B1,C1,D1,E1,F1	1	A1B1C1D1E1F1
C2, E2	2	A1B1C2D1E1F1 A1B1C1D1E2F1
A2, D2, F2	3	A2B1C1D1E1F1 A1B1C1D2E1F1 A1B1C1D1E1F2
B2	4	A1B2C1D1E1F1
B3, E3, F3	5	A1B3C1D1E1F1 A1B1C1D1E3F1 A1B1C1D1E1F3
A3, C3, D3	6	A3B1C1D1E1F1 A1B1C3D1E1F1 A1B1C1D3E1F1

DEFINITION 54 *Alternative a is not less preferable than alternative b, if for each criterion value of a there may be found a not more preferable unique criterion value of alternative b.*

There is an easy way to implement this rule, introduced and proven correct in [39]. Let us substitute a criterion value in each alternative by the corresponding rank in the Joint Ordinal Scale $(JOS(a))$. Then rearrange them in the ascending order (from the most preferred to the least preferred one), so that

$$JOS_1(a) \leq JOS_2(a) \leq ... \leq JOS_n(a)$$

Then the following rule for comparison of two alternatives may be presented.

DEFINITION 55 *Alternative a is not less preferable than alternative b if for each $i = 1, ..., n$ $JOS_i(a) \leq JOS_i(b)$.*

Let us use our Joint Ordinal Scale presented in Table 15.3 to compare the following two applicants, incomparable on the basis of the dominance rule: a=(A1,B2,C1,D1,E1,F2) and b=(A1,B1,C1,D2,E2,F1)

If we substitute each criterion value in alternatives a and b with corresponding rank from the JOS and rearrange them in an ascending order, we will obtain the following two vectors, which can be easily compared: JOS(a)= (1,1,1,1,3,4) and

JOS(b)=(1,1,1,1,2,3). It is clear now that alternative b is preferred to alternative a.

ZAPROS suggests using Joint Ordinal Scale for pairwise comparison of alternatives from A, thus constructing a partial order on this set.

The construction and implementation of Joint Ordinal Scale, as stated above, is based on two assumptions: transitivity of the decision maker's preferences and preferential independence of pairs of criteria (the last condition leads to an additive value function in the decision maker's preferences [33, 34]). This is the basis for the correctness of rule 55.

For the decision making method to be valid within the paradigm of Verbal Decision Analysis it should provide means for verification of underlying assumptions. ZAPROS provides these means as follows.

3.3 Verification of the Structure of the Decision Maker's Preferences

When comparing vectors from Y (for JOS construction) the decision maker can give contradictory responses. In the problem under consideration these responses may be determined as violations of transitivity in the constructed preference relation.

Possible responses of the decision maker in comparison of hypothetical vectors y_i and y_j from Y (see Table 15.2) reflect the binary relation of strict preference ($\succ$) or indifference (~) between these two alternatives. The following conditions should be met as a result of the decision maker's responses:

if $y_i \succ y_j$ and $y_j \succ$ or $\sim y_k$ then $y_i \succ y_k$
if $y_i \sim y_j$ and $y_j \sim y_k$ then $y_i \sim y_k$
if $y_i \sim y_j$ and $y_j \succ y_k$ then $y_i \succ y_k$.

These conditions are checked in the process of preference elicitation, the intransitive pairs are presented to the decision maker for reconsideration.

The procedure for transitivity verification is described in details in [33, 34], is implemented in a corresponding computerized system and was used in a number of different tasks [34, 37, 40].

The next assumption necessary to check is the pairwise preferential independence of criteria.

DEFINITION 56 *Criteria i and j are* preferentially independent *from the other criteria, if preference between vectors with equal values with respect to all criteria but i and j, does not depend on the values of equal components.*

As it is impossible to carry out preference elicitation for all possible combinations of equal values, it was proposed to check preferential independence for pairs of criteria near two very different "reference situations". One variant is based on all the best values for equal components (used in the construction of JOS). The second with the worst possible values for equal components.

If the decision maker's preferences among criterion values are the same when elicited using these two different points, then it is assumed the criteria are preferentially independent.

Although this check is not comprehensive, the preferential stability when using essentially different criterion values as the "reference" point suggests it would hold with the intermediate levels as well [34].

3.4 Contemporary Modifications of ZAPROS

The general direction in enhancing method ZAPROS in recent years [29, 39] was concentrated on the efforts to ensure higher level of compatibility among real alternatives. To achieve that in both publications it was proposed to carry out more ordinal tradeoffs.

For construction of the Joint Ordinal Scale (see section 3.2), only a relatively small number of comparisons are carried out, limited to vectors with all the best criterion values but one.

In general, the decision-maker may be asked to compare any two hypothetical vectors from *X* differing in values with respect to two criteria (with all other values being at the same level).

Larichev [29] proposed just that in a method called ZAPROS III. The method requires comparing all criterion values for all pairs of criteria and using this information for comparison of real alternatives.

As the number of such comparisons may be quite large, it is reasonable to use this approach for relatively small problems (small number of criteria and small number of possible criterion values with relatively large number of real alternatives).

In [39] the authors proposed to use additional comparisons only after applying Joint Ordinal Scale for comparison of real alternatives. The goal is to elicit information necessary to compare alternatives left incomparable only if there is such a need for making the decision. The process is iterative (as needed), that's why it was named STEP-ZAPROS. The authors carried out simulations to evaluate effectiveness of the procedure and the number of additional comparisons carried out by the decision maker for different problem sizes.

Let's look briefly at each of these new methods.

3.4.1 ZAPROS III. ZAPROS III introduces a notion of *Quality Variation (QV)* which is the result of changing one value on the scale of one criterion (e.g., from *Average ability to teach our students* to *Below Average* level).

The decision maker is to compare all possible QVs for each pair of criteria with the assumption that all other criterion values are at the same level. The number of QVs for each scale is $n_i(n_i - 1)/2$, where n_i is the number of values on the criterion scale. In addition, the decision maker is to compare some QVs on the same scale (e.g., QV from Above Average to Average compared to QV from Average to Below Average).

Once all comparisons for two criteria are carried out all QVs for them are rank ordered forming the Joint Scale for Quality Variation (JSQV). For example, let's assume that the JSQV for the first two criteria in applicants' evaluation example, are as follows (we will use A1,A2 to show changing value from A1 to A2):

$$A1, A2 \succ B1, B2 \succ A1, A3 \succ B1, B3 \succ A2, A3 \succ B2, B3.$$

It is proposed to carry out these comparisons at two reference situations (as in ZAPROS): with all the best and all the worst values with respect to other criteria. If the comparisons provide the same JSQV, these criteria are considered to be preferentially independent.

Those rankings are carried out for all pairs of criteria and can be used to construct a Joint Scale of Criteria Variations (JSCV).

Let's look at a simple example for three criteria: a = **(A3, B1, C2)** and b = **(A2, B2, C1)**. Suppose JSQVs for criteria A & B, B & C, and A & C are as follows:

$$A1, A2 \succ B1, B2 \succ A1, A3 \succ B1, B3 \succ A2, A3 \succ B2, B3.$$

$$C1, C2 \succ B1, B2 \succ B1, B3 \succ C1, C3 \succ B2, B3 \succ C2, C3.$$

$$C1, C2 \succ A1, A2 \succ A1, A3 \succ C1, C3 \succ A2, A3 \succ C2, C3.$$

If we combine all this information together the JSCV is:

$$C1, C2 \succ A1, A2 \succ B1, B2 \succ A1, A3 \succ B1, B3 \succ C1, C3$$

$$C1, C3 \succ A2, A3 \succ B2, B3 \succ C2, C3.$$

If in this process violations of transitivity of preferences are discovered, they are presented to the decision maker, and resolved.

Each QV for each criterion gets a rank (e.g., C1,C2 has rank 1, A1,A2 has rank 2, and so on). This rank can be used to compare alternatives. In ZAPROS III [29] it is proposed to present each real alternative as a combination of JSCV ranks. It is not possible, e.g., in alternative a=**(A3,B1,C2)** it is not clear if A3 should be presented as A1,A3 or A2,A3. In ZAPROS we have only information on A1,A2 and A1,A3. We do not have information on A2,A3 and so there is no question about the rank to use. With JSCV we need to differentiate these two cases. To overcome this, ranks describing two alternatives at the same time should be used.

We can rewrite vectors a and b as follows. Criterion A: the change is from A2 to A3, so we change A3 to rank 7 of A2,A3 in the JSCV and A2 to rank 0. Criterion B: change is from B1 to B2, so we change B2 to rank 3 and B1 to rank 0. Criterion C: change is from C1 to C2, so we change C2 to rank 1 and C1 for rank 0. As a result alternative a is presented as (7,0,1) or (0,1,7) and alternative

b is (0,3,0)or (0,0,3). Vector (0,0,3) dominates vector (0,1,7), so alternative ***b*** is preferred to alternative ***a***.

Although the amount of additional information on the decision maker's preferences is rather large, there still may be incomparable alternatives. In ZAPROS III it is proposed to sequentially select non-dominated nuclei (analogous to ZAPROS [33]). Alternatives from the first nucleus are assigned rank 1. An alternative has a rank r if it is dominated by an alternative ranked r-1 and itself dominates alternative ranked r+1. As a result some alternatives can have a "fuzzy" rank (e.g., 5-7).

3.4.2 STEP-ZAPROS. This approach views the general application of ordinal preferences for comparison of real alternatives as a three-step procedure:

1 use rule of dominance to compare real alternatives on the basis of ordinal scales. If required decision accuracy is obtained, stop here

2 construct Joint Ordinal Scale and use it to compare real alternatives. If required decision accuracy is obtained, stop here

3 use additional ordinal tradeoffs to compare real alternatives as necessary. Use restructuring procedures if the necessary accuracy is not achieved.

Additional comparisons are carried out only when necessary and only the necessary comparisons are carried out. Thus, the procedure is oriented on efficient acquisition of necessary information.

When comparing real alternatives using Joint Ordinal Scale, alternatives are presented through JOS ranks: $JOS(a)$ and $JOS(b)$ (see section 3.2). If alternatives a and b have been left incomparable it means we have at least two ranks such that $JOS_i(a) < JOS_i(b)$ while $JOS_j(a) > JOS_j(b)$. These ranks represent some criterion values in JOS.

The idea is to form two vectors from *X* different in values with respect to only two criteria (with all the best values with respect to all other criteria). Different criterion values represent the "contradicting" ranks in **JOS(*a*)** and **JOS(*b*)**.

Let our **JOS(*a*)= (1,1,1,2,3,3)** and **JOS(*b*)= (1,1,1,1,1,5)**. They are incomparable according to JOS as rank 5 is less preferable than rank 2 or 3. If, for example, rank 5 is more preferable than ranks 3 and 3 *together,* then alternative ***b*** would be preferable to alternative ***a***.

Rank 3 is presented in the JOS (see Table 15.3) by corresponding criterion values A2, D2, and F2. Rank 5 corresponds to criterion values B3, E3, and F3. It allows formation of the following vectors, representing combination of ranks (3,3) and (1,5) and differing in only two criterion values: (A1,B1,C1,D2,E1,F2) and (A1,B1,C1,D1, E1,F3). Comparison of these two vectors will compare D1,D2 with F2,F3 (see ZAPROS III).

If the second vector is preferred to the first one then alternative b is preferred to alternative a. If not, they may be left incomparable.

As the comparison of such specially formed vectors reflects comparison of pairs of ranks in the Joint Ordinal Scale, it is referred to as Paired Joint Ordinal Scale (PJOS) and allows the following rule for comparison of real alternatives:

DEFINITION 57 *Alternative a is not less preferable than alternative b if for each pair of criterion values (a_i, a_j) of alternative a there exists a pair of values (b_k, b_l) of alternative b such that $PJOS(a_i, a_j) \leq PJOS(b_k, b_l)$.*

The proof of the correctness of the rule in case of an additive value function is given in [39].

Preferential independence of criteria is checked while constructing the Joint Ordinal Scale (see section 3.2). Transitivity of preferences at the third step is checked only partially in the process of comparisons (as we have previous information on preferences among some of pairs of JOS ranks). It is technically possible to carry out auxiliary comparisons (as in ZAPROS) to ensure transitive closure. It can be applied as necessary at the discretion of the consultant.

To demonstrate the potential of these three steps, simulation results were presented in [39]. Partial information for different problem sizes in presented in Table 15.4.

Table 15.4. Effectiveness of STEP-ZAPROS.

Parameters								
Number of criteria	5	5	5	5	7	7	7	7
Number of criterion values	3	3	5	5	3	3	5	5
Number of alternatives	30	50	30	50	30	50	30	50
% of compared alternatives	76	76	73	74	63	64	56	59
Additional comparisons	14	17	63	96	21	30	86	141

Data show that 1) the number of real alternatives does not influence the efficiency of the procedure very much; 2) the number of criteria to some extent influences overall comparability of alternatives; 3) the number of criterion values has a *crucial* influence on the number of additional comparisons carried out in the third step.

Overall the data show that method ZAPROS is most efficient for tasks where number of criteria is relatively small and number of alternatives for comparison is relatively large.

4. Decision Methods for Multicriteria Alternatives' Classification

Along with multicriteria choice/ranking problems, people may face multicriteria classification problems [9]. Rather a large number of classification tasks in business applications may be viewed as tasks with classes which reflect the levels of the same property. Evaluating creditworthiness of clients is rather often measured on an ordinal level as, e.g., "excellent", "good", "acceptable", or "poor" [6]. Articles submitted to the journals in the majority of cases are divided into four groups: "accepted", "accepted with minor revisions", "may be accepted after revision and additional review", "rejected" [34]. Applicants for a job are divided into accepted and rejected, but sometimes there may be also a pool of applicants left for further analysis as they may be accepted in some circumstances [5, 50].

Multicriteria problems with ordinal criterion scales and ordinal decision classes were named problems of *ordinal classification* (ORCLASS). As with method ZAPROS the ideas of ORCLASS were developed in 80s by a group of Russian scientists under the leadership of Larichev. Journal publications in English appeared only in mid 90s [41, 1].

4.1 Problem Formulation

Formal presentation of the problem under consideration is close to the one in section 3.1 as we use criteria scales with finite set of verbal values and analyze the criterion space. Thus items 1 -4 are the same while item 5 and what is required in the problem differ.

Given:

1 There is a set of n criteria for evaluation of alternatives.

2 X_i is a finite set of possible verbal values on the scale of criterion $i = 1, ..., n$, where $|X_i| = n_i$.

3 $X = \prod_{i=1}^{n} X_i$ is a set of all possible vectors in the space of n criteria.

4 $A = \{a_1, ..., a_i, ..., a_m\} \subseteq X$ is a subset of vectors from X describing real alternatives

5 $C = \{C_1, ..., C_i, ..., C_k\}$ is a set of decision classes.

Required: distribute alternatives from A among decision classes C on the basis of the decision-maker's preferences.

For example, the applicants' problem presented in Table 15.1 may be viewed as a classification problem if we need to divide all applicants into three classes: 1) accepted for an interview, 2) left for further consideration, 3) rejected.

We will use the same notation for preferences as in section 3.1. In addition, notation $C(a)$ means class for alternative a, e.g., $C(a)=C_2$ means alternative a belongs to the second class.

4.2 An Ordinal Classification Approach

As in ZAPROS the VDA framework assumes ordinal criterion scales establishing a *dominance* relationship among vectors from X (see definition 52). In ordinal classification there is an *ordinal relationship among decision classes* as well. This means that alternatives from class C_1 are preferred to alternatives in class C_2 and so on. The least preferable alternatives are presented in class C_k. As a result alternatives with "better" qualities (criterion values) should be placed in a "better" class.

These ordinal qualities allow formation of an effective decision maker's preference elicitation approach [30, 25, 41, 34, 42, 3, 1, 2].

The decision maker is presented with vectors from X and asked directly to define an appropriate decision class. The cognitive validity of this form of preference elicitation was thoroughly investigated and found admissible [32, 36].

It is possible to present the decision maker with all possible vectors from X to construct a universal classification rule in the criterion space. However, it is impractical even for relatively small problem sizes. The ordinal nature of criterion scales and decision classes allows formulation of a strict preference relation: if vector x is placed in a better class than vector y, then vector x is more preferable than vector y.

DEFINITION 58 *For any vectors $x, y \in X$ where $C(x)=C_i$ and $C(y)=C_j$ if $i < j$ then $x \succ y$.*

As a result we can formulate a condition for a non-contradictory classification of vectors x and y: if vector x dominates vector y and is placed into i-th class, then vector y should be placed into a class not more preferable than the i-th class.

DEFINITION 59 *For any vectors $x, y \in X$ if y is dominated by x ($x \succ y$) and $C(x)=C_i$, then $C(y)=C_j$ where $j \geq i$.*

Using this quality we can introduce a notion of *expansion by dominance* [25].

DEFINITION 60 *If vector $x \in X$ is assigned class C_i by a decision maker, then for all $y \in X$ such that $x \succ y$ possible classes are C_j where $j \geq i$. For all $y \in X$ such that $y \succ x$ possible classes are C_j where $j \leq i$.*

Each classification of a vector from X by a decision maker limits possible classes for all dominating it and dominated by it vectors from X. When the number of admissible classes for the vector becomes equal to one, we have a unique class assigned to a vector.

Using expansion by dominance we can obtain classification for some vectors from X not presented to the decision maker (there are some results [25, 41, 34, 42] showing that between 50 and 75% of vectors may be classified indirectly using this rule).

In addition, there is a simple way to discover possible errors in the decision maker's classifications: if an assigned class is outside the admissible range, there is a contradiction in the ordinal classification. Contradictory classifications may be presented to the decision maker for reconsideration.

For more details on the procedure see [41, 34].

The efficiency of the *indirect* classification of vectors from set X depends on the vectors presented to the decision maker as well as on the class assigned [41, 34]. Ideally, we would like to present the decision maker with as few questions as possible and still be able to construct a complete classification of vectors from set X. Different heuristic approaches were proposed to deal with this problem, based on the desire to find the most "informative" vectors to be presented to the decision maker for classification.

4.3 Class Boundaries and Effectiveness of Preference Elicitation

Ordinal classification allows not only a convenient method of preference elicitation, but also an efficient way to present the final classification of set X.

Let assume we have a classification of set X into classes C. We will view C_i as a subset of vectors from X, assigned to the i-th class.

Two special groups of vectors may be differentiated among them: *lower border* of the class LB_i and the *upper border* UB_i. Upper border includes all *non-dominated* vectors in the class, while lower border includes all *non-dominating* vectors in this class.

These two borders accurately represent the i-th class: we can classify any other vector as belonging to class C_i if its criterion values are between values of vectors from LB_i and UB_i.

Let us look at vector $C(x)=C_i$ which is not in the upper or lower border of the class. It means there is a vector $y \in UB_i$ for which $y \succ x$, thus $C(y) \leq C(x)$. Analogously there is object $z \in LB_i$ for which $x \succ z$. Thus $C(x) \geq C(z)$. But $C(y) = C(z) = C_i$. This leads to $C(x) = C_i$.

Borders summarize classification rules. If we know classification of vectors in the class borders only, it would be enough to classify any vector from set X [41, 34, 42]. That is why, heuristic methods are oriented on finding potential "border vectors" for presentation to the decision maker.

The first approach was based on the maximum "informativeness" of unclassified vectors [34]. Each class was presented by its "center" (average of criterion values of vectors already in the class). For each unclassified vector x for each its admissible class "similarity" measure $p_i(x)$ was calculated (it evaluated how probable that class was for that vector). Also, for each admissible class the number of indirectly classified vectors $g_i(x)$ if x is assigned class C_i was evaluated.

Informativeness $F(x)$ for vector x was calculated as $F(x) = \sum p_i(x)\, g_i(x)$ for all admissible classes. The vector with the largest informativeness value was selected for classification by the decision maker. After that the expansion by dominance was carried out and informativeness of all vectors was recalculated.

Simulations showed high effectiveness of the procedure with only 5 to 15% of all vectors from *X* necessary to be classified by the decision maker [34]. The drawback of the approach is its high computational complexity.

Another approach was proposed in [42]. It is based on a maxmin principle. For each unclassified vector the minimum number of indirectly classified vectors in case of admissible classes is defined and the vector with the maximum number is selected for classification by the decision maker. The computational complexity of the approach is a bit lower than in the previous case.

A new algorithm called CYCLE was presented in [25]. The idea is to construct "chains" of vectors between vectors x and y which belong to different classes. The "chain" is constructed sequentially by changing one criterion value in vector x by one level until we obtain criterion values of vector y. Then the most "informative" vector is searched only in the chain, thus essentially lowering the computational complexity of the algorithm.

The effectiveness of the approach was compared to the algorithms of monotone function decoding and appeared much more effective for smaller problems and simpler borders while being somewhat less effective in more complicated cases.

The computation complexity of CYCLE is not stated in this work and there is a question of how we select x and y for the "chain" construction (in the beginning we have only two classified vectors: with the best criterion values and with the worst criterion values, so the chain contains all other vectors from *X*), but the direction seems promising.

5. Place of Verbal Decision Analysis in MCDA

The decision maker is the central figure in decision making based on multiple criteria. Elicitation of the decision makers' preferences should take into account peculiarities of human behavior in the decision processes. This is the goal of Verbal Decision Analysis.

Like outranking methods (e.g. ELECTRE, PROMETHEE) VDA provides outranking relationships among multicriteria alternatives. At the same time, VDA is designed to elicit a sound preference relationship that can be applied to future cases while outranking methods are intended to compare a given set of alternatives. VDA is more oriented on tasks with rather large number of alternatives while number of criteria is usually relatively small. Outranking methods deal mostly with reverse cases.

VDA bases its outranking on axiomatic relationships, to include direct assessment, dominance, transitivity, and preferential independence. Outranking methods use weights as well as other parameters, which serve an operational purpose but also introduce heuristics and possible intransitivity of preferences. VDA is based on the same principles as multiattribute utility theory (MAUT), but is oriented on using the verbal form of preference elicitation and on evaluation of alternative decisions without resort to numbers. That is why we consider that it is oriented on the same tasks as MAUT and will be compared in a more detail to this approach to multicriteria decision making.

5.1 Multi Attribute Utility Theory and Verbal Decision Analysis Methods

The central part of MAUT is in deriving numeric scores for criterion values and relative criterion weights which are combined in an overall evaluation of an alternative's value.

There are a number of methods and procedures for eliciting criterion weights and scores. Some of these methods are based on sound theory, while others use simplified heuristic approaches.

Experiments show that different techniques may lead to different weights [7, 47], but in modelling situations varying criterion weights often does not change the result thus leading to the conclusion that equal weights work sufficiently well [10, 12]. However, the situation may not be the same for real decision tasks when differences between alternatives are small. Slight differences in weights can lead to reversals in the ranking of alternatives [35, 37, 54].

Two approaches (MAUT and VDA) were applied to the same decision making problems [15, 26, 31]. Positive and negative features of each approach were analyzed, the circumstances under which one or the other would be favored were examined.

Three groups of criteria for comparison were considered: methodological, institutional and personal [15, 26].

Methodological criteria characterize an approach from the following perspectives:

- measurements of alternatives with respect to criteria;
- consideration of alternatives;

- complexity reduction;
- quality of output;
- cognitive burden.

Measurements. VDA uses verbal scales, while MAUT is oriented on obtaining numerical values.

People use verbal communication much more readily than quantitative communication. Words are perceived as more flexible and less precise, and therefore seem better suited to describe vague opinions. Erev and Cohen stated that "forcing people to give numerical expressions for vague situations where they can only distinguish between a few levels of probability may result in misleading assessments" [13].

But there are positive factors in utilization of quantitative information: people attach a degree of precision, authority and confidence to numerical statements that they do not ordinarily associate with verbal statements, and it is possible to use quantitative methods of information processing.

The experiments made over many years by Prof. T. Wallsten and his colleagues demonstrated no essential differences in the accuracy of evaluations [8, 13], but there was essential difference in the number of preference reversals. The frequency of reversals was significantly decreased when using the verbal mode [16].

The two methods differ considerably in whether they *force consideration of alternatives.* If the best alternative is not found by using "verbal" comparisons, VDA seeks to form another alternative that has not previously been considered (generating new knowledge) by acknowledging the fact that there is no best alternative among presented. VDA assumes that if it is not possible to find better alternative on an ordinal level, there is either no satisfactory alternative or alternatives are too close in quality to differentiate between them.

The numerical approach does not force thorough consideration of alternatives, as it is capable to evaluate even very small differences among alternatives. It is always possible to find the best alternative in this case. The question is if the result is reliable enough.

Complexity. VDA diminishes complexity of judgments required from the decision maker as it concentrates only on *essential* differences. The MAUT method requires very exact (numerical) comparisons of differences among criteria and/or alternatives in majority of cases.

Quality of output. MAUT provides overall utility value for each alternative. This makes it possible to not only identify the best alternative but also to define the difference in utility between alternatives. This means that the output of MAUT methods is rich enough to give the decision-maker the basis for detailed evaluation and comparison of any set of alternatives.

VDA attempts to construct a binary relation between alternatives which may lead to incomparable alternatives, but assures that comparisons are based on sound information elicitation.

Cognitive burden. A goal of all decision methods is reducing the confusing effect of ambiguity in preferences. Methods deal with this phenomenon in very different ways. VDA alters ambiguity and corresponding compensations into levels (rather than exact numbers).

MAUT attempts to estimate the exact amount of uncertainty. The payoff is that the analysis can derive a single estimate of uncertainty to go with the single estimate of utility.

Institutional criteria include: the ease of using the approach within organizations, and consequences of cultural differences.

Both MAUT and VDA can be considered improvements over confounding cost-benefit analysis based upon data with little hope of shared acceptance. Achieving greater clarity does, to some extent, provide improved communication within organizations. However, the information upon which MAUT develops utility is of suspect reliability.

The VDA approach uses more direct communication and active groups are used to assign the verbal quality grades on criteria scales. The VDA approach does not require the decision-maker or expert to have previous knowledge in decision methods. On the other hand, MAUT findings can be presented graphically and provide sensitivity analysis because of its numerical basis.

Some cultural differences may influence the applicability of different approaches. Americans tend to use numerical evaluations more often than in some other countries (e.g., Russia). American analysts are usually required "to put a price tag on goods not traded in any market place" [14]. That is not always the case in Europe.

Personal criteria include: the educational level required of decision-makers to use methods; and how the professional habits of analysts influence the selection of an approach.

The practical experience and intellectual ability of the decision-maker are presuppositions for the utilization of any analytical technique. MAUT requires more detailed trade-off balancing, calling for deeper ability to compare pairs of criteria performances. VDA is designed to focus on more general concepts.

Training in decision analysis helps decision-makers to understand and accept the MAUT approach. VDA methods do not require any special knowledge in decision analysis on the part of the decision-maker. The VDA approach is especially useful when a decision is made under new circumstances or in conditions of high ambiguity.

Comparison: The MAUT approach has a strong mathematical basis. MAUT provides a strong justification of the type of utility function used for aggregation of single-attribute utilities over criteria. Different kinds of independence conditions can be assumed [21]. In the case of criteria dependence, a nonlinear form quite different from the simple additive linear model is available. The involvement of the decision-maker is needed to elaborate a utility function. But after this is accomplished, it is possible to compare many alternatives. Should a new alternative appear, no additional decision-maker efforts are needed. Possible inaccuracy in the measurements could be compensated for by sensitivity analysis.

Conversely, the questions posed to decision-makers have no psychological justification. Some questions could be very difficult for humans to completely understand. Decision-makers require special training or orientation in order for MAUT methods to be used. Possible human errors in evaluating model parameters are not considered. Sensitivity analysis is recommended to evaluate stability of the result.

Verbal Decision Analysis has both psychological and mathematical basis. In all stages of the method natural language is used to describe concepts and information gathered relating to preference. Preferential criteria independence is checked. If criteria are dependent, we may try to transform the verbal description of a problem to obtain independence [34]. For example, sometimes criteria (or their scales) may be too detailed (not necessary information) or too general (not possible to differentiate). In these cases introducing two or three more detailed criteria instead of one too general for evaluation or collapsing a couple of criteria into one on a more general level may lead to preferential independence. In addition VDA has special procedures for the identification of contradictions in the information provided by the decision-maker.

Conversely, there are some cases when incomparability (due to lack of reliable information) does not guarantee identification of one best alternative. There may be more than one alternative ranked at the best level. The decision rule might not be decisive enough in cases when a decision must be reached quickly. There is no guarantee that experts could find a better alternative after formulation of directions for improvements of existing alternatives.

5.2 Practical Value of the Verbal Decision Analysis Approach

VDA has positive features of:

- Using psychologically valid preference input;
- Providing checks for input consistency;
- It is based on mathematically sound rules.

It was used in a number of applications for different types of decision problems. ZAPROS (and its variations) was used in R&D planning [33, 34], applicants' selection [40], job selection [35, 37], and pipeline selection [15, 26, 31].

R&D planning problem was connected with a state agency financing different research projects. Number of applications for funding was around several thousand each year, approximately 70% of them were awarded required (or reduced) funding. The decisions were to be made rather quickly after the deadline for applications (couple of months). To be able to cope with this level of complexity, it was decided to construct a decision rule in the criterion space and apply it to alternatives' descriptions against the criteria which were obtained through experts. ZAPROS was used to construct Joint Ordinal Scale in the criterion space which was used to form ordered groups of alternatives (for sequential distribution of funds). The number of criteria ranged from 5 to 7 for different subgroups of projects.

The task of applicants selection was implemented in one of the American universities where there could be more than 100 applicants for a faculty position. Six criteria with three level (verbal) scales were used to construct the Joint Ordinal Scale to be used to select s subset of better applicants for further analysis and an interview. The department chair was the decision maker in this case.

Pipeline selection was a somewhat different type of problem where there were relatively small number of very complicated alternatives: possible routes for a new gas pipeline. Modified variant of ZAPROS was used to elicit preferences from the decision maker in this case and use it to analyze the quality of presented alternatives. All alternatives were found out to be not good enough for implementation. The analysis was directed towards "redefining" the problem (through more detailed and/or less detailed criteria) and formation of a new "adjusted" alternative acceptable for the authorities.

The ordinal classification approach was used for R&D planning and journals' evaluation, as well as for job selection [34, 1]. In addition, this approach was found to be very useful in the area of knowledge base construction for expert systems.

Ordinal classification can be rather easily applied to nominal classification tasks if the decision maker (expert) is asked to evaluate the "level of appropriateness" of each nominal class for the presented vector from the criterion space [42]. Quite a number of applications were in the area of medical diagnostics [30, 42]. Ideas of ordinal classification were also implemented within the framework of case-base reasoning [3, 2] and data mining [18]. Transformation of initially nominal classification problems into problems with ordinal classes and ordinal scales enabled more effective procedures for data analysis.

6. Conclusion

MCDA is an applied science. The primary goal of research in MCDA is to develop tools to help people to make more reasonable decisions. In many cases the development of such tools requires combination of knowledge derived from such areas as applied mathematics, cognitive psychology, and organizational behavior. Verbal Decision Analysis is an example of such a combination. It is based on valid mathematical principles, takes into account peculiarities of human information processing system, and places the decision process within the organizational environment of the decision making.

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