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RESEARCH

MCDM Method for Evaluating and Ranking the Online Shopping Websites Based on a Novel Distance Measure Under Intuitionistic Fuzzy Environment

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Abstract

This paper explores the concept of online shopping malls in terms of information technology as a multi-criteria decision-making (MCDM) challenge under an intuitionistic fuzzy environment. The intuitionistic fuzzy sets (IFSs) have a wider range of applications than fuzzy sets (*FSs*) due to their enhanced capability to handle information uncertainty. Despite the usefulness of IFSs, some of the existing measures for IFSs have been found to have shortcomings in handling the complexity of real-world applications. Some of these measures are inadequate and do not yield desirable results. Therefore, there is a need to develop new distance measures based on the Hausdorff metric that can overcome these limitations and provide more reliable and accurate results and also discuss their several mathematical properties. We compare suggested distance/similarity measures with existing measures and find that they are simpler, more intuitive, and better suited for most applications. Furthermore, we proposed an intuitionistic fuzzy linguistic MCDM method for the evaluation of Internet shopping malls. The enhancement of competitive advantage in Internet shopping malls is primarilapplied IFSs to pattern recognition, whiley attributed to factors such as information and e-service dimension, web creativity as well as online reputation. Amazon. in and Myntra. com is recognized as the top two pioneering internet shopping malls. This research introduces an intuitionistic fuzzy MCDM model aimed at aiding decision-makers and planners in evaluating and selecting digital shopping technology. A case study is also discussed for evaluating and ranking internet shopping websites.

Keywords Intuitionistic fuzzy set · Distance measure · Hausdorff metric · Linguistic Variables · Multi-criteria decision making

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1 Introduction

The twenty-first century is witnessing exponential growth in the digital world, and people from all corners of the globe are actively participating in this revolution [1]. As per the study conducted by Kumar and Dash; Tasnim et al. [2–4], individuals are increasingly accessing online platforms to gather information on various topics. In India, more than 205 million people are using the internet, and the growth rate is estimated to exceed 40% in the current year [5]. Lian and Lin [6] have also pointed out that the availability of internet access to a larger population will inevitably result in an increase in online purchases. Online shopping has experienced rapid growth due to an increased focus on providing customers with time-efficient solutions and a growing number of individuals who possess computer literacy [1, 7]. When faced with the task of selecting the most suitable type of online shopping website from a range of options, the MCDM technique is a valuable tool that enables decision-makers and experts to arrive at an optimal decision. Nonetheless, the issue of fuzzy decision-making has been an area of focus for numerous researchers over the past few decades, due to the intrinsic uncertainty and vagueness present in real-world information.

In traditional probability theory, uncertainty is modeled through a set of possible outcomes with assigned probabilities. However, this approach is not always adequate for systems with ambiguous and imprecise boundaries. Zadeh [8] presented a solution to tackle this problem by introducing the concept of fuzzy set theory, which is an extension of traditional set theory. A fuzzy set is a collection of elements in a domain of discourse that possess ambiguous and imprecise boundaries. Each element in a fuzzy set is allocated a membership value between 0 and 1, signifying its degree of association with the set. The application of fuzzy set theory has gained significance in addressing practical and real-world issues, such as pattern recognition, linguistic variables, and MCDM. Furthermore, as an expansion of *FSs*, *IFSs* have even more advanced capabilities for managing uncertainty in decision-making, mathematical programming tasks, pattern recognition, and various other domains.

Several researchers have made significant contributions to the study of *FSs* and *IFSs* [9–14]. Therefore, it is essential to consider their work when investigating the applications of these theories. Overall, fuzzy set theory and IFSs provide a robust framework for managing uncertain information and have broad practical significance.

The study of fuzzy sets has produced various theories including Atanassov's Intuitionistic Fuzzy Sets [15], Gorzalczany [16], Turkson's Interval-Valued Fuzzy Sets (IVESs) [17], Zadeh's Type-2 Fuzzy Sets [18], Tang's Interval Type-2 Fuzzy programming [19], Yeger's Fuzzy Multi-Set and Yanger's Ordered Weighted Average are two aggregation operators in MCDM, as described in the works of Yeger [20] and Yager [21]. The ability of *IFSs* to handle uncertainty has significantly improved decision-making and pattern identification [22–33]. The domain of extended hesitant fuzzy sets is gaining momentum, with research focused on various types of hesitant fuzzy sets such as Dual Hesitant Fuzzy Sets [34–38], Hesitant Fuzzy Linguistic Term Sets [39], Cubic Hesitant Fuzzy Sets [40], and Higher-Order Hesitant Fuzzy Sets [41].

Distance metrics between fuzzy sets are commonly analyzed in the literature, including studies by Chaudhuri and Rosenfield [42] and Diamond and Kloedan [43]. During the same year, two researchers, Hung and Yang, and Gzegozewski, proposed using the Hausdorff metric to design similarity metrics for both *IVFSs* and *IFSs*. Subsequently, Hung and his research team further explored the use of these measures in pattern recognition, as outlined in their publication [44]. Valchos and Sergiadas [45] applied IFSs to pattern recognition, while Chen [46] proposed a score function for *IFSs* and mobile medical app tang [47] in MADM.

The significance of metrics that measure information in describing fuzzy systems and the structures they possess is widely recognized. These measures, which include distance measures and similarity measures, have been extensively utilized across numerous domains, including artificial intelligence and data mining, for decision-making purposes. The utilization of information metrics has been broadened to encompass the area of *IVFSs* theory. A significant number of scholars have made significant contributions to the topic by investigating various aspects of information measures and their applications in the field. As an example, Chen et al. conducted research on the entropy measure of IFS [48, 49] to establish the objective weight. Meanwhile, Xia and Xu [50] explored the use of entropy and cross-entropy of IFS in group decision making, Wang and Xin [51] refined the formula for determining the distance between *IFSs* and utilized it for pattern recognition. Zhang and Yu (cited in [52]) examined the distance metric between *IFSs* and *IVFSs*, while Papakostas et al. [53] and Xu and Chen [54] evaluated and studied the distance and similarity metrics between *IFSs* and proposed various applications. Grzegorzewski first proposed methods to measure the distance between *IFSs* built on the Hausdorff metrics [55]. Although, Chen identified the limitations in Grzegozewski's 2D Hausdorff-based distances [56] through the presentation of counterintuitive cases. Subsequently, Szmidt and Kacprzyk introduced a novel three-dimensional distance measure based on the Hausdorff metric, and they evaluated its mathematical validity and practical applicability in their work [57]. Researchers such as Xu. Z et al., Du WS. et al., Szmidt E. et al., and Li, D et al. have also contributed to the decision making field [58–61].

The proposed work is motivated by the need for an improved distance measure for handling decision-making problems. While there have been many distance measures introduced in recent years, many conventional measures are unable to achieve accurate and classified results when applied to pattern recognition problems [62]. This work aims to address the limitations of existing measures by proposing an optimal measure that provides more classified and accurate results.

In the midst of a highly competitive market, of Indian e-commerce platforms such as Flipkart. com, Amazon, Shopclues, and Paytm are experiencing remarkable success. These online marketplaces offer a diverse selection of merchandise, ranging from electronics, books, and clothing, to accessories and home goods. In addition, service-oriented websites like Makemy Trip.com, skyscanner.net, delta.com, and irctc.co.in have emerged as leaders in the travel and ticketing sectors. To guarantee that customers are satisfied, it is crucial to take into account various factors such as product quality, user experience, ease of access, assurance, and security, as noted by

Hwang and Kim [63]. Consulting firms have conducted research that suggests India possesses significant potential in the online retail sector, and this trend is predicted to persist in the coming years. With the expansion of the smartphone market and the increasing affordability of broadband services, it is anticipated that a larger number of individuals will participate in the online retail community.

Thanks to the internet and information technology, it is now possible to acquire information about any product with just a single click. With the rapid pace of technological change, we are witnessing the introduction of new products and technologies on a monthly basis [1]. The triumph of an online shopping enterprise hinges on comprehending customers' attributes, such as their purchasing patterns, needs, and influencing factors, as highlighted by Kim and Dharni [7, 64]. Customers have become more fastidious and have a plethora of alternatives to traditional offline shopping. Therefore, it is crucial for any e-vendor to attract, engage, convert, and retain customers, as underscored by Tark [65]. This study endeavours to assess the competitive advantages of Internet shopping centers by identifying various dimensions and ranking them based on the viewpoints of experts.

The following are the major contributions of this article:

- Novel distance measure has been proposed which has the ability to contrast *IFSs*.
- The validity of the suggested measures has been established through the verification of their properties.
- In order to assess the efficiency of the suggested method within an IF-environment, experimental analysis has been conducted, which encompasses numerical experiments, pattern recognition, and linguistic variables.
- Lastly, a detailed examination of the MCDM methodology implementation procedure is conducted by thoroughly addressing the issue of internet shopping malls. Additionally, a comparative analysis is performed to evaluate the efficacy of the suggested approach.

The organization of the article is given below:

Section 2, covers the basics elementary of *FSs* and *IFSs*. Section 3, examines the existing distance metrics and introduces the Hausdorff metric. In Section 4, a new measure is introduced, along with its theorem and properties. Section 5, presents similarity measures based on the proposed measure and it's application and explores queries with linguistic variables. Section 6, provides an examination of MCDM using real-life numerical examples. Section 7, gives comparison analysis. Finally, Section 8, conclusion and future prospects.

2 Preliminaries

All over this paper, let $K = \left\{k_1, k_2, ..., k_q\right\}$ be a non-empty finite set. Let B_1, B_2 and B_3 be the intuitionistic fuzzy subsets (*IFSs*) in K.

Definition 2.1 Atanassov [15] An *IFS*, denoted by B_1, in a set K, as follows.

$$B_1 = \left\{(k_q, l_{B_1}(k_q), m_{B_1}(k_q)) \mid k \in K\right\}, q = 1, 2, ..., n. \tag{2.1}$$

where $l_{B_1}(k_q)$ means the degree of membership and $m_{B_1}(k_q))$ means degree of non-membership of the element $k_q \in K$ in the set B_1 such that $0 \leq l_{B_1}(k_q) + m_{B_1}(k_q)) \leq 1$. Here, $r_{B_1}(k_q)) = 1 - l_{B_1}(k_q) - l_{B_1}(k_q)$ denotes the hesitancy degree.

Definition 2.2 Let $B_1 = \left\{(k_q, l_{B_1}(k_q), m_{B_1}(k_q)) \mid k \in K\right\}$ be an *IFSs* in K. $\forall$ positive real number r, the *IFS* B_1^n is explained as:

$$B_1^r = \left\{\left(k_q, (l_{B_1}(k_q)^r, (1-(1-m_{B_1})^r)\right) : k \in K\right\}, r > 0. \tag{2.2}$$

under $0 \leq (l_{B_1}(k_q)^r + (1-(1-m_{B_1})^r) \leq 1$. The concentration and dilation of an *IFS* B_1 can be understood from Eq. (2.2), as follows:

$$CON(B_1) = \left\{(k_q, l_{CON(B_1)}(k_q), m_{CON(B_1)}(k_q)) : k \in K\right\} \tag{2.3}$$

where $l_{CON(B_1)}(k_q) = (l_{B_1}(k_q)^2, m_{CON(B_1)}(k_q) = (1-(1-m_{B_1})^2)$; and

$$DIL(B_1) = \left\{(k_q, l_{DIL(B_1)}(k_q), m_{DIL(B_1)}(k_q)) : k \in K\right\} \tag{2.4}$$

where $l_{DIL(B_1)}(k_q) = (l_{B_1}(k_q)^{\frac{1}{2}}, m_{DIL(B_1)}(k_q) = (1-(1-m_{B_1})^{\frac{1}{2}})$.

Definition 2.3 Atanassov [15] Some operations are given as:

1. $E_1 \subseteq E_2$ iff $\forall\, v_m \in V, r_{E_1}(v_m) \leq r_{E_2}(v_m)$ and $l_{E_1}(v_m) \geq l_{E_2}(v_m)$;
2. $E_1 = E_2$ iff $\forall\, v_m \in V, E_1 \subseteq E_2$ and $E_2 \subseteq E_1$;
3. If ${E_1}^c = \left\{(v_m, l_{E_1}(v_m), r_{E_1}(v_m)) \mid v \in V\right\}$;

Definition 2.4 Wang et al. [66] For any $(B_1, B_2) \in IFSs(K)$ a mapping $D_H^{**}(B_1, B_2) : FSs(K) \times IFSs(K) \rightarrow [0, 1]$ is called the distance measure of *IFSs*, if it holds properties:

(P1) $0 \leq D_H^{**}(B_1, B_2) \leq 1$.
(P2) $D_H^{**}(B_1, B_1) = 0$.
(P3) $D_H^{**}(B_1, B_2) = D^{**}(B_2, B_1)$.
(P4) If $B_1 \subseteq B_2 \subseteq B_3$ then $D^{**}(B_1, B_3) \geq Max\left\{D_H^{**}(B_1, B_2), D^{**}(B_2, B_3)\right\}$.

Definition 2.5 Mitchell [27] and Dengfeng and Chuntian [25] For any $(B_1, B_2) \in IFSs(K)$ a mapping $S^{**}(B_1, B_2) : IFSs(K) \times IFSs(K) \rightarrow [0, 1]$ is called the similarity measure of *IFSs*, if it holds properties:

1. $0 \leq S^{**}(B_1, B_2) \leq 1$.

2. $S^{**}(B_1, B_2) = S^{**}(B_2, B_1)$.
3. $S^{**}(B_1, B_1) = 1$.
4. If $B_1 \subseteq B_2 \subseteq B_3$, then $S^{**}(B_1, B_3) \leq S^{**}(B_1, B_2)$ and $S^{**}(B_1, B_3) \leq S^{**}(B_2, B_3)$.

3 The Existing Distance Measures

Atanassov [15] The distances, represented by Eqs. (3.1) to (3.4), are applied to any two *IFSs*, B_1 and B_2. These distances are specified for the *IFSs* B_1 and B_2.

Hamming Distance

$$e^*_H(B_1, B_2) = \frac{1}{2}\sum_{i=1}^{q}\left(|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)|\right) \tag{3.1}$$

Normalized Distance

$$e^*_{NH}(B_1, B_2) = \frac{1}{2q}\sum_{i=1}^{q}\left(|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)|\right) \tag{3.2}$$

Euclidean Distance

$$e^*_{ED}(B_1, B_2) = \left(\sum_{i=1}^{q}\frac{1}{2}[(l_{B_1}(k_i) - l_{B_2}(k_i))^2 + (m_{B_1}(k_i) - m_{B_2}(k_i))^2]\right)^{\frac{1}{2}} \tag{3.3}$$

Normalized Euclidean Distance

$$e^*_{NED}(B_1, B_2) = \left(\sum_{i=1}^{q}\frac{1}{2q}[(l_{B_1}(k_i) - l_{B_2}(k_i))^2 + (m_{B_1}(k_i) - m_{B_2}(k_i))^2]\right)^{\frac{1}{2}} \tag{3.4}$$

Szmidt and Kacprzyk modified the above distances by adding intuitionistic fuzzy index $r(k_i)$, and the modified distances are given by Eqs. (3.5) to (3.8).

Szmidt and Kaprzyk [67]:

Hamming Distance

$$f^*_H(B_1, B_2) = \frac{1}{2}\sum_{i=1}^{q}\left(|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)| + |r_{B_1}(k_i) - r_{E_2}(k_i)|\right) \tag{3.5}$$

Normalized Hamming Distance

$$f^*_{NH}(E_1, E_2) = \frac{1}{2q}\sum_{i=1}^{q}\left(|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)| + |r_{B_1}(k_i) - r_{B_2}(k_i)|\right) \tag{3.6}$$

Euclidean Distance

$$f_{ED}^{*}(B_1,B_2) = \left(\frac{1}{2}\sum_{i=1}^{q}\left((l_{B_1}(k_i) - l_{B_2}(k_i))^2 + (m_{B_1}(k_i) - m_{B_2}(k_i))^2 + (r_{B_1}(k_i) - r_{E_2}(k_i))^2\right)\right)^{\frac{1}{2}} \tag{3.7}$$

Normalized Euclidean Distance

$$f_{NED}^{*}(B_1,B_2) = \left(\frac{1}{2q}\sum_{i=1}^{q}\left((l_{B_1}(k_i) - l_{B_2}(k_i))^2 + (m_{B_1}(k_i) - m_{B_1}(k_i))^2 + (r_{B_1}(k_i) - r_{B_2}(k_i))^2\right)\right)^{\frac{1}{2}} \tag{3.8}$$

However, there are some limitations to the distances proposed by Szmidt and Kacprzyk [67], and new distances have been introduced by Wang and Xin in their work [66].

$$f_{FW1}^{*}(B_1,B_2) = \frac{1}{q}\sum_{i=1}^{q}\left[\frac{\left(|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)|\right)}{4} + \frac{max\left(|l_{B_1}(k_i) - l_{B_2}(v_i)|, |m_{B_1}(v_i) - m_{B_2}(k_i)|\right)}{2}\right] \tag{3.9}$$

$$f_{FW2}^{*}(B_1,B_2) = \frac{1}{q}\sum_{i=1}^{q}\left(\frac{|l_{B_1}(k_i) - l_{B_2}(k_i)| + |m_{B_1}(k_i) - m_{B_2}(k_i)|}{2}\right) \tag{3.10}$$

Grzegorzewski's [55] distance measure

$$f_{GD}^{*}(B_1,B_2) = \frac{1}{q}\left(\sum_{i=1}^{q} max\left(|l_{B_1}(k_i) - l_{B_2}(k_i)|, |m_{B_1}(k_i) - m_{B_2}(k_i)|\right)\right) \tag{3.11}$$

The Yang and Francisco [68] distance measure

$$f_{YF}^{*}(B_1,B_2) = \frac{1}{q}\left(\sum_{i=1}^{q} max\left(|l_{B_1}(k_i) - l_{B_2}(k_i)|, |m_{B_1}(k_i) - m_{B_2}(k_i)|, |r_{B_1}(k_i) - r_{B_2}(k_i)|\right)\right) \tag{3.12}$$

3.1 Hausdorff Metric

The Hausdorff metric is a commonly used approach to compute the distance between two compact subsets B_1 and B_2 in a Banach space P. The Hausdorff metric is defined as the maximum distance between the forward and backward distances, as explained in several references such as [69–71]. The distance between a point s in B_1 and a point t in B_2 is represented by the formula $d(s, t)$. The definition of the forward distance is $h^{**}(B_1,B_2) = \max_{s\in B_1}\left\{\min_{t\in B_2}(||s-t||)\right\}$, and the definition of backward distance is $h^{**}(B_2, B_1) = \max_{t \in B_2} \{\min_{s \in B_1}(||s - t||)\}$, respectively. The following is the definition of the Hausdorff metric:

$$H^{**}(B_1, B_2) = \max\left\{h^{**}(B_1, B_2), h^{**}(B_2, B_1)\right\} \tag{3.13}$$

The asymmetry of the Hausdorff metric should be mentioned. In general, $h^{**}(B_1, B_2) \neq h^{**}(B_2, B_1)$. e.g., If $I = J$, and the two intervals $B_1 = [\alpha_1, \alpha_2]$ and $B_2 = [\beta_1, \beta_2]$, then from Eq. (3.13), we find that:

$$H^{**}(B_1, B_2) = \max\left\{|\alpha_1 - \beta_1|, |\alpha_2 - \beta_2|\right\}. \tag{3.14}$$

A well-known formula for calculating the distance between two intervals is (3.14). To determine a similarity measure, a distance metric can be employed. Distance measures generally play a significant part in proving the resemblance between two sets. Due to their numerous uses in various fields, they have significantly increased in popularity. Even though there are distance Hausdorff-based *IFS* and Pythagorean fuzzy sets distance measures in the literature [72, 73].

Grzegorzewski [55] presented a novel generalization based on the Hausdorff metric. In this study, we adopt the notion of distance based on the Hausdorff metric.

Hamming Distance

$$i^*_H(B_1, B_2) = \sum_{i=1}^{q} \max\left\{ |l^2_{B_1}(k_i) - l^2_{B_2}(k_i)|, |m^2_{B_1}(k_i) - m^2_{B_2}(k_i)| \right\} \tag{3.15}$$

Normalized Hamming Distance

$$i^*_{NH}(B_1, B_2) = \frac{1}{q}\sum_{i=1}^{q} \max\left\{ |l^2_{B_1}(k_i) - l^2_{B_2}(k_i)|, |m^2_{B_1}(k_i) - m^2_{B_2}(k_i)| \right\} \tag{3.16}$$

Euclidean Distance

$$i^*_{ED}(B_1, B_2) = \left(\sum_{i=1}^{q} \max[(l^2_{B_1}(k_i) - l^2_{B_2}(k_i))^2, (m^2_{B_1}(k_i) - m^2_{B_2}(k_i))^2]\right)^{\frac{1}{2}} \tag{3.17}$$

Normalized Euclidean Distance

$$i^*_{NED}(B_1, B_2) = \left(\frac{1}{q}\sum_{i=1}^{q} \max[(l^2_{B_1}(k_i) - l^2_{B_2}(k_i))^2, (m^2_{B_1}(k_i) - m^2_{B_2}(k_i))^2]\right)^{\frac{1}{2}} \tag{3.18}$$

4 A New Distance Metric for IF-Environment

From the above discussion, numerous distance measures for *IFSs* were proposed by various researchers. Some of them lack the ability to make good decisions. Additionally, we can categorize these shortcomings into two groups: Some of them are yielding identical results, and others are using measurements that produce contradictory and illogical outcomes. It is important to express a new distance metric in a more advantageous way in order to address the limitations of existing measures. Hence, we tried to formulate a novel and reliable distance measure that can better handle different practical applications with greater conviction. We define a Tangent measure for calculating the distance between *IFSs* using the Hausdorff metric. It is suitable to use this measurement technique on intervals, which can be easily used for *IFSs*. Let B_1, B_2 be two *IFSs* and we suppose that two subintervals $I_{B_1}(k_q)$, $I_{B_2}(k_q)$ on [0, 1], denoted by $I_{B_1}(k_q) = \left[l_{B_1}(k_q), 1 - m_{B_1}(k_q)\right]$ and $I_{B_2}(k_q) = \left[l_{B_2}(k_q), 1 - m_{B_2}(k_q)\right]$, where $q = 1, 2, ..., n$. Now, we determine the distance between B_1 and B_2 by computing the distance between the Intuitionistic Fuzzy Sets (*IFSs*). $I_{B_1}(k_q)$ and $I_{B_2}(k_q)$ established on Hausdorff metric with $H^{**}(I_{B_1}(k_q), I_{B_2}(k_q)) = \max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right\}$. Thus, we introduce new Hausdorff metric $D_H^{**}(B_1, B_2)$ between the *IFSs* B_1 and B_2 as follows:

$$D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right\}\right) \tag{4.1}$$

Theorem 4.1 *Let* $K = \left\{k_1, k_2, ..., k_q\right\}$ *be a universal set. This* D_H^{**} *distance measure holds the following properties*:

(A1) (Boundedness) $0 \leq D_H^{**}(B_1, B_2) \leq 1$;
(A2) (Separability) $D_H^{**}(B_1, B_2) = 0$ if and only if $B_1 = B_2$;
(A3) (Symmetric) $D_H^{**}(B_1, B_2) = D_H^{**}(B_2, B_1)$;
(A4) (Containment) $B_1 \subseteq B_2 \subseteq B_3$ then $D_H^{**}(B_1, B_2) \leqslant D_H^{**}(B_1, B_3)$ and $D_H^{**}(B_2, B_3) \leqslant D_H^{**}(B_1, B_3)$;
(A5) (Triangle Inequality) For any B_1, B_2 and B_3, then $D_H^{**}(B_1, B_3) \leqslant D_H^{**}(B_1, B_2) + D_H^{**}(B_2, B_3)$.

Proof Proof of Theorem 4.1 is in Appendix I.

Theorem 4.2 *Some more properties based on proposed distance measure* D_H^{**} *also satisfy properties*:

a) $D_H^{**}(B_1^c, B_2^c) = D_H^{**}(B_1, B_2) \; \forall \; B_1, B_2 \in IFS(K)$.
b) $D_H^{**}(B_1, B_2^c) = D_H^{**}(B_1^c, B_2) \; \forall \; B_1, B_2 \in IFS(K)$.
c) $D_H^{**}(B_1, B_1^c) = 0$ *if and only if* $l_{B_1}(k_q) = m_{B_1}(k_q) \; \forall \; 1 \leq q \leq n$
d) $D_H^{**}(B_1, B_1^c) = 1$, *If* B_1 *is a crisp set.*

Proof Proof of Theorem 4.2 is in Appendix II.

In general, for the each $k_q \in K$, we assign a weight w_q $\{q = 1, 2, ..., n\}$, where $0 \leqslant w_q \leqslant 1$ such that $\sum_{q=1}^{n} w_q = 1$. Consider, how we establish a weighted

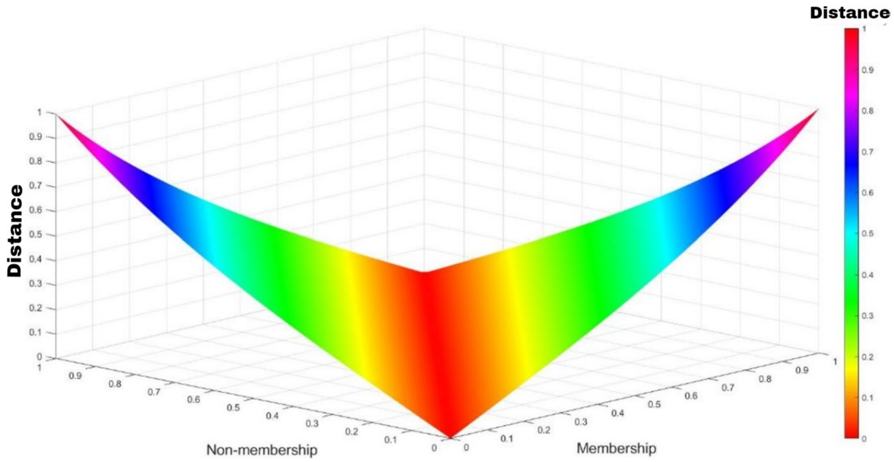


Fig. 1 Graph of proposed distance measure

Hausdorff metric for IFSs: Distance Measures $D_{wH} : IFSs(K) \times IFSs(K) \rightarrow [0,1]$ is given as

$$D_{wH}(B_1, B_2) = \sum_{q=1}^{n} w_q \tan\left(\frac{\pi}{4} \max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right\}\right) \tag{4.2}$$

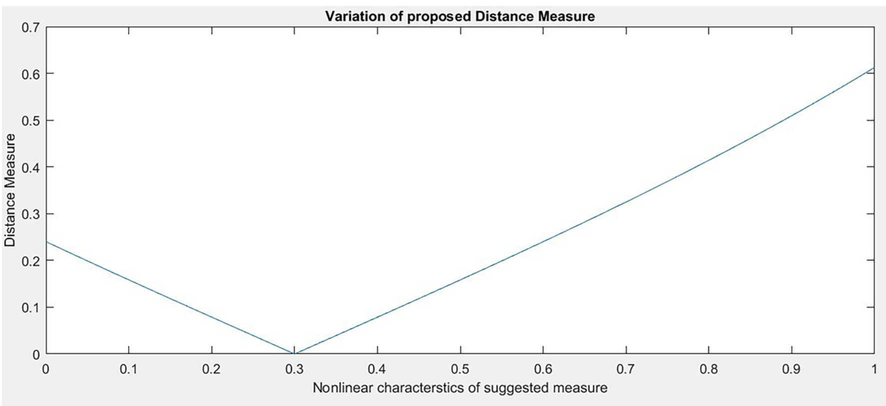


Fig. 2 Graph of non-linear proposed distance measure

Note By setting $w_q = 1/n$ for $q = 1, 2, \ldots, n$, Eq. (4.2) transforms into Eq. (4.1). Thus, Eq. (4.1) is a specific instance of Eq. (4.2).

Example 4.1 Assume that B_1 and B_2 are two IFSs on K, where $B_1 = (l, m)$ and $B_2 = (m, l)$, with l and m being the membership and non-membership degrees satisfying the condition $0 \leq l + m \leq 1$. Figure 1, illustrates the distance measure D_H^{**} concerning the variation in l and m. The Fig. 1, demonstrates that the value of $D_H^{**}(B_1, B_2)$ is bounded as l and m fluctuate within the unit interval i.e. $0 \leq D_H^{**}(B_1, B_2) \leq 1$ and when $B_1 = B_2$ then $D_H^{**}(B_1, B_2) = 0$. Also Fig. 1 clearly indicates the symmetry i.e. $D_H^{**}(B_1, B_2) = D_H^{**}(B_2, B_1)$. When $B_1 = (0, 1)$, $B_2 = (1, 0)$ and vice-versa $D_H^{**}(B_1, B_2) = 1$.

Example 4.2 Let B_1 and B_2 be two *IFSs* on K then for the different choice of B_1 and B_2 as mention below:

$B_1 = (l, 0.3)$ and $B_2 = (0.3, l)$, the boundedness and non-linearity of the proposed outcome are demonstrated in Fig. 2, which portrays the fluctuation of $D_H^{**}(B_1, B_2)$ as l varies between 0 and 1.

5 Similarity Measures for IFS

The concept of the duality between distance and similarity is well-known. To establish the similarity between two *IFSs*, the Hausdorff metric can be utilized to determine the distance between two IFSs. Let u be a decreasing function. Since $0 \leq D_H^{**}(B_1, B_2) \leq 1$, $u(1) \leq u(D_H^{**}(B_1, B_2)) \leq u(0)$. This leads to the conclusion that $0 \leq (u(D_H^{**}(B_1, B_2) - u(1))/(u(0) - u(1)) \leq 1$. Hence, the similarity metrics between *IFSs* B_1 and B_2 can be expressed as below:

Definition 5.1 Let $K = \{k_1, k_2, ..., k_q\}$ be the universal set and $B_1 = \{(k_q, l_{B_1}(k_q), m_{B_1}(k_q)) \mid k \in K\}$, $B_2 = \{(k_q, l_{B_2}(k_q), m_{B_2}(k_q)) \mid k \in K\}$ be two *IFSs* on K. A new similarity metric $S^*(B_1, B_2)$ between *IFSs* B_1 and B_2 can be defined by considering the decreasing function u as follows:

$$S^*(B_1, B_2) = \frac{u(D_H^{**}(B_1, B_2)) - u(1)}{u(0) - u(1)} \tag{5.1}$$

The choice of a suitable function u in Eq. (5.1) allows for the calculation of various similarity measures. One straightforward option is to use the linear function $u(y) = 1 - y$, which will result in a similarity measure between IFSs.

$$S_1^*(B_1, B_2) = 1 - D_H^{**} \tag{5.2}$$

An alternative option is to use the simple rational function $u(y) = 1/(1 + y)$ to determine the similarity between *IFSs* B_1 and B_2. This function is then appealed to measure the similarity between the two *IFSs*.

$$S_2^*(B_1, B_2) = \frac{1 - D_H^{**}}{1 + D_H^{**}} \tag{5.3}$$

$$S_3^*(B_1, B_2) = \frac{e^{-D_H^{**}} - e^{-1}}{1 - e^{-1}} \tag{5.4}$$

5.1 Application of Proposed Distance/Similarity Measures in Pattern Recognition

This section shows how the suggested methods are in pattern recognition.

Algorithm Consider a finite universe $K = \{k_1, k_2, \dots, k_q\}$, and let $T = \{W_1, W_2, \dots, W_n\}$ denote n patterns represented by intuitionistic fuzzy sets (*IFSs*), and i test samples $W = \{w_1, w_2, \dots, w_i\}$, also represented as *IFSs*. The aim is to identify the test samples based on the given patterns. The recognition procedure is outlined below:

Step 1. Find out the distance between the given pattern W_j and test sample w_i using distance/similarity measures.
Step 2. Choose minimum distance for distance measures and maximum distance for similarity measures, then these measures are classified.
Step 3. Unclassified measures if any two distances are equal.

Example 5.1 Let us consider W_1, W_2, W_3 be the three *IFSs* as known patterns:

$W_1 = \{(k_1, 0.5, 0.5), (k_2, 0.6, 0.3), (v_3, 0.9, 0.1)\}$,
$W_2 = \{(k_1, 0.3, 0.7), (k_2, 0.5, 0.4), (k_3, 0.8, 0.0)\}$,
$W_3 = \{(k_1, 0.6, 0.4), (k_2, 0.6, 0.3), (k_3, 0.9, 0.0)\}$.

Consider W, another intuitionistic fuzzy set as unknown pattern given below:

$W = \{(k_1, 0.0, 0.1), (k_2, 0.0, 0.0), (k_3, 0.0, 0.0)\}$

Classification of the unknown pattern W into one of the recognized patterns W_1, W_2, or W_3 is attained through the utilize of multiple intuitionistic fuzzy measures, as demonstrated in Table 1.

The distance measures in Table 1 show that distance measures e^*_{ED}, f^*_H, f^*_{ED}, i^*_H violated the property A_1, because its values are greater than 1. Distance measures e^*_{NED}, f^*_{FW1}, i^*_{ED}, i^*_{NED} fail to recognize W as they show that W is closest to W_1 as well as W_3 i.e. W_1=W_3, there is no information. e^*_H, e^*_{NH}, f^*_{NH}, f^*_{NED}, f^*_{GD}, f^*_{FW2}, f^*_{YF}, i^*_{NH} involving the suggested measures S_1^*, S_2^*, S_3^* and D_H^{**} measure is used to allocate the unknown pattern W to the known pattern W_2. Hence, the suggested measure aligns with existing measures.

Table 1 Comparative analysis

Measures	(W_1, W)	(W_2, W)	(W_3, W)	*Comments*	(*Outcome*)
e^*_H [15]	0.84	0.74	0.78	$W_2 \prec W_3 \prec W_1$	W_2
e^*_{NH} [15]	0.20	0.16	0.18	$W_2 \prec W_3 \prec W_1$	W_2
e^*_{ED} [15]	1.11	1.07	1.11	$W_2 \prec W_1 = W_3$	Violated A(1)
e^*_{NED} [15]	0.85	0.83	0.85	$W_2 \prec W_1 = W_3$	Can't be classified
f^*_H [67]	2.80	2.60	2.70	$W_2 \prec W_1 \prec W_3$	Violated A(1)
f^*_{NH} [67]	0.93	0.87	0.90	$W_2 \prec W_3 \prec W_1$	W_2
f^*_{ED} [67]	1.19	1.07	1.15	$W_2 \prec W_3 \prec W_1$	Violated A(1)
f^*_{NED} [67]	0.63	0.55	0.60	$W_2 \prec W_3 \prec W_1$	W_2
f^*_{GD} [55]	0.67	0.63	0.70	$W_2 \prec W_1 \prec W_3$	W_2
f^*_{FW1} [66]	0.57	0.53	0.57	$W_2 \prec W_1 = W_3$	Can't be classified
f^*_{FW2} [66]	0.23	0.20	0.22	$W_2 \prec W_3 \prec W_1$	W_2
f^*_{YF} [68]	0.93	0.87	0.90	$W_2 \prec W_3 \prec W_1$	W_2
i^*_H [55]	1.42	1.37	1.53	$W_2 \prec W_1 \prec W_3$	Violated A(1)
i^*_{NH} [55]	0.47	0.46	0.51	$W_2 \prec W_1 \prec W_3$	W_2
i^*_{ED} [55]	0.38	0.36	0.38	$W_2 \prec W_1 = W_3$	Can't be classified
i^*_{NED} [55]	0.19	0.17	0.19	$W_2 \prec W_1 = W_3$	Can't be classified
S^*_1	0.64	0.69	0.63	$W_3 \prec W_1 \prec W_2$	W_2
S^*_2	0.41	0.43	0.40	$W_3 \prec W_1 \prec W_2$	W_2
S^*_3	0.45	0.47	0.44	$W_3 \prec W_1 \prec W_2$	W_2
D^{**}_H	0.59	0.55	0.62	$W_2 \prec W_1 \prec W_3$	W_2

Example 5.2 Let us consider W_1, W_2, W_3 be the three *IFSs* as known patterns:

$$W_1 = \{(k_1, 0.5, 0.3), (k_2, 0.2, 0.3), (v_3, 0.4, 0.1)\},$$
$$W_2 = \{(k_1, 0.2, 0.3), (k_2, 0.5, 0.4), (k_3, 0.1, 0.4)\},$$
$$W_3 = \{(k_1, 0.5, 0.3), (k_2, 0.3, 0.2), (k_3, 0.4, 0.1)\}.$$

Consider W, another intuitionistic fuzzy set as unknown pattern given below:

$$W = \{(k_1, 0.0, 0.0), (k_2, 0.1, 0.2), (k_3, 0.4, 0.5)\}$$

The process of identifying the unknown pattern W and assigning it to one of the recognized patterns W_1, W_2, or W_3 is performed using several intuitionistic fuzzy measures. Table 2 displays the results.

The distance measures in Table 2, show that distance measure f^*_H violated the property A_1, because its values are greater than 1. Distance measures e^*_H, e^*_{NH}, e^*_{ED}, e^*_{NED}, f^*_{NH} f^*_{FW2}, f^*_{YF} i^*_{NH}, i^*_{NED} unable to categorised the unknown pattern W as they show that W is closest to W_2 as well as W_3 i.e. W_2=W_3, there is no information. f^*_{ED}, f^*_{NED}, f^*_{GD}, f^*_{FW1}, i^*_H, i^*_{ED} involving the proposed measures S^*_1, S^*_2, S^*_3 and the D^{**}_H measure is used to allocate the unknown pattern W to the known pattern W_1. Thus, the suggested measure aligns with existing measures.

Table 2 Comparative analysis

Measures	(W_1, W)	(W_2, W)	(W_3, W)	*Comments*	(*Outcome*)
e_H^* [15]	0.25	0.35	0.35	$W_1 \prec W_2 = W_3$	Can't be classified
e_{NH}^* [15]	0.04	0.07	0.07	$W_1 \prec W_2 = W_3$	Can't be classified
e_{ED}^* [15]	0.82	0.85	0.85	$W_1 \prec W_2 = W_3$	Can't be classified
e_{NED}^* [15]	0.71	0.72	0.72	$W_1 \prec W_2 = W_3$	Can't be classified
f_H^* [67]	1.30	1.40	1.40	$W_1 \prec W_2 = W_3$	Violated A(1)
f_{NH}^* [67]	0.43	0.47	0.47	$W_1 \prec W_2 = W_3$	Can't be classified
f_{ED}^* [67]	0.64	0.66	0.69	$W_1 \prec W_2 \prec W_3$	W_1
f_{NED}^* [67]	0.28	0.30	0.32	$W_1 \prec W_2 \prec W_3$	W_1
f_{GD}^* [55]	0.27	0.33	0.37	$W_1 \prec W_2 \prec W_3$	W_1
f_{FW1}^* [66]	0.23	0.28	0.30	$W_1 \prec W_2 \prec W_3$	W_1
f_{FW2}^* [66]	0.06	0.09	0.09	$W_1 \prec W_2 = W_3$	Can't be classified
f_{YF} [68]	0.47	0.47	0.47	$W_1 = W_2 = W_3$	Can't be classified
i_H^* [55]	0.44	0.56	0.57	$W_1 \prec W_2 \prec W_3$	W_1
i_{NH}^* [55]	0.15	0.19	0.19	$W_1 \prec W_2 = W_3$	Can't be classified
i_{ED}^* [55]	0.24	0.26	0.25	$W_1 \prec W_3 \prec W_3$	W_1
i_{NED}^* [55]	0.09	0.10	0.10	$W_1 \prec W_2 = W_3$	Can't be classified
S_1^*	0.92	0.89	0.86	$W_3 \prec W_2 \prec W_1$	W_1
S_2^*	0.61	0.57	0.56	$W_3 \prec W_2 \prec W_1$	W_1
S_3^*	0.65	0.61	0.59	$W_3 \prec W_2 \prec W_1$	W_1
D_H^{**}	0.22	0.28	0.30	$W_1 \prec W_2 \prec W_3$	W_1

In light of this, Examples 5.2 and 5.3 demonstrate how the suggested measure outperforms the majority of other existing ones.

5.2 Comparative Study for Linguistic Variables

An illustration is provided to demonstrate the characterization of similarities between linguistic variables through the use of the similarity measures presented in Eqs. (5.2) to (5.4). The concept of a structure for fuzzy query processing using fuzzy sets was first introduced by Tahani [74]. Kacprzyk and Ziolkowski [75] built upon the initial framework by Tahani [74] and further developed the idea of incorporating fuzzy linguistic quantifiers into database queries. Petry [76] later provided an in-depth study on fuzzy databases, including its principles and applications. To query a database effectively, it's important to employ similarity measures (as noted by Candan et al. [77]). To improve the usability of fuzzy queries, it is crucial to clarify the degree of similarity between the fuzzy sets. Hussain and Yang [72] also explored the similarity measures between linguistic variables. In illustration 5.4, the suggested similarity measures between *IFSs* are employed to characterize the similarities between linguistic variables.

Example 5.3 Let $B_1 = \left\{(k_q, l_{B_1}(k_q), m_{B_1}(k_q)) \mid k \in K\right\}, q = 1, 2, ..., n.$ be a *IFS* on K. $\forall$ positive real number n, we have *IFS* B_1^n from the Definition 2.3, with $B_1^r = \left\{ \left(k_q, (l_{B_1}(k_q)^r, (1-(1-m_{B_1})^r\right)^{\frac{1}{2}}) : k \in K\right\}, r > 0.$

The *IFS* B_1 has two linguistic operators, dilation and concentration. The dilation of B_1 is represented as $DIL(B_1) = B_1^{\frac{1}{2}}$ and the concentration of B_1 is $CON(B_1) = B_1^2$. These operations can be thought of as "very (B_1)" and "more or less (B_1)," respectively. In this context, the set $K = \left\{k_1, k_2, k_3, k_4, k_5\right\}$ contains the *IFS*.

$$B_1 = (k_1, 0.5, 0.4), (k_2, 0.6, 0.2), (k_3, 0.4, 0.5), (k_4, 0.7, 0.1), (k_5, 0.0, 1).$$
$$B_1^{\frac{1}{2}} = (k_1, 0.71, 0.23), (k_2, 0.77, 0.11), (k_3, 0.63, 0.29), (k_4, 0.84, 0.05), (k_5, 0.0, 1).$$
$$B_1^2 = (k_1, 0.25, 0.64), (k_2, 0.36, 0.36), (k_3, 0.16, 0.75), (k_4, 0.49, 0.19), (k_5, 0.0, 1).$$
$$B_1^4 = (k_1, 0.06, 0.87), (k_2, 0.13, 0.94), (k_3, 0.02, 0.94), (k_4, 0.24, 0.34), (k_5, 0.0, 1).$$

In the context of the set K, the *IFS* B_1 represents "LARGE." It has been established that operations like $CON(B_1)$ and $DIL(B_1)$ can be used as linguistic hedge expressions such as "Somewhat LARGE," "Extremely LARGE," and "Extremely Extremely LARGE."

Thus, we have

$B_1^{\frac{1}{2}}$ is considered as "Somewhat LARGE,"
B_1^2 is considered as "Extremely LARGE,"
B_1^4 is considered as "Extremely Extremely LARGE."

The terms "LARGE," "Somewhat LARGE," "Extremely LARGE," and "Extremely Extremely LARGE" are abbreviated as L, S.L, E.L., and E.E.L. respectively. To compare the similarity between *IFSs*, the similarity metrics in Eqs. (5.2) to (5.4) are used. The outcomes of this comparison are shown in Table 3, and from these outcomes, conclusions can be drawn regarding the similarities between *IFSs*.

$$B_1(L, E.E.L.) > B_1(L, S.L.) > B_1(L, E.L.), B_1(S.L, E.E.L.) > B_1(S.L., E.L.) > B_1(S.L., L),$$
$$B_1(E.L., S.L.) > B_1(E.L., L) > B_1(E.L., E.E.L.),$$
$$B_1(E.E.L., S.L.) > B_1(E.E.L., L) > B_1(E.E.L., E.L.).$$

The suggested similarity metrics in Eqs. (5.2) to (5.4) have been shown to meet all necessary criteria and provide accurate comparisons between L, S.L., E.L., and E.E.L. in a compound linguistic variable setting, making them useful and applicable.

Figure 3 shows the graph of linguistic variables, which is obtained by calculated values of similarity measures.

Table 3 Computation of similarity metrics

	L	*S.L.*	*E.L.*	*E.E.L.*
	1.0000	0.9938	0.9914	0.9943
L	1.0000	0.7493	0.7542	0.7694
	1.0000	0.7794	0.7834	0.7960
	0.9938	1.0000	0.9950	0.9951
S.L	0.7493	1.0000	0.7602	0.7766
	0.7794	1.0000	0.7885	0.8021
	0.9914	0.9950	1.0000	0.9905
E.L.	0.7542	0.7602	1.0000	0.7583
	0.7834	0.7885	1.0000	0.7886
	0.9943	0.9951	0.9905	1.0000
E.E.L.	0.7694	0.7766	0.7583	1.0000
	0.7960	0.8021	0.7866	1.0000

6 MCDM Method Based on IF-Distance Measures

The recent surge in the popularity of online shopping can be attributed to the widespread usage and development of the internet, which has made it easily accessible to the general public. As a result, traditional shopping is being gradually replaced by the convenience and ease of online shopping. Online shopping primarily takes place on the internet and social networks. Among scholars, MCDM is one of the most extensively researched topics in the literature. MCDM is the process of identifying the optimal solution from a set of possible alternatives. The suggested approach in this study consists of four main steps.

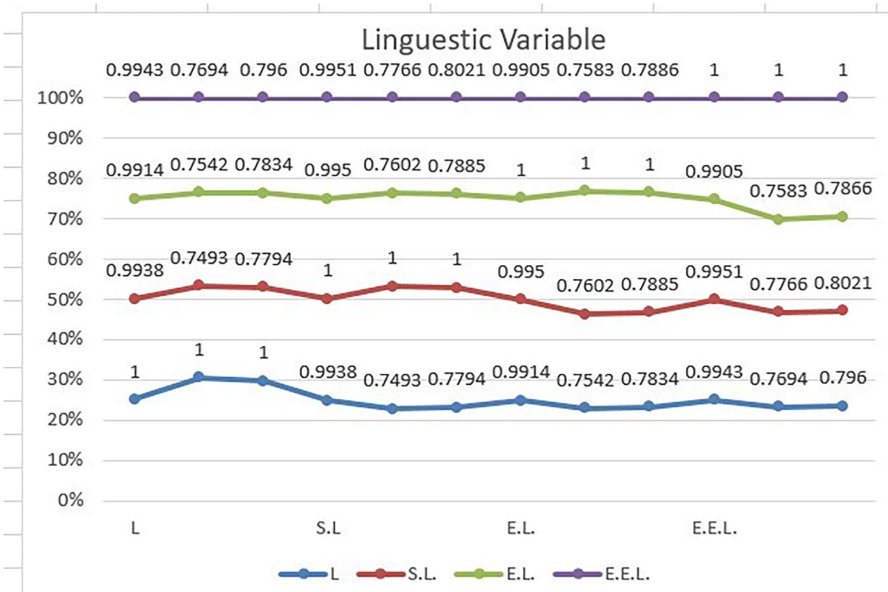


Fig. 3 Graph of linguistic variables

6.1 Algorithm for Multi-Criteria Decision Making

let $K = \{k_1, k_2, ..., k_q\}$ be a set of attributes, there are n alternatives.

$R_i = R_{iq} = \{(k_q, l_{R_i}(k_q), m_{R_i}(k_q)) \mid k_q \in K\}, i = 1, 2, ..., n; q = 1, 2, ..., t$. To determine the best alternative, a decision-making process is followed which involves several steps. These steps include:

Step 1. Normalize decision alternatives. Criteria in MCDM situations can often be split into two categories: cost type and benefit type. To turn the cost attribute into a benefit attribute during the decision-making process, the following formula should be employed. If all criteria are already benefits, there is no requirement for any conversion.

$$\overline{R_{im}} = \begin{cases} R_{iq}, & \text{for benefit attribute } k_q, \\ R^c_{iq}, & \text{for cost attribute } k_q, \end{cases} \tag{6.1}$$

$R^c_{iq} = \{(k_q, m_{R_i}(k_q), l_{R_i}(k_q)) \mid k_q \in K\}, q = 1, 2, ..., t.$

The alternative R_i can be rewritten as $R_i = \overline{R_{iq}}$, which is based on the transformation formula above.

Step 2. Calculate the distance measure $D^*(R_i, R)$, $(i = 1, 2, ..., n)$, where $R = \overbrace{(1,0), (1,0), ..., (1,0)}^{q}$ is an optimal alternative with q criteria.

Step 3. Choose the minimum one $D^*(R_{i0}, D)$ from $D^*(R_i, R)\,(i = 1, 2, 3, ..., n)$, i.e., $D^*(R_{i0}, D) = \min\{D^*(R_i, D)\}$.

The alternative R_{i0} is determined to be the optimal choice based on the principle of minimum distance measure.

Step 4. Calculate the degree of confidence (DOC), $DOC^{i0} = \sum_{i=1}^{n} |D^*(R_{i0}, D) - D(R_i, D)|$ [32]. If the DOC^{i0} is highest, the result of the distance measure is more credible. See more details in Appendix III. The effectiveness of a metric is determined by the magnitude of its DOC values. Therefore, this parameter holds greater significance in assessing the performance of different metrics.

7 Case Study

India is a swiftly expanding retail market, and global behemoth Amazon has inaugurated its website specifically for the Indian market. Indian websites like JioMart, AJIO.com, Nykaa, Amazon, and Flipkart are doing well in fashion, lifestyle, electronics, and books, whereas Skyscanner. in, Yatra.com, and Makemytip.com are leading in travel and ticket-related business. For fuelling e-commerce in India there are so many causes:

1. Money-saving- Due to direct purchasing
2. 24*7 Availability

3. Large range of variety in every product
4. Less timing consuming
5. Busy life style
6. Flexibility for customers

The domain of online retail has observed remarkable growth in India. Though there are many significant players in this industry, the present study considers the following e-commerce websites in Table 4.

Research compiled from the literature has identified numerous competitive advantages that can assist organizations in the pursuit of success. A brief description of each of these factors is outlined in Table 5, along with their associated citations.

A research framework proposed model is presented in Fig. 4 and is based on the competitive advantages components discussed above with their citations as shown in Table 5.

7.1 Methodology in Internet Shopping Malls

A survey was conducted on the age groups represented among online customers and the results were quite telling. It showed that the majority of shoppers belonged to the young demographic, as 35% were aged between 18 and 25 years old, 55% between 26 and 35, and only 8% and 2%, respectively, in the age groups of 36–45 and 45 and above. Moreover, it revealed that males made up 65% of online shoppers, while females represented the remaining 35%. This survey shows that 90% of online customers. Our survey was conducted among experts from both industry and academia, who were chosen based on their relevant experience. Table 6 presents the details of the experts selected for our survey, including their expertise, experience, gender, and designation.

7.2 Analysis for Selection of Best Online Shopping Mall

Tzeng and Tang et al. proposal [86, 87], the suggestion was made to select the best option that is the furthest away from the negative perfect solution and the closest to

Table 4 Description of alternatives

Internet shopping websites	Description
Amazon.in (R_1)	Amazon. com is widely recognised as an e-commerce giant. It recently made a foray into the Indian market, offering a platform for sellers to sell their products
AJIO. com (R_2)	Online products related to lifestyle and fashion. Furthermore, it broadened its array of products to incorporate global brands such as Nike, Puma, Sketchers, and so on
Myntra. com (R_3)	The website is a well-known and highly reliable source for beauty products and it attracts a large number of visitors
Nykaa (R_4)	Nykaa is an e-commerce platform located in India, which specializes in offering fashion and beauty products that cater to both genders
Firstcry. com (R_5)	Firstcry.com is mainly known for kid’s fashion

Table 5 Description of criteria

Criteria	Description of Criteria
E-reputation (R_1^*)	The degree of consideration given by consumers to the reputation of the goods and the corporations offered by online shopping malls is determined by various factors, such as centralized reputation, faith value, supplier's rating, consumer relationship, and social obligation [2, 78–81].
Information and Electronic Service Dimension (R_2^*)	This criterion helps determine the concerns of consumers who are aware about their privacy, reliability, price sensitivity, third-party seals, and the reliability of online service providers [79, 82–84].
Web performance metrics (R_3^*)	This factor assesses the extent to which consumers value the quality of web-related dimensions offered by online shopping malls. These dimensions include web quality, web design, ease of navigation, and responsiveness [2, 7, 63, 85].
Incentives and Post-Purchase Service (R_4^*)	This criterion measures how well-informed buyers are about the rewards and post-purchase services offered by online malls, including discount coupons, cash-back offers, free home delivery, payment upon delivery, and return policies [2, 7].
Digital creativity (R_5^*)	This criterion pertains to consumers who prioritize personal innovativeness and keeping up-to-date with information technology. It includes their willingness to experiment with new technologies, adopt new technology, and try out new information technologies [2, 6, 7, 85].

the positive ideal solution. In our study, we administered a questionnaire to collect expert opinions on five shopping websites, evaluating them based on the five factors we identified.

Figure 5 shows how experts collected data. We conducted a survey among experts to analyse their views on five shopping websites based on five factors and the data given below in Table 7, which is taken from [88]

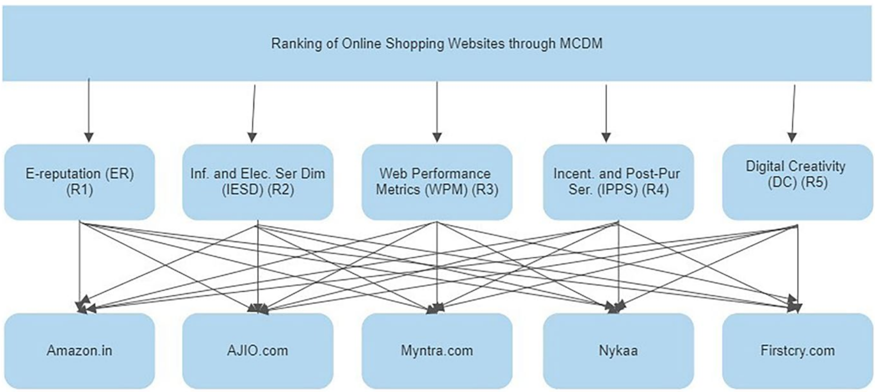


Fig. 4 Proposed research model

Table 6 Record of specialist and their specialization

Sr.No	Designation	Gender	Experience	Expertise
1.	Computer Engineer	female	20 yrs	Digital marketing
2.	Soft Engineer	male	10 yrs	Coder
3.	Business Analyst	male	20 yrs	Data analyst
4.	Principal	male	15 yrs	Econometrics
5.	Professor	male	12 yrs	E-customer behaviour
6.	Business Analyst	male	12 yrs	Feedback evolution
7.	Computer engineer	female	7 yrs	Internet security
8.	Soft engineer	female	6 yrs	System design
9.	Faculty	male	15 yrs	Social media marketing
10.	Soft engineer	male	6 yrs	Online marketing

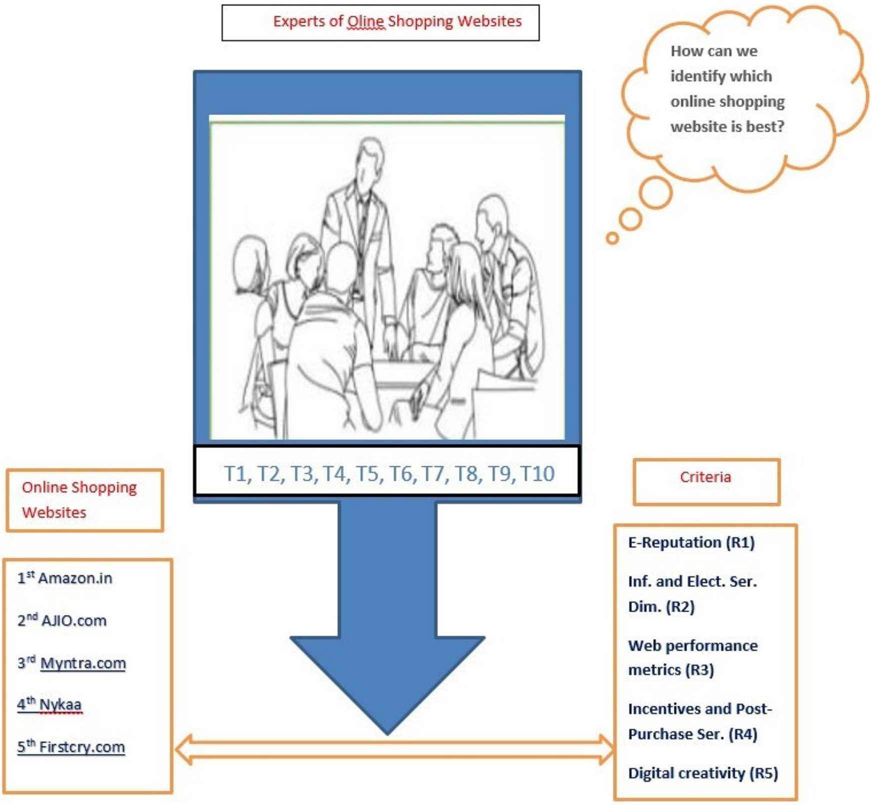


Fig. 5 MCDM problem to find internet shopping malls

Table 7 Decision matrix

Internet Websites	ER R_1^*	IESD R_2^*	WPM R_3^*	RPPS R_4^*	DC R_5^*
Amazon. in	8.06	8.79	8.06	8.58	6.82
AJIO. com	8.08	7.28	7.46	7.57	7.77
Myntra. com	7.72	8.47	8.02	7.50	8.05
Nykaa	6.971	6.85	7.13	7.16	7.28
Firstcry. com	7.45	7.46	7.46	7.98	7.47

The values presented in Table 7 cannot be directly applied to existing or proposed metrics, thus necessitating their transformation into the IF domain. To achieve this, a conversion formula is formulated as follows:

$$c^*(k_{ij}) = (0.5)(c(k_{ij})), \text{ where } c(k_{ij}) = 1 - exp\left(-\frac{k_{ij}-\min(k_{ij})}{\max(k_{ij})-\min(k_{ij})}\right)$$
$$d^*(k_{ij}) = \left(1-(k_{ij})^{0.5}\right)^2$$

where $c^*(k_{ij})$ =Membership degree and $d^*(k_{ij})$ =Non-membership degree [89]

By applying the provided formula [89], specifically the membership $c^*(k_{ij})$ and non-membership $d^*(k_{ij})$ expressions, to the data presented in Table 7, a transformation is carried out. This transformation yields the corresponding IF values, as illustrated in Table 8.

Figure 6 shows the graph of DOC, which is obtained by Table 9. Figure 6 shows the highest DOC value of the proposed measure, which is the best.

7.3 Implications

The implications of this research are significant for both academic and practical purposes. This study addresses gaps in previous research by concentrating on evaluating the competitive advantages of e-commerce giants in India. The researchers developed a model of ranking of internet shopping websites, as shown in Fig. 7. The

Table 8 Normalized decision matrix

Internet Websites	ER R_1^*	IESD R_2^*	WPM R_3^*	RPPS R_4^*	DC R_5^*
Amazon. in R_1	(0.32,0.19)	(0.32,0.19)	(0.32,0.19)	(0.32,0.19)	(0.98,0.00)
AJIO. com R_2	(0.32,0.19)	(0.10,0.47)	(0.15,0.37)	(0.12,0.42)	(0.20,0.30)
Myntra. com R_3	(0.25,0.25)	(0.28,0.21)	(0.31,0.20)	(0.11,0.46)	(0.32,0.19)
Nykaa R_4	(0.00,1.00)	(0.00,1.00)	(0.00,1.00)	(0.00,1.00)	(0.18,0.33)
Firstcry. com R_5	(0.18,0.33)	(0.14,0.40)	(0.15,0.37)	(0.22,0.28)	(0.00,1.00)

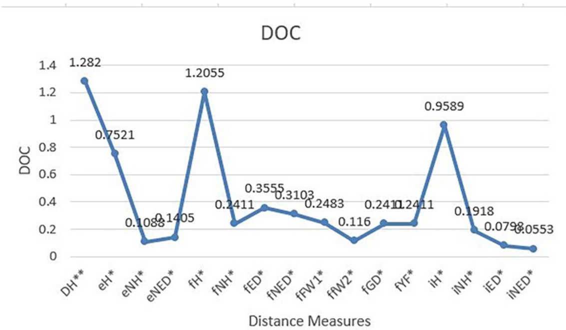
Table 9 Results of distance measures

D	$D(R_1, R)$	$D(R_2, R)$	$D(R_3, R)$	$D(R_4, R)$	$D(R_5, R)$	*Ranking*	Best	*Doc*
D_H^{**}	0.4763	0.7436	0.7016	0.9502	0.7920	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	1.282
e_H^* [15]	0.1040	0.2557	0.2325	0.3494	0.3285	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.7521
e_{NH}^* [15]	0.0036	0.0267	0.0223	0.0303	0.0439	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.1088
e_{ED}^* [15]	0.9540	1.0195	1.0090	1.1026	1.0772	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	/
e_{NED}^* [15]	0.8820	0.8986	0.8998	0.9500	0.9201	$R_1 \prec R_2 \prec R_3 \prec R_5 \prec R_4$	R_1	0.1405
f_H^* [67]	0.5480	0.8105	0.7745	0.9640	0.8485	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	1.2055
f_{NH}^* [67]	0.1096	0.1621	0.1549	0.1928	0.1697	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.2411
f_{ED}^* [67]	0.5571	0.6653	0.6452	0.7163	0.7296	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.3555
f_{NED}^* [67]	0.1427	0.2091	0.1968	0.2253	0.2499	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.3103
f_{FW1}^* [66]	0.0898	0.1382	0.1313	0.1879	0.1501	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.2483
f_{FW2}^* [66]	0.0075	0.0319	0.0275	0.0384	0.0482	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.116
f_{GD}^* [55]	0.1096	0.1621	0.1549	0.1928	0.1697	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.2411
f_{YF}^* [68]	0.1096	0.1621	0.1549	0.1928	0.1697	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.2411
i_H^* [55]	0.7260	0.9575	0.9413	0.9935	0.9706	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.9589
i_{NH}^* [55]	0.1452	0.1915	0.1883	0.1987	0.1941	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.1918
i_{ED}^* [55]	0.2088	0.2287	0.2265	0.2302	0.2296	$R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$	R_1	0.0798
i_{NED} [55]	0.0514	0.0654	0.0638	0.0658	0.0659	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.0553

overall ranking is $R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$. Moreover, the research highlights the significance of the online service provider's reputation to customers, which implies that internet shopping websites need to undertake actions to improve their reputation on online platforms.

The present study employs MCDM to rank internet shopping malls by evaluating their competitive advantages based on an extensive literature review. The opinions of experts were also taken into consideration, and the outcomes are presented

Fig. 6 Graph of DOC



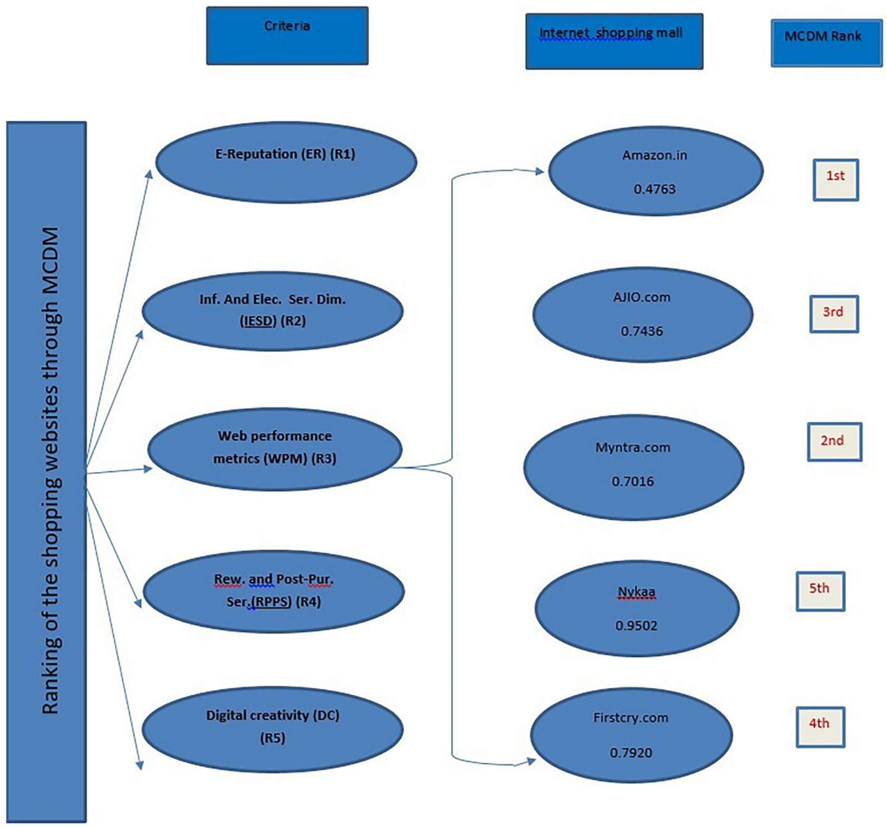


Fig. 7 Evaluating and ranking of internet shopping websites

in Table 9. By using step 4 of the algorithm (If the DOC^{i0} is highest, the result of distance measure is more credible). From Table 9, we get 1.282 (DOC of proposed measure) which is greater than all existing measures. According to the highest DOC, through the ranking, we conclude that Amazon. in and Myntra.com are the top two online shopping malls. According to the findings, Amazon is ranked first, followed by Myntra. com, Azio.com, Firstcry, and Naykaa. Amazon. in, popularly referred to as the "Amazon of Indian e-commerce," has established a noteworthy reputation for delivering exceptional services. Myntra.com, an e-commerce platform specializing in beauty products, was ranked second in the study. Given Amazon. 's extensive product offerings, it attracts a larger number of visitors and customers.

Example 6.1 If we add four more factors i.e. customer service, delivery speed, pricing, and product range then what is the effect of that example?

If we added four more criteria in Table 7, then we get Table 10 and by applying the provided formula [89], specifically the membership $c^*(k_{ij})$ and non-membership $d^*(k_{ij})$ expressions, to the data presented in Table 10, a transformation is carried out. This transformation yields the corresponding IF values, as illustrated in Table 11.

Explanation of 6.1 example By applying all the steps of Algorithm 6.1 we obtained Table 12. Table 12 shows that R_1 is the best option. In this example, which is that ranking is the same $R_1 \prec R_3 \prec R_2 \prec R_5 \prec R_4$ of the last case study. Amazon. com has an Ist ranking.

8 Comparison Analysis

To ensure the authenticity and soundness of the MCDM method, we refer to Table 13, where it can be seen that the optimal choice recommended by the suggested method aligns with the optimal choices suggested by previously established methods [37, 38, 58, 90–96]. Specifically, all the existing methods suggest that R_1 is the best option, which coincides with the recommendation of the proposed method. This similarity between the outcomes of suggested method and the established methods confirms the reliability of the proposed MCDM approach within the IF domain.

Table 10 Decision matrix

Internet Websites	ER R_1^*	IESD R_2^*	WPM R_3^*	RPPS R_4^*	DC R_5^*	CS R_6^*	DS R_7^*	PG R_8^*	PR R_9^*
Amazon. in R_1	8.06	8.79	8.06	8.58	6.82	8.05	8.61	8.75	7.06
AJIO. com R_2	8.08	7.28	7.46	7.57	7.77	7.01	6.89	7.23	6.52
Myntra. com R_3	7.72	8.47	8.02	7.50	8.05	7.65	8.35	8.46	7.89
Nykaa R_4	6.971	6.85	7.13	7.16	7.28	6.85	6.72	7.25	7.14
Firstcry. com R_5	7.45	7.46	7.46	7.98	7.47	7.25	7.48	6.53	7.67

* *CS* customer service, *DS* delivery speed, *PG* pricing, *PR* Product Range

Table 11 Normalized decision matrix

Internet	ER	IESD	WPM	RPPS	DC	CS	DS	PG	PR
Websites	R_1^*	R_2^*	R_3^*	R_4^*	R_5^*	R_6^*	R_7^*	R_8^*	R_9^*
Amazon. in R_1	(0.32,0.19)	(0.32,0.19)	(0.32,0.19)	(0.32,0.19)	(0.98,0.00)	(0.32,0.19)	(0.32,0.19)	(0.32,0.19)	(0.16,0.36)
AJIO. com R_2	(0.32,0.19)	(0.10,0.47)	(0.15,0.37)	(0.12,0.42)	(0.20,0.30)	(0.06,.57)	(0.00,1.00)	(0.14,0.39)	(0.00,1.00)
Myntra. com R_3	(0.25,0.25)	(0.28,0.21)	(0.31,0.20)	(0.11,0.46)	(0.32,0.19)	(0.24,0.26)	(0.29,0.21)	(0.29,0.21)	(0.32,0.19)
Nykaa R_4	(0.00,1.00)	(0.00,1.00)	(0.00,1.00)	(0.00,1.00)	(0.18,0.33)	(0.00,1.00)	(0.05,0.60)	(0.14,0.39)	(0.18,0.33)
Firstcry. com R_5	(0.18,0.33)	(0.14,0.40)	(0.15,0.37)	(0.22,0.28)	(0.00,1.00)	(0.14,0.39)	(0.14,0.39)	(0.00,1.00)	(0.28,0.22)

Table 12 Result of distance measures

D	$D(R_1,R)$	$D(R_2,R)$	$D(R_3,R)$	$D(R_4,R)$	$D(R_5,R)$	*Ranking*	Best	*Doc*
D_H^{**}	0.5479	0.8332	0.6506	0.9141	0.8089	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	1.0152
e_H^* [15]	1.0067	1.7281	0.9046	1.3008	1.3001	$R_3 \prec R_1 \prec R_5 \prec R_4 \prec R_2$	R_3	/
e_{NH}^* [15]	0.0659	0.1153	0.0513	0.0680	0.0586	$R_3 \prec R_5 \prec R_4 \prec R_1 \prec R_2$	R_3	0.1071
e_{ED}^* [15]	1.2704	1.5523	1.2152	1.3332	1.3485	$R_3 \prec R_1 \prec R_4 \prec R_5 \prec R_2$	R_3	/
e_{NED}^* [15]	1.0297	1.0792	1.0239	1.0483	1.0722	$R_3 \prec R_1 \prec R_4 \prec R_5 \prec R_2$	R_3	/
f_H^* [67]	5.6200	7.9100	6.5900	8.4500	7.7650	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	/
f_{NH}^* [67]	0.6244	0.8789	0.7322	0.9389	0.8574	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	0.9098
f_{ED}^* [67]	0.5571	0.6653	0.6452	0.7163	0.7296	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.3555
f_{NED}^* [67]	0.1427	0.2091	0.1968	0.2253	0.2499	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$	R_1	0.3103
f_{FW1}^* [66]	0.5153	0.7900	0.6097	0.8889	0.7675	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	0.9949
f_{FW2}^* [66]	0.0727	0.1203	0.0547	0.0728	0.0655	$R_3 \prec R_5 \prec R_1 \prec R_4 \prec R_2$	R_3	0.1125
f_{GD}^* [55]	0.6244	0.8789	0.7322	0.9389	0.8611	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	0.2367
f_{YF}^* [68]	0.6244	0.8789	0.7322	0.9389	0.8611	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	0.2367
i_H^* [55]	7.2972	8.7875	8.3203	8.9131	8.7595	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	/
i_{NH}^* [55]	0.9131	0.9764	0.9245	0.9903	0.9733	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$	R_1	0.2121
i_{ED}^* [55]	0.1745	0.1793	0.1704	0.1771	0.1743	$R_3 \prec R_5 \prec R_1 \prec R_4 \prec R_2$	R_3	0.0236
i_{NED} [55]	0.0424	0.0436	0.0401	0.0426	0.0413	$R_3 \prec R_5 \prec R_1 \prec R_4 \prec R_2$	R_1	0.0095

Table 13 Comparative study results

Methods	Ranking Results
TODIM method [90]	$R_1 \prec R_4 \prec R_5 \prec R_3 \prec R_2$
PF similarity measure [91]	$R_1 \prec R_4 \prec R_2 \prec R_5 \prec R_3$
EDAS method [92]	$R_1 \prec R_5 \prec R_4 \prec R_3 \prec R_2$
PFDWG operator [37]	$R_1 \prec R_3 \prec R_2 \prec R_4 \prec R_5$
PFDWA operator [38]	$R_1 \prec R_4 \prec R_5 \prec R_2 \prec R_3$
Generalized PF distance measure [93]	$R_1 \prec R_4 \prec R_2 \prec R_3 \prec R_5$
VIKOR method [94]	$R_1 \prec R_4 \prec R_2 \prec R_5 \prec R_3$
PF projection method [95]	$R_1 \prec R_2 \prec R_3 \prec R_4 \prec R_5$
IFS-TOPSIS [58]	$R_1 \prec R_5 \prec R_2 \prec R_4 \prec R_3$
IFS-CODAS [96]	$R_1 \prec R_5 \prec R_2 \prec R_4 \prec R_3$
D_H^{**} (Proposed Measure)	$R_1 \prec R_3 \prec R_5 \prec R_2 \prec R_4$

9 Conclusion

In this paper, Tangent distance measures for *IFSs* are presented and their properties are discussed. With real data from the machine learning repository, we presented their applications in pattern recognition and online shopping website problems. We also suggested a conversion formula from real data to *IFSs* data and explore a new method of MCDM. Furthermore, we have introduced a performance index, referred to as the degree of confidence (DOC), which demonstrates that the proposed metric has yielded superior outcomes in comparison to its contenders. Finally, a real application in the field of online shopping websites is presented to demonstrate the potential use of the proposed methodology. The researchers employed MCDM to verify and assess the sensitivity of the results. The findings indicate that except for the first two online shopping websites, Amazon. in and Myntra. com, there were no significant variations in the results, as presented in Table-9. These outcomes validate that the suggested methods are feasible, relevant, and highly suitable for addressing problems related to pattern recognition, linguistic variables, and MCDM. To acknowledge the limitations of this study, it is essential to note that the research was restricted to the northern region of India, and the sample size used by the researchers was limited. In the future, we can extend the proposed measures and MCDM method in different environments such as q-rung orthopair fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets, and T-spherical fuzzy sets.

Appendix I

Proof of Theorem 4.1

Proof

(A1) For $B_1, B_2 \in IFS(K)$, $B_1 = (l_{B_1}(k_q), m_{B_1}(k_q))$ and $B_1 = (l_{B_2}(k_q), m_{B_2}(k_q))$ it is evident that $0 \le l_{B_1}(k_q) \le 1,\ 0 \le l_{B_2}(k_q) \le 1,\ 0 \le m_{B_1}(k_q) \le 1,\ 0 \le m_{B_2}(k_q) \le 1.$ $0 \leqslant |l_{B_1}(k_q) - l_{B_2}(k_q)| \leqslant 1, 0 \leqslant |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))| \leqslant 1.$ $0 \leqslant |l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))| \leqslant 1.$ $0 \leqslant \max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right) \leqslant 1.$ $0 \leqslant \left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant \frac{\pi}{4}.$
This implies that $0 \leqslant \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant \tan\frac{\pi}{4}$ $0 \leqslant \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant 1$ $0 \leqslant \sum_{i=1}^{n}\tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant \sum_{i=1}^{n} 1$ $0 \leqslant \sum_{i=1}^{n}\tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant n$ $0 \leqslant \frac{1}{n}\tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) \leqslant 1.$

Therefore by Eq. (4.1), we get $0 \leq D_H^{**}(B_1, B_2) \leq 1$.

(A2) $D_H^{**}(B_1, B_2) = 0$ if and only if $B_1 = B_2 \Rightarrow D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right)$.

Suppose $B_1 = B_2$ then $l_{B_1}(k_q) = l_{B_2}(k_q), m_{B_1}(k_q) = m_{B_2}(k_q)$ for all k_q. $l_{B_1}(k_q) - l_{B_2}(k_q) = 0, (1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q)) = 0$ for all k_q.

This implies, $|l_{B_1}(k_q) - l_{B_2}(k_q)| = |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))| = 0$, $D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4}\max(0,0)\right)$ $D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{n} \tan(0)$ $D_H^{**}(B_1, B_2) = \frac{1}{n} \times 0$ $D_H^{**}(B_1, B_2) = 0$.

Conversely, $D_H^{**}(B_1, B_2) = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) = 0.$ $\Rightarrow \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right) = 0$ $|l_{B_1}(k_q) - l_{B_2}(k_q)| = |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))| = 0$, $l_{B_1}(k_q) = l_{B_2}(k_q), m_{B_1}(k_q) = m_{B_2}(k_q)$ for all k_q. $l_{B_1}(k_q) - l_{B_2}(k_q) = 0$, $(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q)) = 0$ for all k_q. $l_{B_1}(k_q) = l_{B_2}(k_q), m_{B_1}(k_q) = m_{B_2}(k_q)$ for all k_q.

Therefore, we obtain $B_1 = B_2$, and the property A_2 is proved.

(A3) $D_H^{**}(B_1, B_2) = D_H^{**}(B_2, B_1)$ $D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4}\max\left(|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right)\right)$ $D_H^{**}(B_2, B_1) = \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4}\max\left(|l_{B_2}(k_q) - l_{B_1}(k_q)|, |(1 - m_{B_2}(k_q)) - (1 - m_{B_1}(k_q))|\right)\right)$

This is evident that $D_H^{**}(B_1, B_2) = D_H^{**}(B_2, B_1)$ holds $\forall\, k_q \in K$, $|l_{B_1}(k_q) - l_{B_2}(k_q)| = |l_{B_2}(k_q) - l_{B_1}(k_q)|$ and $|m_{B_1}(k_q) - m_{B_2}(k_q)| = |m_{B_2}(k_q) - m_{B_1}(k_q)|$ are held. Therefore, $A3$ is proved.

(A4) If $B_1 \subseteq B_2 \subseteq B_3$ then $l_{B_1}(k_q) \leq l_{B_2}(k_q) \leq l_{B_3}(k_q)$, $m_{B_1}(k_q) \geq m_{B_2}(k_q) \geq m_{B_3}(k_q)$, for all $k_q \in K$. Thus, we can get $H^{**}(B_1, B_2) = \max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |m_{B_1}(k_q) - m_{B_2}(k_q)|\right\}$, $H^{**}(B_1, B_3) = \max\left\{|l_{B_1}(k_q) - l_{B_3}(k_q)|, |m_{B_1}(k_q) - m_{B_3}(k_q)|\right\}$, and $H^{**}(B_2, B_3) = \max\left\{|l_{B_2}(k_q) - l_{B_3}(k_q)|, |m_{B_2}(k_q) - m_{B_3}(k_q)|\right\}$. We consider the following two cases: If (i) $|\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \geq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$, then $H^{**}(B_1, B_3) = |l_{B_1}(k_q) - l_{B_3}(k_q)|$. However, we have $|\, m_{B_1}(k_q) - m_{B_2}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,|$ and $|\, m_{B_2}(k_q) - m_{B_3}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,|$. On the other hand, we have $|\, l_{B_1}(k_q) - l_{B_2}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,|$ and $|\, l_{B_2}(k_q) - m_{B_3}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,|$. By combining the previous results, we are able to derive a new result, $H^{**}(I_{B_1}, I_{B_2}) \leq H^{**}(I_{B_1}, I_{B_3})$ and $H^{**}(I_{B_2}, I_{B_3}) \leq H^{**}(I_{B_1}, I_{B_3})$. Hence, we have $D_H^{**}(B_1, B_2) \leq D_H^{**}(B_1, B_3)$. Now, we consider second case.

(ii) If $|\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$, then $H^{**}(B_1, B_3) = |m_{B_1}(k_q) - m_{B_3}(k_q)|$. However, we have $|\, l_{B_1}(k_q) - l_{B_2}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$ and $|\, l_{B_2}(k_q) - l_{B_3}(k_q)\,| \leq |\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$. On the other hand, we have $|\, m_{B_1}(k_q) - m_{B_2}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$ and $|\, m_{B_2}(k_q) - m_{B_3}(k_q)\,| \leq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$. By combining the previous results, we are able to derive a new result, $H^{**}(I_{B_1}, I_{B_2}) \leq H^{**}(I_{B_1}, I_{B_3})$ and $H^{**}(I_{B_2}, I_{B_3}) \leq H^{*}(I_{B_1}, I_{B_3})$. Hence, we have $D_H^{**}(B_1, B_2) \leq D_H^{**}(B_1, B_3)$. Therefore, cases (i) and (ii) fulfill the validation of (A4).

(A5) Now, let's examine two distinct cases.: (i) $|\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \geq |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$, then $H^{**}(B_1, B_3) = |l_{B_1}(k_q) - l_{B_3}(k_q)|$, $H^{**}(B_1, B_3) = |l_{B_1}(k_q) - l_{B_2}(k_q) + l_{B_2}(k_q) - l_{B_3}(k_q)|$, and $H^{**}(B_1, B_3) = |l_{B_1}(k_q) - l_{B_2}(k_q)| + |l_{B_2}(k_q) - l_{B_3}(k_q)|$

$= \max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |m_{B_1}(k_q) - m_{B_2}(k_q)|\right\} + \max\left\{|l_{B_2}(k_q) - l_{B_3}(k_q)|, |m_{B_2}(k_q) - m_{B_3}(k_q)|\right\} = H^{**}(I_{B_1}, I_{B_2}) + H^{**}(I_{B_2}, I_{B_3}) = D_H^{**}(B_1, B_2) + D_H^{**}(B_2, B_3)$
(ii) Similarly, $|\, l_{B_1}(k_q) - l_{B_3}(k_q)\,| \le |\, m_{B_1}(k_q) - m_{B_3}(k_q)\,|$, then $H^{**}(B_1, B_3) =$, $|m_{B_1}(k_q) - m_{B_2}(k_q) + m_{B_2}(k_q) - m_{B_3}(k_q)|$, and $H^{**}(B_1, B_3) \le |m_{B_1}(k_q) - m_{B_2}(k_q)| + |m_{B_2}(k_q) - m_{B_3}(k_q)| = \max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |m_{B_1}(k_q) - m_{B_2}(k_q)|\right\} + \max\left\{|l_{B_2}(k_q) - l_{B_3}(k_q)|, |m_{B_2}(k_q) - m_{B_3}(k_q)|\right\} = H^{**}(I_{B_1}, I_{B_2}) + H^{**}(I_{B_2}, I_{B_3}) = D_H^{**}(B_1, B_2) + D_H^{**}(B_2, B_3)$. Thus, we get $D_H^{**}(B_1, B_2) \le D_H^{**}(B_1, B_2) + D_H^{**}(B_2, B_3)$.

Appendix II

Proof of Theorem 4.2

Proof

a) $D_H^{**}(B_1^c, B_2^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|, |l_{B_1}(k_q) - l_{B_2}(k_q)|\right\}\right)$
$= \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right\}\right)$
$= D_H^{**}(B_1, B_2)$

b) $D_H^{**}(B_1, B_2^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - m_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - l_{B_2}(k_q))|\right\}\right)$
$= \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|(1 - m_{B_1}(k_q)) - (1 - l_{B_2}(k_q))|, |l_{B_1}(k_q) - m_{B_2}(k_q)|\right\}\right)$
$= D_H^{**}(B_1^c, B_2)$

c) $D_H^{**}(B_1, B_1^c) = 0 \quad \Leftrightarrow \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - m_{B_1}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - l_{B_1}(k_q))|\right\}\right) = 0 \quad \Leftrightarrow \quad \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - m_{B_1}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - l_{B_1}(k_q))|\right\}\right) = 0 \Leftrightarrow \left\{|l_{B_1}(k_q) - m_{B_1}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - l_{B_1}(k_q))|\right\} = 0 \Leftrightarrow |l_{B_1}(k_q) - m_{B_1}(k_q)| = 0\, \forall q \Leftrightarrow l_{B_1}(k_q) - m_{B_1}(k_q) = 0\, \forall q \Leftrightarrow l_{B_1}(k_q) = m_{B_1}(k_q)\, \forall q$

d) $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - m_{B_1}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - l_{B_1}(k_q))|\right\}\right)$. Let $B_1 = (l_1, m_1), B_1^c = (m_1, l_1) D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_1 - m_1|, |m_1 - l_1|\right\}\right)$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\left\{|l_1 - m_1|\right\}\right)$. If we take B_1 as a crisp set i.e. either 1 or 0.
Let $B_1 = (0, 1)$, $B_1^c = (1, 0)$. $D_H^{**}(B_1, B_2) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\left\{|l_{B_1}(k_q) - l_{B_2}(k_q)|, |(1 - m_{B_1}(k_q)) - (1 - m_{B_2}(k_q))|\right\}\right)$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\{|0 - 1|, |1 - 0|\}\right)$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\left(\frac{\pi}{4}\max\{1, 1\}\right)$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} \tan\frac{\pi}{4}$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n}\sum_{q=1}^{n} 1$ $D_H^{**}(B_1, B_1^c) = \frac{1}{n} \times n$. Then, we get $D_H^{**}(B_1, B_1^c) = 1$.

Appendix III

Explanation of DOC

From Table 9, in D_H^{**} $|0.4763 - 0.7436| = 0.2673$, $|0.4763 - 0.7016| = 0.2253$, $|0.4763 - 0.9502| = 0.4739$, $|0.4763 - 0.7920| = 0.3157$.

Now, $0.2673 + 0.2253 + 0.4739 + 0.3157 = 1.282$.

Similarly, we can find DOC for other distance measures.

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Declarations

Conflict of Interest The authors declare no competing interests.

References

1. Latha TJ, Suganthi L (2015) An empirical study on creating software product value in India-an analytic hierarchy process approach. Int J Bus Inf Syst 18(1):26–43
2. Kumar A, Dash MK (2015) Effectiveness of electronic service dimensions on consumers' electronic buying behaviour and exploration of different groups. Int J Bus Innov Res 9(1):81–99
3. Tasnim Z, Shamrat FMJM, Allayear SM, Ahmed K, Nobel NI (2021) Implementation of an intelligent online job portal using machine learning algorithms. In Emerging Technologies in Data Mining and Information Security: Proceedings of IEMIS 2020, Springer Singapore. 3, 591-603
4. Shamrat FMJM, Tasnim Z, Ghosh P, Majumder A, Hasan MZ (2014) Personalization of job circular announcement to applicants using decision tree classification algorithm. In 2020 IEEE International Conference for Innovation in Technology (INOCON). 1-5 IEEE
5. Chakravarti N (2014) Internet users in India crosses 200 million mark
6. Lian JW, Lin TM (2008) Effects of consumer characteristics on their acceptance of online shopping: comparisons among different product types. Comput Hum Behav 24(1):48–65
7. Kim EY, Kim YK (2004) Predicting online purchase intentions for clothing products. Eur J Mark 38(7):883–897
8. Zadeh LA (1965) Fuzzy sets. Inform Control 8(3):338–353
9. Arya V, Kumar S (2021) Knowledge measure and entropy: a complementary concept in fuzzy theory. Granul Comput 6(3):631–643
10. Arya V, Kumar S (2020) Fuzzy entropy measure with an applications in decision making under bipolar fuzzy environment based on TOPSIS method. Int J Inf Manag Sci 31(2):99–121
11. Arya V, Kumar S (2021) Multi-criteria decision making problem for evaluating ERP system using entropy weighting approach and q-rung orthopair fuzzy TODIM. Granul Comput 6(4):977–989. https://doi.org/10.1007/s41066-020-00242-2
12. Bajaj PK, Hooda DS (2010) Generalized measures of fuzzy directed-divergence, total ambiguity and information improvement. J Appl Math Stat Inform JAMSI 6(2):31–44

13. Montes S, Couso I, Gil P, Bertoluzza C (2002) Divergence measure between fuzzy sets. Int J Approx Reason 30(2):91–105
14. Bhatia PK, Singh S (2013) On some divergence measures between fuzzy sets and aggregation operations. AMO-Advanced Modeling and Optimization 15(2):235–248
15. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
16. Gorzalczany B (1983) Approximate inference with interval-valued fuzzy sets-an outline. In Proceedings of the Polish Symposium on Interval and Fuzzy Mathematics 89-95
17. Turksen IB (1986) Interval valued fuzzy sets based on normal forms. Fuzzy Set Syst 20(2):191–210
18. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning. Inform Sci 8:199–249
19. Tang G, Long J, Gu X, Chiclana F, Liu P, Wang F (2022) Interval type-2 fuzzy programming method for risky multicriteria decision-making with heterogeneous relationship. Inform Sci 584:184–211
20. Yager RR (1986) On the theory of bags. Int J Gen Syst 13:23–37
21. Yager RR (1988) On ordered weighted averaging aggregation operators in multi-criteria decision making. IEEE Trans Syst Man Cybern 18:183–190
22. Arefi M, Taheri SM (2014) Weighted similarity measure on interval-valued fuzzy sets and its application to pattern recognition. Iran J Fuzzy Syst 11(5):67–79
23. Boran FE, Akay D (2014) A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition. Inform Sci 255:45–57
24. Chachi J, Taheri SM (2013) A unified approach to similarity measures between intuitionistic fuzzy sets. Int J Intell Syst 28(7):669–685
25. Cheng DF, Li CT (2002) New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recogn Lett 23:221–225
26. Jin HP, Lim KM, Kwun YC (2009) Distance measure between intuitionistic fuzzy sets and its application to pattern recognition. South Korean Intelligence System of the Society 19:556–561
27. Mitchell HB (2003) On the Dengfeng-Chuntian similarity measure and its application to pattern recognition. Pattern Recogn Lett 24(16):3101–3104
28. Mondal K, Pramanik S (2015) Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Glob J Adv Res 2(2):464–471
29. Nguyen H (2015) A new knowledge-based measure for intuitionistic fuzzy sets and its application in multiple attribute group decision making. Expert Syst Appl 42(22):8766–8774
30. Nguyen H (2016) A novel similarity/dissimilarity measure for intuitionistic fuzzy sets and its application in pattern recognition. Expert Syst Appl 45:97–107
31. Xu ZS (2007) Models for multiple attribute decision making with intuitionistic fuzzy information. Internat J Uncertain Fuzziness Knowledge-Based Systems 15(03):285–297
32. Garg H (2017) Some picture fuzzy aggregation operators and their applications to multicriteria decision-making. Arab J Sci Eng 42(12):5275–5290
33. Xu ZS (2007) Intuitionistic preference relations and their application in group decision making. Inform Sci 177:2363–2379
34. Singh P (2014) A new method for solving dual hesitant fuzzy assignment problems with restrictions based on similarity measure. Appl Soft Comput 24:559–571
35. Qiu W, Zhang G, Zhu L (2016) Cosine similarity measures for dual hesitant fuzzy sets. In Proc. 4th Int. Conf. Machinery, Mater. Comput. Technol., Hangzhou, China. 511-514
36. Liu X, Shi YY, Zou L, Luo SY (2017) Similarity degree for multi-attribute decision making with incomplete dual hesitant fuzzy sets. In Proc. 7th Int. Conf. Intell. Sci. Big Data Eng. (IScIDE), vol. 10559. Cham, Switzerland: Springer. 113-122
37. Jana J, Roy SK (2019) Dual hesitant fuzzy matrix games: based on new similarity measure. Soft Comput 23(18):8873–8886
38. Jana C, Senapati T, Pal M, Yager RR (2019) Picture fuzzy Dombi aggregation operators: application to MADM process. Appl Soft Comput 74:99–109
39. Hesamian G, Shams M (2015) Measuring similarity and ordering based on hesitant fuzzy linguistic term sets. J Intell Fuzzy Syst 28(2):983–990
40. Yong R, Zhu A, Ye J (2019) Multiple attribute decision method using similarity measure of cubic hesitant fuzzy sets. J Intell Fuzzy Syst 37(1):1075–1083
41. Farhadinia B (2014) Distance and similarity measures for higher order hesitant fuzzy sets. Knowl Based Syst 55:43–48

42. Chaudhuri BB, Rosenfeld A (1999) A modified Hausdorff distance between fuzzy sets. Inf Sci 118:59–171
43. Diamond P, Kloeden P (1994) Metric spaces of fuzzy sets: theory and applications. World Scientific Publishing, Singapore
44. Hung WL, Yang MS (2008) On the J-divergence of intuitionistic fuzzy sets with its application to pattern recognition. Inform Sci 178(6):1641–1650
45. Vlachos IK, Sergiadis GD (2007) Intuitionistic fuzzy information: applications to pattern recognition. Pattern Recogn Lett 28:197–206
46. Chen TY (2011) A comparative analysis of score functions for multiple criteria decision making in intuitionistic fuzzy settings. Inform Sci 181(17):3652–3676
47. Tang G, Yang Y, Gu X, Chiclana F, Liu P, Wang F (2022) A new integrated multi-attribute decision-making approach for mobile medical app evaluation under q-rung orthopair fuzzy environment. Expert Syst Appl 200:117034
48. Chen TY, Li CH (2010) Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis. Inform Sci 180(21):4207–4222
49. Chen TY, Li CH (2011) Objective weights with intuitionistic fuzzy entropy measures and computational experiment analysis. Appl Soft Comput 11(8):5411–5423
50. Xia MM, Xu ZS (2012) Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment. Inf Fusion 13:31–47
51. Wang WQ, Xin XL (2005) Distance measure between intuitionstic fuzzy sets. Pattern Recogn Lett 26:2063–2069
52. Zhang HM, Yu LY (2013) New distance measures between intuitionistic fuzzy sets and interval-valued fuzzy sets. Inform Sci 245:181–196
53. Papakostas GA, Hatzimichailidis AG, Kaburlasos VG (2013) Distance and similarity measures between intuitionistic fuzzy sets: a comparative analysis from a pattern recognition point of view. Pattern Recogn Lett 34:1609–1622
54. Xu ZS, Chen J (2008) An overview of distance and similarity measures of intuitionistic fuzzy sets. Int J Uncertainty Fuzziness Knowledge Based Syst 16:529–555
55. Grzegorzewski P (2004) Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. Fuzzy Set Syst 148(2):319–328
56. Chen TY (2007) A note on distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. Fuzzy Sets Syst 158:2523–2525
57. Own CM (2009) Switching between type-2 fuzzy sets and intuitionistic fuzzy sets: an application in medical diagnosis. Appl Intell 31:283–291
58. Xu Z (2007) Some similarity measures of intuitionistic fuzzy sets and their application to multi-attribute decision making. Fuzzy Optim Decision Making 6:109–121
59. Du WS, Hu BQ (2015) Aggregation distance measure its induced similarity measure between intuitionistic fuzzy sets. Pattern Recogn Lett 60:65–71
60. Szmidt E, Kackprzyk J (2002) using intuitionistic fuzzy sets in group decision making. Control Cybern 31:1055–1057
61. Li D (2005) Multiattribute decision making models and methods using intuitionistic fuzzy sets. J Comput Syst Sci 70(1):73–85
62. Shamrat FJM, Chakraborty S, Billah MM, Kabir M, Shadin NS, Sanjana S (2021) Bangla numerical sign language recognition using convolutional neural networks. Indones J Electr Eng Comput Sci 23(1):405–413
63. Hwang Y, Kim DJ (2007) Customer self-service systems: the effects of perceived web quality with service contents on enjoyment, anxiety, and e-trust. Decis Support Syst 43(3):746–760
64. Dharni K (2014) Exploring information system evaluation in Indian manufacturing sector. Int J Bus Inf Sys 17(4):453–468
65. Tarkiainen A, Arminen H, Kuivalainen O (2014) Online content: who is willing to pay and for what? Int J Bus Inf Sys 17(3):283–305
66. Wang C, Zhou X, Tu H, Tao S (2017) Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. Ital J Pure Appl Math 37:477–492
67. Szmidt E, Kackprzyk J (2000) Distances between intutionistic fuzzy sets. Fuzzy Sets Syst 114:505–518
68. Yang Y, Chiclana F (2012) Consistancy of 2D and 3D distances of intuitionistic fuzzy sets. Expert Syst Appl 39:8665–8670

69. Huttenlocher DP, Klanderman GA, Rucklidge WJ (1993) Comparing images using the Hausdorff distance. IEEE Trans Pattern Anal Mach Intell 15:850–863
70. Nalder JS (1978) Hyperspaces of Sets. N. Y. Marcel Dekker 49:635–663
71. Olson CF (1998) A probabilistic formulation for Hausdorff matching. In Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition (Cat. No. 98CB36231), Santa Barbara, CA, USA 150-156
72. Hussian Z, Yang MS (2019) Distance and similarity measures of Pythagorean fuzzy sets based on the Hausdorff metric with application to fuzzy TOPSIS. Int J Intell Syst 34(10):2633–2654
73. Yang MS, Hussian Z (2019) Distance and similarity measures of hesitant fuzzy sets based on Hausdorff metric with applications to multi-criteria decision making and clustering. Soft Comput 23:5835–5848
74. Tahani V (1977) A conceptual framework for fuzzy query processing-a step toward very intelligent database systems. Inf Process Manag 13:289–303
75. Kacprzyk J, Ziolkowski A (1986) Database queries with fuzzy linguistic quantifiers. IEEE Trans Syst Man Cybern 16:474–479
76. Petry FE (1996) Fuzzy database: principles and applications; Kluwer: Dordrecht. The Netherlands
77. Candan KS, Li WS, Priya ML (2000) Similarity-based ranking and query processing in multimedia databases. Data Knowl Eng 35:259–298
78. Fan M, Tan Y, Whinston AB (2005) Evaluation and design of online cooperative feedback mechanisms for reputation management. IEEE Trans Knowl Data Eng 17(2):244–254
79. Hsu MH, Ju TL, Yen CH, Chang CM (2007) Knowledge sharing behavior in virtual communities: the relationship between trust, self-efficacy, and outcome expectations. Int J Hum Comput Stud 65(2):153–169
80. Hung YH, Huang TL, Hsieh JC, Tsuei HJ, Cheng CC, Tzeng GH (2012) Online reputation management for improving marketing by using a hybrid MCDM model. Knowl-Based Syst 35:87–93
81. Jøsang A, Ismail R, Boyd C (2007) A survey of trust and reputation systems for online service provision. Decis Support Syst 43(2):618–644
82. Ho R, Vogel D (2014) The impact of social networking functionalities on online shopping: an examination of the web's relative advantage. Int J Bus Inf Sys 16(1):25–41
83. Kim DJ, Ferrin DL, Rao HR (2008) A trust-based consumer decision-making model in electronic commerce: the role of trust, perceived risk, and their antecedents. Decis Support Syst 44(2):544–564
84. Mann BJS, Sahni SK (2013) Role of trust and customer loyalty in reducing perceived security risk in internet banking. Int J Electron Bus 10(4):331–354
85. Keisidou E, Sarigiannidis L, Maditinos D (2011) Consumer characteristics and their effect on accepting online shopping, in the context of different product types. International Journal of Business Science and Applied Management 6(2):31–51
86. Tzeng GH, Huang JJ (2011) Multiple attribute decision making: methods and applications. CRC Press, USA
87. Tang G, Zhang X, Zhu B, Seiti H, Chiclana F, Liu P (2023) A mathematical programming method based on prospect theory for online physician selection under an R-set environment. Information Fusion 93:441–468
88. Kumar DS, Dash MK, Kumar A (2016) Using TOPSIS and modified TOPSIS methods for evaluating the competitive advantages of internet shopping malls. International Journal of Business Information Systems 22(4):476–494
89. Gupta R, Kumar S (2022) Intuitionistic Fuzzy similarity-based information measure in the application of pattern recognition and clustering. Int J Fuzzy Syst 24(5):2493–2510
90. Wei G (2018) TODIM method for picture fuzzy multiple attribute decision-making. Informatica 29:555–566
91. Wei G (2018) Picture fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. Fund Inform 157(3):271–320
92. Zhang S, Wei G, Gao H, Wei C, Wei Y (2019) EDAS method for multiple criteria group decision making with picture fuzzy information and its application to green suppliers' selections. Technol Econ Dev Econ 25:1123–1138
93. Son LH (2016) Generalized picture distance measure and applications to picture fuzzy clustering. Appl Soft Comput 46:284–295
94. Joshi R (2020) A novel decision-making method using R-Norm concept and VIKOR approach under picture fuzzy environment. Expert Syst Appl 147:113228

95. Chen CT (2000) Extensions of the TOPSIS for group decision making under fuzzy environment. Fuzzy Sets Syst 114:1–9
96. Mishra AR, Mardani A, Rani P, Kamyab H, Alarsheedi M (2021) A new intuitionistic fuzzy combinative distance based assesment framework to asses low carbon sustainable suppliers in the maritime sector. Energy 237:121500

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