Chapter 11

DEALING WITH UNCERTAINTIES IN MCDA

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Abstract Many MCDA models are based on essentially deterministic evaluations of the consequences of each action in terms of each criterion, possibly subjecting final results and recommendations to a degree of sensitivity analysis. In many situations, such an approach may be justified when the primary source of complexity in decision making relates to the multicriteria nature of the problem rather than to the stochastic nature of individual consequences. Nevertheless, situations do arise, especially in strategic planning problems, when risks and uncertainties are as critical as the issue of conflicting management goals. In such situations, more formal modelling of these uncertainties become necessary.

In this paper, we start by reviewing the meaning and origin of risk and uncertainty. We recognize both internal uncertainties (related to decision maker values and judgements) and external uncertainties (related to imperfect knowledge concerning consequences of action), but for this paper focus on the latter. Four broad approaches to dealing with external uncertainties are discussed. These are multiattribute utility theory and some extensions; stochastic dominance concepts, primarily in the context of pairwise comparisons of alternatives; the use of surrogate risk measures as additional decision criteria; and the integration of MCDA and scenario planning. To a large extent, the concepts carry through to all schools of MCDA. A number of potential areas for research are identified, while some suggestions for practice are included in the final section.

Keywords: Multicriteria analysis, multiobjective programming, uncertainty, risk, utility theory.

1. What is Uncertainty?

The term uncertainty can have many different meanings. The Chambers Dictionary (1998 edition) defines "uncertain" as not definitely known or decided; subject to doubt or question. Klir and Folger [30] quote six different definitions for "uncertainty" from Webster's Dictionary. In the context of practical applications in multicriteria decision analysis, however, the definition given by Zimmermann [59] would appear to be particularly appropriate. With minor editing, this is as follows:

> Uncertainty implies that in a certain situation a person does not possess the information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behaviour or other characteristics.

At a most fundamental level, uncertainty relates to a state of the human mind, i.e. lack of complete knowledge about something. Many writers also use the term "risk", although the definition of the term varies widely. Some earlier work tended to apply the term "risk" to situations in which probabilities on outcomes are (to a large extent) known objectively (cf. Goicoechea et al. [16], p. 389, and Millet and Wedley [37] for some reference to this view). More recently, the concept of risk has come to refer primarily to the desirability or otherwise of uncertain outcomes, in addition to simple lack of knowledge. Thus, for example, Fishburn [13] refers to risk as "a chance of something bad happening", and in fact separates uncertainty (alternatives with several possible outcome values) from the fundamental concept of risk as a bad outcome. Sarin and Weber [45] state that "judgements about riskiness depend on both the probability and the *magnitude of adverse effects*" (my emphasis), while Jia and Dyer [25] also discuss the psychological aspects of establishing a preference order on risks.

For the most part in this chapter, we shall make use of the value-neutral term "uncertainty", referring to "risk" only when direct preference orderings of the uncertainty *per se* are relevant (for example, in Section 4). It is interesting to note in passing that while the thrust of the present discussion is to give consideration to the effects of uncertainty on MCDA, there has also been work on applying multicriteria concepts to the measurement of risk for other purposes, as for example in credit risk assessment (Dimitras et al. [12], who make use of a rough sets approach).

A number of authors (e.g. French [14], Zimmermann [59]) have attempted to categorize types or sources of uncertainty in the context of decision making. French [14], for example, identifies no less than 10 different sources of uncertainty which may arise in model building for decision aid, which he classifies into three groups referring broadly to uncertainties in the modelling (or problem structuring) process, in the use of models for exploring trends and options, and in interpreting results. The common theme underlying such categorizations, as well as those of other authors, such as Friend [15] and Levary and Wan [34], is the need at very least distinguish between *internal uncertainty,* relating to the process of problem structuring and analysis, and *external uncertainty,* regarding the nature of the environment and thereby the consequences of a particular course of action which may be outside of the control of the decision maker. Let us briefly examine each of these broad categories of uncertainty.

Internal uncertainty. This refers to both the structure of the model adopted and the judgmental inputs required by those models, and can take on many forms, some of which are resolvable and others which are not. Resolvable uncertainties relate to imprecision or ambiguity of meaning – for example, what exactly may be meant by a criterion such as "quality of life"? Less easily resolvable problems may arise when different stakeholders generate different sets of criteria which are not easily reconciled; or perceive alternatives in such different ways that they differ fundamentally on how they contribute to the same criterion.

Imprecisions in human judgments, whether these relate to specifications of preferences or values (for example importance weights in many models), or to assessments of consequences of actions, have under certain circumstances been modelled by fuzzy set (see, for example, Chapters 4 and 5 of Klir and Folger [30]) and related approaches (such as the use of rough sets as described by Greco et al. [20, 19, 21]. From the point of view of practical decision aid, such models of imprecision add complexity to an already complex process, and the result may often be a loss of transparency to the decision maker, contrary to the ethos of MCDA. For this reason, the view espoused here is that internal uncertainties should ideally be resolved as far as is possible by better structuring of the problem (cf. Belton and Stewart [6], Chapter 3) and/or by appropriate sensitivity and robustness analysis where not resolvable.

External uncertainty. This refers to lack of knowledge about the consequences of a particular choice. Friend [15] and French [14] both recognize a further distinction between uncertainty about the environment and uncertainty about related decision areas, as described below.

- *Uncertainty about the environment* represents concern about issues outside the control of the decision maker. Such uncertainty may be a consequence of a lack of understanding or knowledge (in this sense it is similar to uncertainty about related decision areas) or it may derive from the randomness inherent in processes (for example the chance of equipment failure, or the level of the stock market). For example, the success of an investment in new production facilities may rest on the size of the potential market, which may depend in part on the price at which the good will be sold, which itself depends on factors such as the cost of raw materials and labour costs. A decision about whether or not to invest in the new facilities must take all of these factors into account. This kind of uncertainty may be best handled by responses of a technical nature such as market research, or forecasting.

- *Uncertainty about related decision areas* reflects concern about how the decision under consideration relates to other, interconnected decisions. For example, suppose a company which supplies components to computer manufacturers is looking to invest in a management information system. They would like their system to be able to communicate directly with that of their principal customers; however, at least one of these customers may be planning to install a new system in the near future. This customer's decision could preclude certain of the options open to the supplier and would certainly have an impact on the attractiveness of options. The appropriate response to uncertainty of this kind may be to expand the decision area to incorporate interconnected decisions, or possibly to collaborate or negotiate with other decision makers.

Under many circumstances, both internal and external uncertainties can be treated in much the same manner, for example by appropriate sensitivity analyses *post hoc*. In other words, the approach might be to make use of a crisp deterministic MCDA methodology, and to subject the results and conclusions to extensive sensitivity studies. Indeed, we would assert that such sensitivity studies should routinely be part of any MCDA application.

Where uncertainties are of sufficient magnitude and importance to be modelled explicitly as part of the MCDA methodology, however, the modelling approaches for internal and external uncertainties may often become qualitatively different in nature. It seems, therefore, that the treatment of the two types of uncertainty should preferably be discussed in separate papers or chapters. In order to provide focus for the present paper, our attention will be focussed primarily on consideration of the *external uncertainties* as defined above. Without in any way minimizing the importance of dealing with internal uncertainties, our choice of the problem of external uncertainties as the theme for this chapter is in part due to the present author's practical experience, which suggests that it is the external uncertainties which are often of sufficient magnitude and importance to require more explicit modelling.

Admittedly, the boundary between external uncertainty and imprecision is, well, fuzzy! To this extent, at least some of the material in this chapter may well be appropriate to internal uncertainties as well, while some methods formulated to deal with human imprecision might equally well be useful in dealing with external uncertainties. We leave it to the reader to decide where this may be true. We do not attempt here a comprehensive review of literature related primarily to internal uncertainties, but the interested reader may wish to consult some of the following references:

- Fuzzy set approaches: Klir and Folger [30]; Chang et al. [9, 8]; Yeh et al. [57]; (Some of these do partially relate to external uncertainties as well.)

- Rough set approaches: Greco et al. [20, 19, 21, 22].

- Identifying potentially optimal solutions amongst uncertainty ranges: Cook and Kress [10]; Lahdelma and Salminen [32]; Lahdelma et al. [33].

Our approach will also be pragmatic, motivated by practical needs of real-world decision analysis. In particular, the fundamental philosophical point of departure is a belief in the over-riding need for *transparency* in any MCDA: it is vitally and critically important that any approaches to MCDA are fully understandable to all participants in the process. Elegant mathematical models which are inaccessible to such participants are of very little practical value.

Within the context of the opening discussion, let us now define a notational framework within which to consider MCDA under uncertainty (primarily "external uncertainty" as defined earlier). Let X be the set of actions or decision alternatives. When there is no uncertainty about the outcomes, there exists a one-to-one correspondence between elements of X and consequences in terms of the criteria, and X may written as the product space $\prod_{i=1}^{n} X_i$, where X_i is the set of evaluations with respect to criterion i. In other words, any $x \in X$ may be viewed as an n-dimensional vector with elements $x_i \in X_i$, where x_i represents the evaluation of x with respect to the criterion i.

Under uncertainty, however, the one-to-one correspondence between actions and evaluations or consequences breaks down. It may be possible to postulate or to conceptualize an ultimate set of consequences $Z_1(x), Z_2(x), \ldots, Z_n(x)$ corresponding to each of the criteria, but at decision time there will still exist many possible values for each $Z_i(x)$. For ease of notation, we shall use $\mathbf{Z}(x)$ to indicate the vector of $Z_i(x)$ values.

In some cases, it may be possible and useful to structure $Z_i(x)$ (or $\mathbf{Z}(x)$) in the form $Z_i(x, \xi)$ (or $\mathbf{Z}(x, \xi)$), where $\xi \in \Xi$ fully characterizes the external conditions, sometimes termed the "states of nature", and Ξ represents the set of all possible states of nature. The assumption is then that once ξ (the state of nature) is established or revealed, then the consequences in terms of each criterion will also be known. We observe, however, that even Ξ might not be fully known or understood at decision time, and that Ξ could possibly depend upon the action x (although, for ease of notation, we shall not show this explicitly).

The question to be addressed in this chapter is that of constructing some form of (possibly partial) preference ordering on X, when the consequences are incompletely known or understood in the sense described in the previous paragraph.

As indicated earlier, one approach may be initially to ignore the uncertainty, and to conduct the analysis on the basis of a nominal set of consequences $x_1, x_2, \ldots, x_n$ chosen to be representative of the possible $Z_i(x)$, followed by extensive sensitivity analysis which takes into account the range of uncertainty in each $Z_i(x)$. Under many circumstances this may be adequate. Care needs to be exercised in undertaking sensitivity analyses, however, as simple "one-at-a-time" variations in unknown parameter values may fail to identify effects of higher order interactions. Some of the complications inherent in undertaking properly validated sensitivity analyses, and suggestions as to how these may be addressed, are discussed by Rios Insua [41], Parnell et al. [39] and Saltelli et al. [44]. In the remainder of this chapter, our focus will be on situations in which the ranges of uncertainty are simply too large to be handled purely by such sensitivity analysis.

In Section 2 we discuss the use of probability models to represent the uncertainties, emphasizing particularly the comprehensively axiomatized approach of multiattribute utility theory. The potential for relaxing the needs to specify complete utility functions are addressed in Section 3, which leads naturally to the use of pairwise comparison models for MCDA. In many practical situations, decision maker preferences for various types of risk (magnitude and impact of the uncertainties) may be modelled by defining explicit risk-avoidance criteria, and these are discussed in Section 4. Finally, links between MCDA and scenario planning for dealing with uncertainties are presented in Section 5, before concluding with some general implications for practice.

2. Probabilistic Models and Expected Utility

The most thoroughly axiomatized mathematical treatment of uncertainty is that of probability theory. The application of probability concepts would require the specification of a (multivariate) probability distribution on $\mathbf{Z}(x)$ for each action x, so that in effect the decision requires a comparison of probability distributions (sometimes called "lotteries" in this context). Let $\P^x(\mathbf{z})$ denote the probability distribution function on $\mathbf{Z}(x)$, i.e.:

$$\P^x(\mathbf{z}) = \Pr[Z_1(x) \leq z_1, Z_2(x) \leq z_2, \ldots, Z_n(x) \leq z_n].$$

Define $P_i^x(z_i)$ as the corresponding marginal probability distribution function for $Z_i(x)$.

Where uncertainties are structured in terms of "states of nature", the probability distributions may be defined on the ξ (rather than on the $\mathbf{Z}(x)$ directly). In some situations, the probability distribution on ξ may be independent of the action which would make the application of probability models much more tractable, but this will not necessarily always be the case.

A possibility at this stage is to construct a deterministic MCDA model based only on expectations, and to subject the results to some form of (possibly interactive) sensitivity analysis guided by the broader distributional properties. Examples of this are in the PROTRADE method described by Goicoechea et al. [16], Chapter 7, dealing with an interactive method for multiobjective mathematical programming problems, and in the stochastic extensions to outranking proposed by Mareschal [35].

Simple expectation models do not, however, take full account of the ranges of outcome which may occur. Multiattribute utility theory (MAUT) extends the concept of expectation to include explicit modelling of risk preferences, i.e. of the magnitudes of dispersion that may occur. MAUT is discussed by Dyer in Chapter 7 of this volume, and also more comprehensively in the now classic text of Keeney and Raiffa [27] and by von Winterfeldt and Edwards [53]. In essence, MAUT seeks to construct a *"utility function"* $U(\mathbf{Z})$, such that for any two actions x and y in X, $x \succsim y$ if and only if $\mathrm{E}[U(\mathbf{Z}(x))] \geq \mathrm{E}[U(\mathbf{Z}(y))]$, where expectations are taken with respect to the probability distributions on $\mathbf{Z}(x)$ and on $\mathbf{Z}(y)$ respectively.

Practically, the construction of the global utility function $U(\mathbf{Z})$ starts with the construction of partial or marginal utility functions individually for each attribute, say $u_i(Z_i)$, satisfying the expected utility hypothesis for variations in Z_i only. The axioms underlying the existence of such marginal utility functions and the methods for their construction are well-known from univariate decision analysis (see, for example, Chapter 7, or Goodwin and Wright[17], Chapter 5). It is well-established that these axioms are not descriptively valid, in the sense that decision makers do systematically violate them (see, for example, the various paradoxes described by Kahnemann and Tversky [26], or in the text of Bazerman [4]). Attempts have been made to extend the utility models to account for observed behaviour (see, for example, Miyamoto and Wakker [38] for a review of such extensions in the multicriteria context). Nevertheless, as we have argued elsewhere (e.g., Belton and Stewart [6], Section 4.3.1), descriptive failures do not lessen the value of the simpler axiomatically based theory of MAUT as a coherent discipline within which to construct preferences in a simple, transparent and yet defensible manner.

The real challenge relates to the aggregation of the $u_i(Z_i)$ into a $U(\mathbf{Z})$ still satisfying the expected utility hypothesis for the multivariate outcomes. The two simplest forms of aggregation are the *additive* and *multiplicative,* which we shall now briefly review (although a full description can be found in Chapter 7).

Additive aggregation. In this case, we define:

$$U(\mathbf{Z}) = \sum_{i=1}^{n} k_i u_i(Z_i). \tag{11.1}$$

This model is only justifiable if the criteria are *additive independent,* i.e. if preferences between the multivariate lotteries depend only on the marginal probability distributions. That this is not an entirely trivial assumption may be seen by considering two-dimensional lotteries ($n = 2$) in which there are only two possible outcomes on each criterion, denoted by z_i^0 and z_i^1 for $i = 1, 2$. Suppose that $z_i^1 \succ z_i^0$. Then without loss of generality, the partial utility functions can be standardized such that $u_1(z_1^0) = u_2(z_2^0) = 0$ and $u_1(z_1^1) = u_2(z_2^1) = 1$. Consider then a choice between two lotteries defined as follows:

- The lottery giving equal chances on $(z_1^0\ ;\ z_2^0)$ and $(z_1^1\ ;\ z_2^1)$; and
- The lottery giving equal chances on $(z_1^0\ ;\ z_2^1)$ and $(z_1^1\ ;\ z_2^0)$.

We note that both lotteries give the same marginal distributions on each Z_i, i.e. equal chances on each of z_i^0 and on z_i^1 for each i. It is easily verified that with additive aggregation defined by (11.1), both of these lotteries yield an expected utility of $(k_1 + k_2)/2$. The additive model thus suggests that the decision maker should always be indifferent between these two lotteries. There seems, however, to be no compelling axiomatic reason for forcing indifference between the above two options. Where there is some measure of compensation between the criteria (in the sense that good performance on one can compensate for poorer outcomes on the other), the second option may be preferred as it ensures that one always gets some benefit (a form of multivariate risk aversion). On the other hand, if there is need to ensure equity between the criteria (if they represent benefits to conflicting social groups, for example), then the first lottery (in which loss or gain is always shared equally) may be preferred.

Multiplicative aggregation. Now we define $U(\mathbf{Z})$ such that:

$$1 + kU(\mathbf{Z}) = \prod_{i=1}^{n} [1 + kk_i u_i(Z_i)] \qquad (11.2)$$

where the multivariate risk aversion k parameter satisfies:

$$1 + k = \prod_{i=1}^{n} [1 + kk_i] . \qquad (11.3)$$

Use of the multiplicative model requires that the condition of *mutual utility independence* be satisfied. A subset of criteria, say $C \subset \{1, 2, \ldots, n\}$ is set to be utility independent of its complement $\bar{C} = \{1, 2, \ldots, n\} \setminus C$, if preferences for lotteries involving only Z_i for $i \in$ *Cforfixed values* of Z_i for $i \in \bar{C}$ are independent of these fixed values. The criteria are said to be mutually utility independent if every subset of the criteria is utility independent of its complement.

In principle, however, there are no good reasons why criteria *should necessarily be* mutually utility independent, and in fact it can be difficult in practice to verify that the condition holds. Good problem structuring for MCDA would seek to ensure preferential independence of some form between criteria (for example, such that trade-offs between pairs of criteria are independent of outcomes on other criteria), but mutual utility independence is a much stronger assumption and a more elusive concept than this.

Models based on weaker preference assumptions have been developed, such as the multilinear model given by:

$$U(\mathbf{Z}) = \sum_{i=1}^{n} k_i u_i(Z_i) + \sum_{i=1}^{n} \sum_{i<j\leq n} k_{ij} u_i(Z_i) u_j(Z_j) + \ldots + k_{12\ldots n} u_1(Z_1) u_2(Z_2) \ldots u_n(Z_n). \quad (11.4)$$

The large number of parameters which have to fitted to decision maker preferences is prohibitive in most real world applications. Even the multiplicative model is far from trivial to apply in practice. Its construction involves the following steps:

- *Assessment of the partial utilities* $u_i(Z_i)$ by standard single attribute lottery procedures.
- *Parameter estimation:* The multiplicative model includes $n + 1$ parameters which have in principle to be estimated. In the light of (11.3), however, only n independent parameters need estimation. Estimates thus require at least n preference statements concerning hypothetical choices to be made by the decision maker. Some of these can be based on deterministic trade-off assessments, but at least one of the hypothetical choices must involve consideration of preferences between multivariate lotteries.

In exploring the literature, it is difficult to find many reported applications even of the multiplicative model, let alone the multilinear model. Some of the practical complications of properly implementing these models are illustrated by Rosqvist [42] and Yilmaz [58].

Such difficulties of implementation raise the question as to how sensitive the results of analysis may be to the use of the additive model (11.1) instead of the more theoretically justifiable aggregation models given by (11.2) or (11.4). We have seen earlier that situations can be constructed in which the additive model may generate misleading results. But how serious is this in practice? Construction of the additive model requires much less demanding inputs from the decision maker, and it may be that the resultant robustness or stability of the model will compensate for biases introduced by use of the simpler model. In Stewart [46] a number of simulation studies are reported in which the effects are studied of using the additive aggregation model when "true preferences" follow a multiplicative aggregation model. Details may be found in the cited reference, but in essence it appeared that the errors introduced by using the additive model were generally extremely small for realistic ranges of problem settings. The errors were in any case substantially smaller than those introduced by incorrect modelling of the partial utility functions (such as by over-linearization of the partial functions which appears to be a frequent but erroneous simplification). Related work (Stewart [47]) has also demonstrated that more fundamental violations of preferential independence may also introduce substantial errors.

Concerns about the validity of the axiomatic foundations of utility theory have led other writers to formulate alternative models to circumvent these. Miyamoto and Wakker [38] review generalizations to utility theory, while others (e.g. Beynon et al. [7] and Yang [56]) relax the demands of probability theory by invoking concepts from Dempster-Shafer theory of evidence. Unfortunately, these generalizations tend often to make the models even more complex and thus less transparent to decision makers, further aggravating difficulties of implementation.

Our overall conclusion is thus that in the practical application of expected utility theory to decision making under uncertainty, the use of the additive aggregation model is likely to be more than adequate in the vast majority of settings. The imprecisions and uncertainties involved in constructing the partial utilities, which need in any case to be addressed by careful sensitivity analysis, are likely to far outweigh any distinctions between the additive and multiplicative models. In fact, given that marginal utility functions based on preferences between hypothetical lotteries may generally not differ markedly from deterministic value functions based on relative strengths of preference (e.g. von Winterfeldt and Edwards [53], Chapter 10), we conjecture that even the first step of the model construction could be based on the latter (e.g. by use of the SMART methodology, von Winterfeldt and Edwards [53], Section 8.2).

3. Pairwise Comparisons

As indicated in the previous section, the requirements of fitting a complete utility function can be extremely demanding both for the decision maker (in providing the necessary judgemental inputs) and for the analysts (in identifying complete multivariate distributions). We have seen how the assumption of a simple additive model may substantially reduce these demands without serious penalty in many practical situations. Nevertheless, other attempts at avoiding the construction of the full utility model have been made.

Even for single criterion models, the construction and validation of the complete utility model may be seen as too burdensome. Quite early work recognized, however, that it may often not be necessary to construct the full utility function in order to confirm whether one alternative is preferred to another. The conclusions may be derived from the concepts of *stochastic dominance* introduced by Hadar and Russell [23], and extended (to include third order stochastic dominance) by Whitmore [55].

For purposes of defining stochastic dominance, suppose for the moment that there is only one criterion which we shall denote by $Z(x)$ (i.e. unsubscripted). Then let $P^x(z)$ be the (univariate) probability distribution function of $Z(x)$, i.e.: $P^x(z) = \Pr[Z(x) \leq z]$. With some abuse of notation, we shall use P^x (without argument) to denote the probability distribution described by the function $P^x(z)$. Suppose also that values for $Z(x)$ are bounded between z^L and z^U.

Three degrees of stochastic dominance may then be defined as follows.

First degree stochastic dominance (*FSD*): P^x stochastically dominates P^y in the *first degree* if and only $P^x(z) \leq P^y(z)$ for all $z \in [z^L, z^U]$ (Hadar and Russell [23]).

Second degree stochastic dominance (*SSD*): P^x stochastically dominates P^y in the *second degree* if and only:

$$\int_{z^L}^{\zeta} P^x(z)dz \leq \int_{z^L}^{\zeta} P^y(z)dz$$

for all $\zeta \in [z^L, z^U]$ (Hadar and Russell [23]).

Third degree stochastic dominance (*TSD*): P^x stochastically dominates P^y in the *third degree* if and only $\mathrm{E}[Z(x)] \geq \mathrm{E}[Z(y)]$ and:

$$\int_{z^L}^{\eta} \int_{z^L}^{\zeta} P^x(z)dzd\zeta \leq \int_{z^L}^{\eta} \int_{z^L}^{z} P^y(z)dzd\zeta$$

for all $\eta \in [z^L, z^U]$ (Whitmore [55]).

In this single-criterion case, the standard axioms of expected utility theory imply the existence of a utility function $u(z)$ such that $x \succ y$ if and only if:

$$\int_{z^L}^{z^U} u(z)dP^x(z > \int_{z^L}^{z^U} u(z)dP^y(z).$$

Without having explicitly to identify the utility function, however, considerations of stochastic dominance allow us to conclude the following (Bawa [3]):

1 If P^x stochastically dominates P^y in the first degree $(P^x \;\; FSD\, P^y)$, then $x \succ y$ provided that $u(z)$ is an increasing function of z (which can be generally be assumed to be true in practical problems).

2 If P^x *SSD* P^y, then $x \succ y$ provided that $u(z)$ is a concave increasing function of z (i.e. the decision maker is risk averse).

3 If P^x *TSD* P^y, then $x \succ y$ provided that $u(z)$ is a concave increasing function of z with positive third derivative (corresponding to a risk averse decision maker exhibiting decreasing absolute risk aversion).

The potential importance of the above results lies in the claim that has been made that in practice some form of stochastic dominance may hold between many pairs of probability distributions. In other words, we may often be able to make pairwise comparisons between alternatives according to a particular criterion on the basis of stochastic dominance considerations, without needing to establish the partial value function for comparison of lotteries. In fact, we may often argue that FSD provides a strict pairwise preference, while SSD and TSD provide weaker forms of pairwise preference. Only in the absence of any stochastic dominance would we be unable to determine a preference without obtaining much stronger preference information from the decision maker.

The existence of pairwise preferences at the level of a single criterion under uncertainty suggests that some form of outranking approach may be appropriate to aggregation across multiple criteria under uncertainty. D'Avignon and Vincke [11] did in fact propose an outranking approach to dealing with uncertainty, in which they started by comparing univariate probability distributions for each criterion in order to obtain "preference indices" measuring degree of preference for one lottery over another in terms of one criterion, which were then aggregated according to an outranking philosophy. Their preference indices may not be easily interpretable by many decision makers however, and perhaps with this problem in mind, Martel and Zaras [36] (but see also Azondékon and Martel [1]) suggested an alternative outranking approach in which preferences according to individual criteria were established as far as possible by stochastic dominance considerations.

Martel and Zaras found it useful to introduce two forms of concordance index, which they term "explicable" and "non-explicable". For the "explicable" concordance, x is judged at least as good as y according to criterion i if P_i^x stochastically dominates P_i^y at first, second or third degrees. This is quite a strong assumption, as it implies decreasing absolute risk aversion. The "non-explicable" concordance arises if neither of P_i^x or P_i^y stochastically dominates the other. The authors concede that in this case it is not certain that x is at least as good as y, but they do combine the two indices under certain conditions. The discordance when comparing x to y is only non-zero in their model if P_i^y *FSD* P_i^x.

Although some of the implementation details are not clear from the paper, the method of Martel and Zaras does appear to offer potential as an approach to dealing with uncertainty in MCDA using quite minimal preference information from the decision maker. This might at least be valuable for a first-pass screening of alternatives. Two problems may, however, limit wide applicability:

- Strong independence assumptions are implicitly made: The approach is based entirely on the marginal distributions of the elements of $\mathbf{Z}(x)$. This would only be valid if these elements (i.e. the criteria) were stochastically independent, or if the decision maker's preferences were additively independent in the sense of Keeney and Raiffa [27]. Either assumption would need to be carefully justified.

- Strong risk aversion assumptions are made: As indicated above, the method as proposed bases concordance measures on risk aversion and on decreasing absolute risk aversion. Especially the latter assumption may not always be easy to verify. The method can be weakened by basing concordance either only on FSD or on FSD and SSD, but this may not generate such useful results.

There is clear scope for further research aimed at addressing the above problems.

4. Risk Measures as Surrogate Criteria

In this and the next sections, we move to more pragmatic approaches to dealing with uncertainty in the multicriteria context.

One obvious modelling approach is to view avoidance of risks as decision criteria in their own right. For example, the standard Markowitz portfolio theory (cf. Jia and Dyer [25]) represents a risky single-criterion objective (monetary reward) in terms of what are effectively two non-stochastic measures, namely expectation and standard deviation of returns. In this sense a single criterion decision problem under uncertainty is structured as a deterministic bi-criterion decision problem. The extension to risk components for each of number of fundamental criteria is obvious (see, for example, Millet and Wedley [37], p. 104, in the context of AHP).

There has, in fact, been a considerable literature on the topic of measuring risk for purposes of decision analysis, much of it motivated by the descriptive failures of expected utility theory. Papers by Sarin and Weber [45], and by Jia and Dyer [25] contain many useful references. This literature is virtually entirely devoted to the single criterion case (typically financial returns), but it is worth recalling some of the key results with a view to extending the approaches to the multicriteria case.

The common theme has been that of developing axiomatic foundations for representation of psychological perceptions of risk (including consideration of importance and impact in addition to simple uncertainty), often based on some form of utility model. For example, Bell [5] considers situations in which, if a decision maker switches from preferring one (typically more risky) lottery to another as his/her wealth increases, then he/she never switches back to preference for the first as wealth further increases. This he terms the "one-switch" rule for risk preferences, and demonstrates that if the decision maker is decreasingly risk averse, obeys the one switch rule, and approaches risk neutrality as total wealth tends to infinity, then the utility as a function of wealth w must take on the form $w - be^{-cw}$ for some positive parameters b and c. Taking expectations results in an additive aggregation of two criteria, namely:

- The expectation of wealth (to be maximized); and
- The expectation of be^{-cw} (to be minimized), which can be viewed as a measure of risk.

Sarin and Weber [45] and Jia and Dyer [25] provide arguments for general moments of the distribution of returns (including but not restricted to variance) and/or expectations of terms such as be^{-cw}, as measures of risk. While these may be useful as descriptive measures of risk behaviour, from the point of view of practical decision aid it is doubtful whether the decision maker would be able to interpret anything but variance (or standard deviation) for purposes of providing necessary preference information (to establish tradeoffs, relative weights, goals, etc.).

Limited empirical and simulation work which we have undertaken in the context of fisheries management (Stewart [48]) suggested that perceptions of risk of fishery collapse might be modelled better by probabilities of achieving one or more goals (in that case, periods of time before a collapse of the fishery). One advantage of such measures is that they might be much more easily interpreted by decision makers for purposes of expressing preferences or value judgements.

Given the modelling success in representing preferences under uncertainty by simple additive models of expected return and one or more risk measures, there seems to be no reason why such results should not be extended to the general multicriteria problem under uncertainty. In other words, each criterion (not necessarily financial) for which there exists substantial uncertainties might be restructured in terms of two separate criteria, viz. expected return and risk. Many of the above results produce an axiomatic justification for an additive aggregation of expected return and risk, so that these sub-criteria would be preferentially independent under the same axiomatic assumptions.

In spite of how obvious such multicriteria extensions might be, there seems to be little reference in the literature to explicit multicriteria modelling of returns and risks. It is this author's experience, however, that various risk-avoidance criteria arise almost naturally during the structuring phase of decision modelling, so that in practice risk avoidance criteria may in fact be more common than is apparent from the literature.

Some of the few explicit references to multicriteria modelling in terms of a risk-return decomposition appear in the context of goal programming. For example, Ballestero [2] expresses a stochastic multicriteria problem in terms of goals on combinations of risks and returns which are then solved by goal programming, but he does not separate out the risk and return components which may have led to a simpler model structure. Korhonen [31] develops a multicriteria model for financial management, in which a number of different financial performance measures are used as criteria, some of which have a risk interpretation. Details of the solution procedure are not given, but the formulation clearly lends itself to a goal programming structure.

A somewhat earlier paper by Keown and Taylor [28] describes an integer goal programming model for capital budgeting, which can be viewed (together with the STRANGE method of Teghem et al. [50]) as an extension of chance-constrained stochastic programming (see Ruszczyński and Shapiro [43] for a broad introduction to stochastic programming). Keown and Taylor define goals in terms of desired probability levels, which may generically be expressed in the form:

$$\Pr\left[g(Z) \leq \beta\right] \geq \alpha$$

where $g(Z)$ is some performance function based on the unknown attribute values, β the desired level of performance, and α a desired probability of achieving such performance. By using normal approximations, however, Keown and Taylor reduce the probability goal to one expressed in terms of a combination of mean and standard deviation which is subsequently treated in a standard goal programming manner. This suggests opportunity for research into investigation of generalized goal programming models which deal directly with deviations from both the desired performance levels (b, above) and the desired probability levels (α, above).

Some work on fuzzy multiobjective programming (e.g. Chang et al. [9] and Chang and Wang [8]) can be viewed in a similar manner, in the sense that a degree of anticipated level of goal achievement, measured in a fuzzy membership sense, may be interpreted as a risk measure.

More generally, the structuring of MCDA problems under uncertainty in terms of expected value and risk sub-criteria for each main criterion does have the advantage of being relatively simple and transparent to users. Such an approach appears to be easily integrated into any of the main MCDA methodologies, namely value measurement, outranking and goal programming/reference point methods. As indicated earlier, however, a decidedly open research question relates to the manner in which risk is most appropriately measured *for this purpose.*

A further practical issue is the extent to which the necessary independence properties can be be verified. In other words, to what extent can "risk" on one criterion be measured and assessed without taking into consideration ranges of uncertainties on the other criteria. Once again, this offers much scope for further research.

5. Scenario Planning and MCDA

Scenario planningscenario planning (van der Heijden [52]) was developed as a technique for facilitating the process of identifying uncertain and uncontrollable factors which may impact on the consequences of decisions in the strategic management context. Scenario analysis has been widely accepted as an important component of strategic planning, and it is thus somewhat surprising how little appears to have been written concerning links between MCDA and scenario planning. A discussion of the link between scenario planning and decision making is provided by Harries [24], but does not place this in an MCDA framework.

Scenario planning may be described as a process of organizational learning, distinguished by an emphasis on the explicit and ongoing consideration of multiple futures. The scenarios themselves are constructed as stories which describe the current and plausible, but challenging, future states of the organizational environment. They provide alternative perspectives that will challenge an organization in viewing the future and in evaluating its strategies and action plans. The primary goal of scenario planning is in the first instance to provide a structured "conversation" to sensitize decision makers to external and uncontrollable uncertainties, and to develop a shared understanding of such uncertainties. The approach is, however, naturally extended to the more analytical process of designing, evaluating and selecting courses of action on the basis of robustness to these uncertainties, which suggests close parallels with MCDA (as discussed, for example, by Goodwin and Wright [18]). We shall explore these parallels shortly.

Scenarios are meant to represent fairly extreme futures than can still be viewed as plausible. As to what constitutes sufficiently "extreme" would depend on the facilitator, as in a very real sense, there will always be a possible future more extreme (and thus with greater potential impact on the consequences of decisions) than any which is incorporated into formal scenarios.

Van der Heijden suggests five principles which should guide scenario construction:

- At least two scenarios are required to reflect uncertainty, but more than four has proved (in his experience) to be impractical;

- Each scenario must be plausible, meaning that it can be seen to evolve in a logical manner from the past and present;

- Each scenario must be internally consistent;

- Scenarios must be relevant to the client's concerns and they must provide a useful, comprehensive and challenging framework against which the client can develop and test strategies and action plans;

- The scenarios must produce a novel perspective on the issues of concern to the client.

Once scenarios are constructed, they may be used to explore and to evaluate alternative strategies for the organization. Most proponents of scenario planning seem to avoid formal evaluation and analysis procedures, preferring to leave the selection of strategy to informed judgement. For example, van der Heijden [52] (pp. 232–235) rejects "traditional rationalistic decision analysis" as an approach which seeks to find a "right answer". This, however, represents are rather limited and technocratic view of decision analysis, contrary to the constructive and learning view espoused by most in the MCDA field. The constructivist perspective is discussed at a number of places by Belton and Stewart [6] (see particularly Chapters 3, 4 and 11), where it is argued that the underlying axioms are not meant to suggest a "right answer", but to provide a coherent discipline within which to construct preferences and strategies. Within such a view, the aims of scenario planning and MCDA share many commonalities, suggesting the potential for substantial synergies in seeking to integrate MCDA and scenario planning. On the one hand, MCDA can enrich the evaluation process in scenario planning, while the scenario planning approach can contribute to deeper understanding of the effects of external uncertainties in MCDA.

Various authors have hinted at the concept of scenarios in MCDA. These include, for example, Klein et al. [29], although this is largely in the context of a two state stochastic programming model; Watkins et al. [54], also in a stochastic programming context; Millet and Wedley [37], Section 3, who refer to "states of nature"; Urli and Nadeau [51] in the context of multiple objective linear programming. These authors do not refer directly to the philosophical basis of scenario planning, however, and in many senses the models are structured to suggest that the scenarios or states of nature constitute a complete sample space (see later).

Pomerol [40] is one of the few to discuss scenario planning in the context of decision theory or decision analysis, but without substantive link to MCDA. He does however warn (page 199) of the danger that what might appear to be a robust choice of action (perhaps through unstructured and unsupported use of scenarios) may in fact be an illusion resulting from the fact that some events have simply been ignored. Such a danger suggests another perspective on the potential for two-way synergistic advantage between scenario planning and formal decision analysis: not only may scenario planning provide a means of dealing with uncertainties in MCDA, but decision analysis might contribute to avoiding of illusions of robustness or control in decision making. In the latter context, MCDA might contribute to the choice of scenarios as well as to the formal analysis of alternative courses of action.

Preliminary suggestions for such integration of scenario planning and MCDA is made on pages 312–315 of Belton and Stewart [6], which extended an earlier discussion in Chapter 14 of of Goodwin and Wright [17]. In the remainder of this section, we seek to explore these potentialities in greater detail. For this purpose, suppose that a set of scenarios $Y = \{y_1, \ldots, y_s\}$ have been selected for purposes of evaluating alternatives. Let us then define $z_i(x, y_r)$ (expressed by a lower case letter to emphasize that this is no longer viewed as a random variable) as the consequence of action x in terms of criterion i, under the conditions defined by scenario y_r. As before, $\mathbf{z}(x, y_r)$ will represent the corresponding vector of consequences.

Standard assumptions of MCDA imply that it should be possible for each individual criterion, to obtain at least partial preference orderings on any given set of specific (deterministic) consequences, independently of any other criteria, whether or not these outcomes refer to real or hypothetical alternatives. This observation forms the basis of a scenario-based approach to MCDA under uncertainty.

A direct MAUT approach would presumably still strive to establish a preference ordering of the alternatives in terms of an "expected" utility defined by:

$$\sum_{r=1}^{s} p_r U(\mathbf{z}(x, y_r))$$

where p_r represent the "probability" associated with scenario y_r. There is, however, an immediate theoretical problem concerning the definition and interpretation of p_r. The set of scenarios Y does not constitute a complete probability space. More importantly, each element of this set, y_r, cannot in general be expected to represent the same hypervolume in probability space, so that even a relative probability density (or "likelihood") at the point in probability space represented by y_r cannot be used as a surrogate for p_r. Thus both the practical and theoretical questions regarding the assessment of the p_r remain fundamentally unanswered, and alternative procedures need to be defined.

It will simplify further discussion (and often the implementation) of the models to be discussed if now restrict consideration to the case in which the space of alternatives is also discrete, i.e. the alternatives belong to the set $A = \{a_1, \ldots, a_m\}$. With some abuse of notation we shall then use z_i^{kr} to denote the performance level of alternative a_k in terms of criterion i under the conditions of scenario y_r. The vector $\mathbf{z}^{kr}$ will be interpreted in a similar manner.

In searching for an appropriate and broadly applicable theoretical basis for modelling preferences in this context, two approaches immediately suggest themselves as an extension of the approach discussed by Goodwin and Wright [17], Chapter 14:

Model A: Apply a standard MCDA approach, to construct a preference model (ordinal or cardinal) across all ms possible outcomes (combinations of alternatives and scenarios) given by the performance level vectors $\mathbf{z}^{kr}$. This process involves aggregation across the original n criteria. Goodwin and Wright [17] adopt this model, making use of an n-dimensional additive value function to generate preference values for each $\mathbf{z}^{kr}$. Other MCDA approaches may equally well be employed, however, such as outranking (to generate a classification into preference classes) or goal programming (to measure achievements in terms of distance from a goal or reference level). An $m \times s$ table can then be constructed, giving for each alternative an aggregate measure of performance or goal satisfaction under each scenario. A second evaluation is then required to select the alternative which is "best" in some sense across all scenarios.

Model B: Treat each of the ns criterion-scenario combinations as *metacriteria* (much as in Teghem et al. [50]), and apply some form of MCDA to the problem of comparing m alternatives in terms of the $n \times s$ metacriteria.

Let us now explore the above two possibilities in somewhat greater detail.

5.1 Model A

Here the first step is to evaluate the $m \times s$ distinct "outcomes" in terms of the n criteria by some form of MCDA process, to provide an aggregate comparative evaluation of each outcome. As indicated above, Goodwin and Wright [17] suggested such an approach, and applied a simple value measurement model (SMART) to this step. In other words, the approach adopted was as follows:

1 A value function $v_i(z_i)$ was constructed for each criterion, standardized (e.g. to a 0–100 scale) over an appropriate range of performance levels covering at the least the ms outcomes.

2 Swing weights w_i were assessed by considering the ranges of outcomes used to standardize the scale for each criterion.

3 An overall value for each outcome was computed as:

$$V(\mathbf{z}^{kr}) = \sum_{i=1}^{n} w_i v_i(z_i^{kr}).$$

As an alternative to the value measurement suggested by Goodwin and Wright, the analyst might:

- Use an outranking method to construct a valued pairwise preference relation as done by Mareschal [35], or a (perhaps partial) preference ordering of the full set of ms outcomes; or

- Apply a goal programming method obtain an aggregate distance measure between each of the ms outcomes $\mathbf{z}^{kr}$ and a pre-specified set of goals for each of the n criteria.

Whichever methodology of MCDA is applied, the result will be some numerical scoring, say ζ_{kr} indicating a level of performance or goal satisfaction achieved by each alternative k under the conditions of each scenario y_r. The ζ_{kr} scores can be represented in a two-dimensional matrix, to give a form of "pay-off" table. The places the problem into a framework which can be viewed either as a standard monocriterion decision problem under uncertainty, or as an MCDA problem with aggregate performances under each scenario playing the role of "criteria". The final step is to select the alternative i which is robust against the uncertainties (according to the first view), or which best satisfies these "criteria" (according to the second view).

Goodwin and Wright leave this second phase selection problem to direct holistic judgement, and this does indeed seem to be consistent with the usual scenario planning philosophy. Nevertheless, if a value function approach is adopted and properly implemented in the first step, then the ζ_{kr} values should constitute an interval preference scale. It should then be permissible to construct an additive aggregation of the form $\sum_{r=1}^{s} \omega_r \zeta_{kr}$, where the ω_r represent relative weights on the scenarios. It may be difficult to elicit appropriate values for the scenario weights, however, as these may not be intuitively self-evident. Certainly, as we have indicated earlier, an assumption that ω_r should be equated to a "probability" for scenario y_r cannot really be supported. Some form of "swing-weighting" approach would perhaps be more justifiable.

An alternative approach may be to adopt a "max-min" strategy, i.e. to select alternative k which maximizes the worst aggregate performance given by $\min_{r=1}^{s} \zeta_{kr}$. This could plausibly be construed as the most robust solution, but is unsatisfying from an MCDA perspective, as no consideration is given to possibilities of trade-offs between performances under different scenarios. For example, if one alternative is very good under all but one scenario, but marginally worst on the remaining scenario, should it summarily be rejected? The second level MCDA problem thus poses some challenging questions to the MCDA research community.

5.2 Model B

In this model, the approach is of a standard MCDA form, treating all $n \times s$ combinations of criteria and scenarios as "metacriteria" (where each represents the desire of the decision maker to achieve satisfactory performance according to a particular criterion under a particular scenario).

At the outset, this formulation fits neatly into the MCDA framework, as the operational requirement of being able to compare alternatives in terms of each criterion without reference to performance on other criteria, will typically be satisfied if true for the original criteria. Even the stricter preferential independence condition of additive value function models may be expected to apply if satisfied for the original criteria. This would follow, provided that tradeoffs between outcomes under two scenarios do not depend on how well the alternative performs under other scenarios. On *prima facie* grounds it is difficult to conceive of situations in which such independence would not apply.

The process would then follow standard MCDA procedures, and any of the well-known MCDA methodologies (e.g. value measurement, goal programming or outranking) should in principle all be applicable (not necessarily equally easily or transparently, however). An important point distinguishing model B from model A, is that preference structures across criteria would be allowed to differ across scenarios, in the sense that (a) relative tradeoffs between criteria (importance weights) and (b) intensities of preference for different increments in performance on any one criterion may differ from scenario to scenario. It is an open question as to whether such changes may or should be expected.

Perhaps the most critical question would relate to importance weights placed on each of the metacriteria, as required in some or other sense by most MCDA methods. In principle, we require a relative weight, say w_{ir} to be placed on each metacriterion. There seems to be no difficulty in principle in establishing ratios $w_{ir}/w_{\ell r}$ for any pair of criteria under the assumption of the same scenario r. This would correspond exactly to standard MCDA considerations (e.g. swing weights for value functions). If decision makers can also express the relative importance of changes in performance level for the same criterion under different scenarios, by considering the question as to whether the same range of outcomes on the criterion would have a more or less important impact on the final decision under one scenario than another, this would generate estimates of $w_{ir}/w_{i\rho}$ for the pair of scenarios. From the two sets of ratios, it would be possible to infer relative weights for all combinations. In fact, by repeating the $w_{ir}/w_{i\rho}$ assessments for two or more criteria, some evaluation of the consistency of the estimates would also be possible.

5.3 The Way Forward

A formal integration of MCDA and scenario planning would thus appear to offer substantial potential benefits, and anecdotal evidence suggests that something along this line is done from time to time. At the present time, however, a completely integrated procedure would require answers to the following research questions:

- Do preference structures tend to change from scenario to scenario? If so, then this might better be handled by Model B.
- Are particular MCDA methods more appropriate to one model or the other?
- How many scenarios are needed for effective application of MCDA?
- How should these scenarios be constructed? Should the primary emphasis be on plausibility of the scenarios (as in standard scenario planning) or on achieving representivity of ranges of variation that can occur?
- How should weights be assessed?

At time of writing, a series of simulation studies are under way (based broadly on the approach described by Stewart [47, 49]) to address some of the above questions, especially those related to the number and selection of scenarios. Definitive results are not yet available, but early indications are extremely encouraging, in the sense that good results can be obtained with as few as 3–5 scenarios.

6. Implications for Practice

It should be evident from the preceding discussion that there still remains considerable scope for research into the treatment of substantive external uncertainties within an MCDA framework. It is hoped that such research will lead to ever-improved methodologies. Nevertheless, for the practitioner, certain guidelines can be given at the present time. These may be summarized as follows.

1 For those working within a value or utility function framework, the expectation of a simple additive value function can generate quite useful insights for the decision maker, *provided that* due attention is given to the shape (changing marginal values) of the function (cf. Stewart [46]). On the other hand, complete multiplicative or multilinear multiattribute utility functions may be difficult to implement correctly.

2 With any MCDA approach, there is value and some theoretical justification in decomposing those criteria for which there is substantial uncertainty regarding outcomes, into two subcriteria of expected value and a risk measure respectively. An open question remains as to whether variance or standard deviation (which are conventionally used in this context) are the most appropriate risk measures for all problem types.

3 The integration of MCDA and scenario planning is relatively easy to apply in at least two different ways, and may be particularly transparent to many decision makers. Once again, there do remain some open questions, especially as regards the number of scenarios to be used and the means by which they are constructed or selected.

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