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REVIEW ARTICLE

Measurement of strong phases, D–$\bar{D}$ mixing, and CP violation using quantum correlation at charm threshold

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We review the measurements of the D decay strong-phase parameters based on quantum-correlated $D^0\bar{D}^0$ pairs produced in the $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ process, and we discuss their role in the measurements of Cabibbo-Kobayashi-Maskawa angle γ and D–$\bar{D}$ mixing. In addition, we present estimates of the size of quantum-correlated datasets necessary to support the γ and charm mixing measurements conducted at the LHCb and Belle II experiments. Finally, we review the methods for measuring the D–$\bar{D}$ mixing and CP violation parameters at a high-luminosity charm factory, giving sensitivity estimates.

Keywords quantum correlation, charm mixing, CP violation

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Contents

1 Introduction

The quantum-correlated $D^0\bar{D}^0$ pairs produced at an e^+e^- collider operating at the charm threshold allow the measurement of D decay strong-phase parameters, which are important input information for the measurement of the Cabibbo–Kobayashi–Maskawa (CKM) angle γ and D^0–$\bar{D}^0$ mixing. Large samples of correlated $D^0\bar{D}^0$ data may also be used to directly measure the charm mixing and CP violation parameters. In this paper, we review these aspects of quantum correlation at the charm threshold. Section 2 introduces the formalism that describes the $D^0\bar{D}^0$ quantum correlation in the $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ process. In Section 3, we discuss γ measurements that take the D decay parameters measured at the charm threshold as input, and we present the current constraints on those parameters as provided by the CLEO-c and BESIII experiments. The same constraints are also useful for improving the understanding of charm mixing, as shown in Section 4. In turn, charm mixing measurements can be used to improve the strong-phase parameter precision, with a positive impact on the extraction of γ (Section 5). In Section 6, the projected uncertainty of γ measurements over the next decade is presented, and estimates are given on the volume of charm threshold data required to fulfill these expectations. Analogous estimates are given for charm mixing. In Sections 7, 8, and 9, we discuss the measure-

*Special Topic: Potential Physics at a Super τ-Charm Factory (Ed. Hai-Bo Li).

ment of D–$\bar{D}$ mixing and CP violation parameters using time-integrated and time-dependent analyses at a hypothetical charm factory and, in Section 10, we compare the precision expected from the upgraded LHCb experiment with that expected from Belle II. Finally, although unrelated to quantum correlation, we present sensitivity estimates for measurements of time-integrated CP asymmetries in a selection of D decays at LHCb, Belle II, and a charm factory in Section 11.

2 Quantum correlation

In this section we introduce the concepts underlying the majority of the measurements that will be discussed throughout this review. We define the mass eigenstates in the neutral D meson system as

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle\,, \tag{1}$$

with eigenvalues $\lambda_k = m_k - \frac{\mathrm{i}}{2}\Gamma_k$ $(k=1,2)$, where $m_{1,2}$ and $\Gamma_{1,2}$ are the mass and width of the two neutral D meson states. We adopt the convention $CP|D^0\rangle = |\bar{D}^0\rangle$ and we require $CP|D_{1,2}\rangle = \pm|D_{1,2}\rangle$ if CP is conserved. This condition implies $q/p = 1$ if the CP is a symmetry of the Hamiltonian governing the D–$\bar{D}$ mixing. The mixing is parameterized by the quantities x and y, which are defined as $x = (m_1 - m_2)/\Gamma$ and $y = (\Gamma_1 - \Gamma_2)/2\Gamma$, with $\Gamma = (\Gamma_1 + \Gamma_2)/2$. Experimentally, it is found that $x = \left(0.41^{+0.14}_{-0.15}\right) \times 10^{-2}$ and $y = \left(0.63^{+0.07}_{-0.08}\right) \times 10^{-2}$ [1]. From Eq. (1), it follows that

$$|D^0(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\bar{D}^0\rangle\,, \tag{2}$$

$$|\bar{D}^0(t)\rangle = g_+(t)|\bar{D}^0\rangle + \frac{p}{q}g_-(t)|D^0\rangle\,, \tag{3}$$

where $g_\pm(t) = \left(\mathrm{e}^{-\mathrm{i}\lambda_1 t} \pm \mathrm{e}^{-\mathrm{i}\lambda_2 t}\right)/2$ and t is time. In the $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ process, the $D\bar{D}$ pair is produced in a coherent angular momentum $L=1$ state that is antisymmetric under the $D \leftrightarrow \bar{D}$ exchange, following the conservation of the quantum number $C=-1$. Thus,

$$|\Psi(3770)\rangle \to |D^0\bar{D}^0\rangle - |\bar{D}^0 D^0\rangle\,. \tag{4}$$

Both of the particles evolve in time. However, this occurs in phase, so that the quantum state remains that described in Eq. (4) until one of the particles decays. Once the first particle has decayed, the second particle continues to evolve in time from the state projected when the first decay occurred. If the second D decays at a given t after the first particle, the amplitude A of the process is given by

$$A(\Psi(3770) \to D^0\bar{D}^0 \to f_1 f_2)$$
$$\propto \langle f_1|D^0\rangle\langle f_2|\bar{D}^0(t)\rangle - \langle f_1|\bar{D}^0\rangle\langle f_2|D^0(t)\rangle\,, \tag{5}$$

where $f_{1,2}$ indicate the final states of the two D and the time evolution of the D mesons is given by Eqs. (2) and (3). The time-dependent rate calculated from Eq. (5) is

$$\begin{aligned}\frac{\mathrm{d}\Gamma(\Psi(3770) \to f_1 f_2)/\mathrm{d}t}{\mathrm{e}^{-\Gamma|t|}} &\propto \left(|a_+|^2 + |a_-|^2\right)\cosh(y\Gamma t)\\ &+ \left(|a_+|^2 - |a_-|^2\right)\cos(x\Gamma t)\\ &-2\Re[a_+^* a_-]\sinh(y\Gamma t) - 2\Im[a_+^* a_-]\sin(x\Gamma t)\,, \end{aligned}\tag{6}$$

where a_+ and a_- are defined as

$$a_+ = \bar{A}_{f_1}A_{f_2} - A_{f_1}\bar{A}_{f_2}\,, \tag{7}$$

$$a_- = \frac{p}{q}A_{f_1}A_{f_2} - \frac{q}{b}\bar{A}_{f_1}\bar{A}_{f_2}\,, \tag{8}$$

and A_f ($\bar{A}_f$) is the decay amplitude of D^0 ($\bar{D}^0$) to the final state f. Neglecting terms of $\mathcal{O}(x^4, y^4)$, the time-integrated rate is

$$\begin{aligned}\Gamma(\Psi(3770) \to f_1 f_2) &\propto |a_+|^2\left(1 + \frac{y^2 - x^2}{2}\right) + |a_-|^2\left(\frac{x^2+y^2}{2}\right)\\ &= |a_+|^2 + \mathcal{O}(x^2, y^2)\,.\end{aligned}\tag{9}$$

Eq. (9) gives access to the interference between the D decay amplitudes. As an example, let us consider the selection of CP-tagged $D^0 \to K^-\pi^+$ decays, where one D is selected in a CP-even final state (e.g., $f_1 = K^+K^-$) and the other in $K^-\pi^+$. Assuming CP conservation, $\bar{A}_{f_1} = A_{f_1}$. Introducing $r_{K\pi}\mathrm{e}^{-\mathrm{i}\delta_{K\pi}} = \bar{A}_{K^-\pi^+}/A_{K^-\pi^+}$, $a_+ = A_{K^+K^-}A_{K^-\pi^+}(1 - r_{K\pi}\mathrm{e}^{-\mathrm{i}\delta_{K\pi}})$ from Eq. (7), and from Eq. (9)

$$\Gamma(\Psi(3770) \to f_1 f_2) \propto 1 + r_{K\pi}^2 - 2r_{K\pi}\cos\delta_{K\pi}\,. \tag{10}$$

Hence, it is possible to directly access the relative phase $\delta_{K\pi}$ from the CP-tag rate. This is used as input in γ and D–$\bar{D}$ mixing measurements, as discussed in Sections 3 and 4, respectively.

The same concept can be applied to any combination of double-tagged events, namely, flavor-specific semileptonic modes ($K^-\mu^+\nu_\mu$, $K^-e^+\nu_e$), quasi flavor-specific modes ($K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^+\pi^-$), CP-even/odd modes (e.g. $K_S^0\pi^0$), self-conjugate multi-body channels ($K_{S,L}^0 h^+h^-$ and $h^+h^-\pi^0$ $(h=\pi,K)$, $K^+K^-\pi^+\pi^-$), and others (e.g., $K_S^0K^-\pi^+$). In the following sections, we discuss a number of measurements involving many of these decay channels.

3 Input to CKM angle γ measurement

The most sensitive method for the measurement of the angle γ of the unitarity triangle exploits the interference between the $b \to c\bar{u}s$ and $b \to u\bar{c}s$ amplitudes in the $B^- \to D^{(*)0}K^{(*)-}$ decays. The amplitude of the $B^- \to [f]_D K^-$ decay can be expressed as $A(B^- \to [f]_D K^-) = A_D A_f + A_{\bar{D}} \bar{A}_f$, where $[f]_D$ represents any final state originating from the decay of D^0 or $\bar{D}^0$, $A_D \equiv \langle D^0 K^-|H|B^-\rangle$, $A_{\bar{D}} \equiv \langle \bar{D}^0|H|B^-\rangle$, $A_f \equiv \langle f|H|D^0\rangle$, and $\bar{A}_f \equiv \langle f|H|\bar{D}^0\rangle$. Defining the hadronic parameters r_B and δ_B as $r_B \mathrm{e}^{\mathrm{i}(\delta_B - \gamma)} \equiv A_{\bar{D}}/A_D$, the rate of $B^- \to DK^-$ can be expressed as

$$\Gamma(B^- \to [f]_D K^-) \propto |A_f|^2 + r_B^2 |\bar{A}_f|^2 + 2\cos(\delta_B - \gamma)\Re[A_f \bar{A}_f^*] + 2\sin(\delta_B - \gamma)\Im[A_f \bar{A}_f^*]. \quad (11)$$

Assuming that CP is conserved in the decay of D^0, the rate of the charge-conjugate mode is obtained from Eq. (11) by replacing γ with $-\gamma$. The weak-phase γ, r_B and δ_B are extracted from the measurement of the $B^\pm$ yields, provided that the quantities $\Re[A_f \bar{A}_f^*]$ and $\Im[A_f \bar{A}_f^*]$ are constrained with independent measurements. We discuss cases in which this is achieved by exploiting the $D\bar{D}$ quantum coherence at the charm threshold in the remainder of this section.

3.1 The model-independent Dalitz method

In the Dalitz method, the D meson is reconstructed in a 3-body final state such as $D^0 \to K_S^0 \pi^+\pi^-$ [2, 3]. The decay amplitudes of D^0 and $\bar{D}^0$ can be defined as $A_f = f_- = f(m_-^2, m_+^2)$ and $\bar{A}_f = f_+ = f(m_+^2, m_-^2)$, where m_-^2 and m_+^2 are the squared masses of $K_S^0\pi^-$ and $K_S^0\pi^+$, respectively. In the model-dependent approach, the amplitudes $f_\pm$ are determined through a Dalitz plot analysis performed on a large sample of flavor-tagged D^0 decays, which are selected from $D^{*+} \to D^0\pi^+$, for example. The unknown γ, r_B, and δ_B are extracted by measuring the $B^\pm$ yields as functions of the position in the D Dalitz plot, using Eq. (11). The systematic uncertainty associated with the parameterization of the $f_\pm$ amplitudes varies significantly among different experiments (9° at Belle [4], 3° at BaBar [5], smaller at LHCb [6]). Thus, it is difficult to predict the extent to which this error can be safely reduced in future measurements. However, it may be assumed that this error will limit the precision of the model-dependent approach over the next decade, when the overall uncertainty of γ is expected to reach the 1° level (see Section 6).

The alternative is the model-independent approach [2, 3], which is free from the uncertainty associated with the Dalitz model description. The Dalitz plot is divided into $2N$ bins that are chosen to be symmetric under the $m_+^2 \leftrightarrow m_-^2$ exchange, and the number of $B^\mp \to [K_S^0\pi^+\pi^-]_D K^\mp$ decays is measured in each bin. From Eq. (11), the number of $B^\mp$ decays in each bin i can be expressed as

$$N_i^\mp \propto K_{\pm i} + r_{B\mp}^2 K_{\mp i} + 2\sqrt{K_i K_{-i}}\,(x_{B\mp} C_i \pm y_{B\mp} S_i)\,, \quad (12)$$

where $x_{B\mp} \equiv r_B\cos(\delta_B \mp \gamma)$, $y_{B\mp} \equiv r_B \sin(\delta_B \mp \gamma)$, and $r_{B\mp}^2 = x_{B\mp}^2 + y_{B\mp}^2$. K_i is proportional to $|A(D^0 \to K_S^0\pi^+\pi^-)|^2$ integrated over bin i. The parameter C_i (S_i) is the average cosine (sine) of the strong-phase difference between $A(D^0 \to K_S^0\pi^+\pi^-)$ and $A(\bar{D}^0 \to K_S^0\pi^+\pi^-)$ over bin i, such that

$$C_i = \frac{\int_{D_i} \Re[A_f \bar{A}_f^*]\mathrm{d}D}{\sqrt{\int_{D_i}|A_f|^2\mathrm{d}D}\sqrt{\int_{D_i}|\bar{A}_f|^2\mathrm{d}D}}\,. \quad (13)$$

S_i is defined as in Eq. (13) with $\Re$ replaced by $\Im$.

K_i can be measured at both e^+e^- and hadronic B-factories using flavor-tagged $D^0 \to K_S^0\pi^+\pi^-$ decays. On the other hand, C_i and S_i can only be measured from the interference between D^0 and $\bar{D}^0$ decaying in the same final state. This condition is fulfilled in the $\Psi(3770) \to D^0\bar{D}^0$ process, through exploitation of the quantum coherence of the $D^0\bar{D}^0$ system. The statistical precision in the extraction of γ depends on the bin shape and N. For a limit of infinite $\Psi(3770)$ sample size, the precision achieved with very large N is equal to that obtained in the unbinned case. The CLEO-c detector has provided measurements of the C_i and S_i parameters for four different binnings, each with $N = 8$ [7–9]. The loss of statistical precision compared to the unbinned case is estimated to be approximately 10%–20% [7]. Additionally, the experimental error on C_i and S_i translates into a systematic contribution to the measurement of γ in the 2–4° range.

Figure 1 shows the $D \to K_S^0\pi^+\pi^-$ Dalitz plot binning adopted in the CLEO-c experiment in order to extract the C_i and S_i parameters. Figure 2 shows the measured C_i and S_i values, which have been used in the Belle [10] and LHCb [11] measurements. An analogous procedure has been applied to $D^0 \to K_S^0 K^+K^-$ and used at LHCb to constrain γ, together with $D^0 \to K_S^0\pi^+\pi^-$. Because of the smaller size of the CLEO-c $D^0 \to K_S^0K^+K^-$ sample, the Dalitz plot was partitioned into 4 bins ($N = 2$) [7].

Using a sample of 772 million $B\bar{B}$ pairs, the Belle experiment has measured [10]

$$\gamma = (77.3^{+15.1}_{-14.9} \pm 4.1 \pm 4.3)^\circ, \quad (14)$$

where the first error is statistical, the second incorporates the experimental error without the C_i and S_i uncertainty, and the third is due to the C_i and S_i uncertainty.

Combining the $D^0 \to K_S^0\pi^+\pi^-$ and $D^0 \to K_S^0K^+K^-$ channels for a dataset of 3.0 fb^{-1}, the LHCb experiment has found [11]

$$\gamma = (62^{+15}_{-14})^\circ, \tag{15}$$

where the total error includes a contribution of 2°–3° from the uncertainty of the C_i and S_i parameters.

In Section 6, we will discuss the quantity of charm threshold data required in order to limit the systematic uncertainty of γ associated with the C_i and S_i parameters to a small value, compared to the expected overall uncertainty.

3.2 The ADS method

In the Atwood, Dunietz, and Soni (ADS) method [12], the D^0 is reconstructed into a doubly Cabibbo-suppressed (DCS) decay, such as $D^0 \to K^+\pi^-$. The decay rate of the $B^- \to [K^+\pi^-]_D K^-$ process is the result of the interference between $B^- \to D^0K^-$ followed by the DCS $D^0 \to K^+\pi^-$, and the suppressed $B^- \to \bar{D}^0K^-$ followed by the Cabibbo-allowed $\bar{D}^0 \to K^+\pi^-$. From Eq. (11), it follows that

$$\begin{aligned} R^{\mp} &\equiv \frac{\Gamma(B^{\mp} \to [K^{\pm}\pi^{\mp}]_D K^{\mp})}{\Gamma(B^{\mp} \to [K^{\mp}\pi^{\pm}]_D K^{\mp})} \\ &= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \mp \gamma)\,, \end{aligned} \tag{16}$$

where $r_D \mathrm{e}^{-\mathrm{i}\delta_D} \equiv \langle K^+\pi^-|H|D^0\rangle / \langle K^+\pi^-|H|\bar{D}^0\rangle$. The measurement of $R^{\mp}$ can be used to constrain γ, provided r_D and the strong-phase difference δ_D are known. r_D can be measured with very good precision at both e^+e^- and hadronic machines in analysis of D–$\bar{D}$ mixing with $D \to K^+\pi^-$. Note that δ_D has been constrained by both the CLEO-c and BESIII experiments, exploiting the quantum coherence in the $\Psi(3770) \to D^0\bar{D}^0$ decay. Here, one D is reconstructed into $K^{\pm}\pi^{\mp}$ while the other forms a CP-eigenstate final state [13, 14] [see Eq. (10)]. To fully exploit the available dataset, a number of additional double-tag combinations have also been reconstructed at CLEO-c (see the end of Section 2). Using a dataset of 818 pb^{-1}, the CLEO-c detector has yielded results of

$$\cos\delta_D = 0.81^{+0.22+0.07}_{-0.18-0.05}\,, \tag{17}$$

$$\sin\delta_D = -0.01 \pm 0.41 \pm 0.04\,, \tag{18}$$

$$|\delta_D| = (10^{+28+13}_{-53-0})^\circ\,. \tag{19}$$

Using a dataset of 2.92 fb^{-1}, the BESIII experiment has measured the asymmetry $A^{CP}_{K\pi}$ of CP-tagged D decay rates to $K^-\pi^+$, where

$$A^{CP}_{K\pi} = \frac{\mathcal{B}_{D_{CP-}\to K^-\pi^+} - \mathcal{B}_{D_{CP+}\to K^-\pi^+}}{\mathcal{B}_{D_{CP-}\to K^-\pi^+} + \mathcal{B}_{D_{CP+}\to K^-\pi^+}}\,, \tag{20}$$

finding that $A^{CP}_{K\pi} = (12.7 \pm 1.3 \pm 0.7) \times 10^{-2}$ [14]. From $A^{CP}_{K\pi}$ and using the relation $2r_D\cos\delta_D + y = (1+R_{WS})A^{CP}_{K\pi}$, where R_{WS} is the decay rate ratio of the wrong- (WS) and right-sign (RS) processes $\bar{D}^0 \to K^-\pi^+$ and $D^0 \to K^-\pi^+$, respectively, the BESIII experiment has yielded

$$\cos\delta_D = 1.02 \pm 0.11 \pm 0.06 \pm 0.01\,. \tag{21}$$

The first and second uncertainties in this expression are

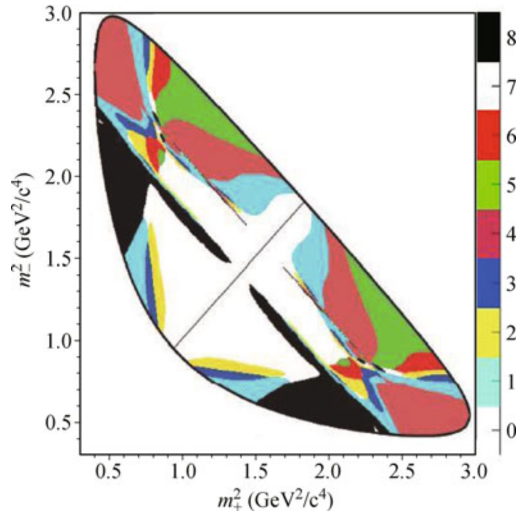

Fig. 1 Binning of the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot that was found to exploit best the B-statistics according to the BaBar Dalitz plot model. It was used by CLEO-c [7] to extract the C_i, S_i parameters adopted by Belle and LHCb as baseline for their model-independent γ measurement [10, 11].

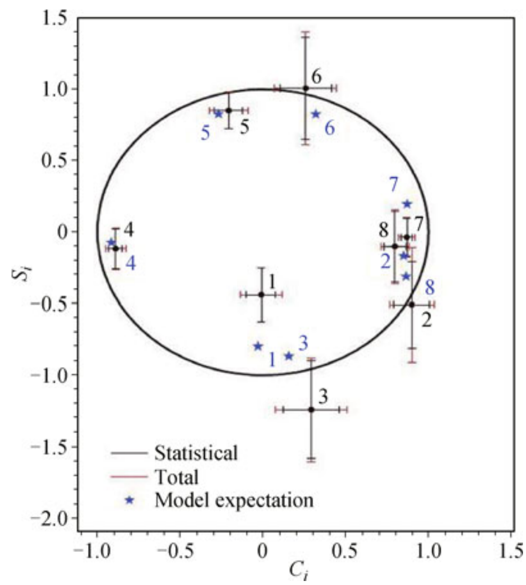

Fig. 2 CLEO-c measurement of the C_i, S_i parameters for the $D \to K_S^0\pi^+\pi^-$ Dalitz plot binning displayed in Fig. 1 [7]. This binning and the corresponding set of C_i, S_i parameters have been used by the Belle and LHCb experiments in their model-independent measurement of γ [10, 11].

statistical and systematic, respectively, while the third uncertainty arises from the world-average values used for r_D, y, and R_{WS}.

3.3 The ADS-like method

In Section 3.1, we discussed the measurement of γ when the D particle is selected in the $K^0_S h^+ h^-$ ($h = \pi, K$) final states. In principle, it is possible to apply the same technique to all fully reconstructible three- or even four-body decays, such as $D^0 \to K^\pm \pi^\mp \pi^0$, $D^0 \to K^\pm \pi^+ \pi^- \pi^\mp$ and $D^0 \to K^0_S K^\pm \pi^\mp$. However, the Dalitz analysis is complex (extremely complex in the four-body case) and requires large control samples in order to model the Dalitz structure or to allow extraction of the C_i and S_i parameters. A significantly simpler means of using these channels exists, although it is employed at the expense of the statistical power. Specifically, the $D \to f$ decay is treated inclusively, with no attempt being made to separate the intermediate resonances contributing to the final state. Then, the of $B^- \to [f]_D K^-$ yield can be expressed as

$$\begin{aligned} &\Gamma(B^- \to [f]_D K^-) \\ &\quad \propto r_B^2 + r_f^2 + 2 r_B R_f r_f \cos(\delta_B + \delta_f - \gamma)\,, \end{aligned} \tag{22}$$

where r_f is the ratio of the magnitude of the suppressed and favored D decay amplitudes and δ_f is the strong-phase difference between the amplitudes, averaged over the phase space of the final state. The coherence factor R_f is defined by

$$R_f \mathrm{e}^{-\mathrm{i}\delta_f} = \frac{\int A_f(\boldsymbol{m}) A_{\bar{f}}(\boldsymbol{m}) \mathrm{e}^{-\mathrm{i}\delta(\boldsymbol{m})} \mathrm{d}\boldsymbol{m}}{\sqrt{\int A_f^2(\boldsymbol{m}) \mathrm{d}\boldsymbol{m} \int A_{\bar{f}}^2(\boldsymbol{m}) \mathrm{d}\boldsymbol{m}}}\,, \tag{23}$$

where $\boldsymbol{m}$ is the vector of the variables describing the position in the Dalitz plot. R_f takes a value between 0 and 1 and accounts for possible dilution effects arising from the interference of the resonances in the D decay. It can be measured in the interference of $D \to f$ and $\bar{D} \to f$ using quantum-correlated $D^0 \bar{D}^0$ data. The R_f and the average strong-phase differences of $D^0 \to K^+ \pi^- \pi^0$ and $D^0 \to K^+ \pi^- \pi^+ \pi^-$ have been measured using CLEO-c data [15] through the analysis of double-tagged $\Psi(3770)$ decays, where one D is reconstructed in the signal mode and the other in the $K^0_S \pi^+ \pi^-$ final state. The parameters of interest are then extracted from a χ^2 fit to the double-tag rates. The $\Delta\chi^2$ scan in the $R_{K3\pi}$, $\delta_{K3\pi}$ parameter space is shown in Fig. 3. CLEO-c measurements of $R_{K^0_S K\pi}$ and $\delta_{K^0_S K\pi}$ have also been reported [16], and these results have been used together with $R_{K3\pi}$ and $\delta_{K3\pi}$ in the combined measurement of γ performed at LHCb [17]. The results are summarized in Table 1.

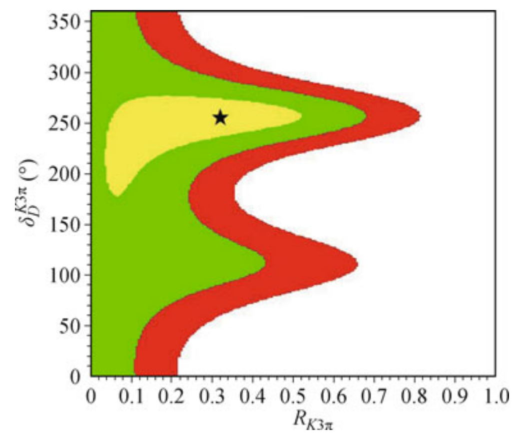

Fig. 3 Scan of $\Delta\chi^2$ in the $R_{K3\pi}, \delta_{K3\pi}$ parameter space obtained from the analysis of the CLEO-c data [15].

Table 1 Measurement of the average strong phase difference and coherence factor of $D^0 \to K^+ \pi^- \pi^0$ [15], $K^+ \pi^+ \pi^- \pi^-$ [15] and $K^0_S K^+ \pi^-$ [16] using $\Psi(3770) \to D\bar{D}$ CLEO-c data.

D decay	R_D	$\delta_D(°)$
$K^+\pi^-\pi^0$	0.82 ± 0.07	164^{+20}_{-14}
$K^+\pi^+\pi^-\pi^-$	$0.32^{+0.20}_{-0.28}$	255^{+21}_{-78}
$K^0_S K^+ \pi^-$	0.73 ± 0.08	8.3 ± 15.2

3.4 The GLW-like method

In the Gonau, London, and Wyler (GLW) method [18, 19], the D meson is reconstructed in CP-eigenstate final states, such as K^+K^-, $\pi^+\pi^-$ (CP-even) or $K^0_S \pi^0$ (CP-odd). The $B^\mp \to [f]_D K^\mp$ yield can be expressed in terms of r_B and γ using the same expression derived for the ADS method but, in this case, $r_f \mathrm{e}^{-\mathrm{i}\delta_f} = \eta_{CP}$, where $\eta_{CP} = \pm 1$ is the CP-eigenvalue of the final state, such that

$$\Gamma(B^\mp \to [f]_D K^\mp) \propto 1 + r_B^2 + 2\,\eta_{CP}\, r_B \cos(\delta_B \mp \gamma)\,. \tag{24}$$

The GLW method can be extended to D decays that are self-conjugate but not exactly CP-eigenstates, such as $D \to \pi^+\pi^-\pi^0$. Obviously, insofar as they are three-body decays, these decays can be used to constrain γ through a Dalitz analysis, as discussed in Section 3.1. Alternatively, it is possible to measure their CP content by utilizing the quantum-correlated $D^0\bar{D}^0$ decays and adopting a modified GLW approach. Let us consider a $\Psi(3770) \to D\bar{D}$ analysis, where the self-conjugate signal decay mode is $D \to f_{s.c.}$. Following the notation in Ref. [20], let M^+ indicate the number of background-subtracted double-tagged candidates, where one D is reconstructed in the signal mode and the other is in a CP-odd tag mode. Let S^+ indicate the number of background-subtracted single-tagged CP-odd candidates with no requirement on the other D. The CP fraction is defined as $F_+ = N^+/(N^+ + N^-)$, where $N^\pm = M^\pm/S^\pm$ and M^- and S^- are defined analogously to M^+ and S^+ by replacing CP-odd with CP-even. $F_+ = 1(0)$ for a signal mode that is fully CP-even (CP-odd). It can be shown [20] that

$$\Gamma(B^\mp \to [f_{s.c.}]_D K^\mp) \propto 1 + r_B^2 + (2F_+ - 1)2r_B\cos(\delta_B \mp \gamma)\,. \quad (25)$$

The CP-even fractions for $D^0 \to \pi^+\pi^-\pi^0$, $D^0 \to K^+K^-\pi^0$, and $D^0 \to \pi^+\pi^-\pi^-\pi^+$ have been measured [21] using 818 pb^{-1} of data collected by the CLEO-c experiment at the $\Psi(3770)$ resonance, yielding

$$F_+(\pi^+\pi^-\pi^0) = 0.973 \pm 0.017\,, \quad (26)$$

$$F_+(K^+K^-\pi^0) = 0.732 \pm 0.055\,, \quad (27)$$

$$F_+(\pi^+\pi^-\pi^-\pi^+) = 0.737 \pm 0.028\,, \quad (28)$$

where the errors include the statistical and systematic contributions. The value very close to 1 for $D \to \pi^+\pi^-\pi^0$ makes this channel particularly interesting. Together with $D \to K^+K^-$ and $D \to \pi^+\pi^-$, it can be treated as an additional CP-even mode, but with a branching fraction of 1.43×10^{-2}, which is between 3 and 10 times larger than the two-body modes [22].

4 Input to D–$\bar{D}$ mixing measurement

The Heavy Flavor Averaging Group (HFAG) computes world averages of the D–$\bar{D}$ mixing and CP violation parameters through a global fit, which takes as input the available charm mixing measurements [1]. The mixing analysis of the $D^0 \to K^\pm\pi^\mp$ decays makes an important contribution, from which the y' and x'^2 parameters are measured. Both y' and x' are expressed in terms of x, y, and the strong-phase difference between the D^0 and $\bar{D}^0$ amplitudes as $y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$ and $x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}$[1)], respectively. To obtain x and y from y' and x'^2, it is necessary to know $\delta_{K\pi}$, which has been constrained by both the CLEO-c and BESIII experiments, as shown in Section 3.2. The CLEO-c measurement is currently taken as input in the HFAG charm mixing fit. As the precision of the y' and x'^2 results will be further improved in future, it will be necessary to also improve the precision of $\delta_{K\pi}$ in order to fully exploit the information obtained from the $D \to K^\pm\pi^\mp$ decays. Considering the problem from a different perspective, it is apparent that y' can be used to improve knowledge of $\delta_{K\pi}$, which is an important component in γ measurement (Section 3.2). This topic is further discussed in Section 5.

The measurement of the CP-even fractions, which is discussed in Section 3.4 in the context of γ, provides new approaches to improving sensitivity to CP violation in D–$\bar{D}$ mixing measurements. Two important probes in the Belle II and LHCb charm physics programs are y_{CP} and A_Γ, which are respectively defined as

$$2y_{CP} = y\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right), \quad (29)$$

$$2A_\Gamma = y\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right). \quad (30)$$

y_{CP} and A_Γ are measured from the lifetime ratio of the $D^0 \to h^+h^-$ ($h = K, \pi$) CP-even decays with respect to the $D^0 \to K^+\pi^-$ CP-mixed state, and from the lifetime difference of $D^0 \to h^+h^-$ and $\bar{D}^0 \to h^+h^-$, respectively. In Ref. [23] it is noted that a significant improvement in the experimental sensitivity to y_{CP} and A_Γ can be obtained by adding self-conjugate multi-body decays such as $D^0 \to \pi^+\pi^-\pi^0$ and $D^0 \to \pi^+\pi^-\pi^+\pi^-$, provided the fractional CP-even content of the final state is known. The CLEO-c measurement of the CP fractions for the $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$, and $\pi^+\pi^-\pi^+\pi^-$ final states are reported in Eqs. (26)–(28).

In Section 3.1, it was shown that the model-independent measurement of γ with $B \to DK$ and $D \to K_S^0h^+h^-$ decays ($h = \pi, K$) requires knowledge of the C_i and S_i parameters, which describe the average cosine and sine of the strong-phase difference of the $\bar{D}^0$ and D^0 amplitudes over the $K_S^0h^+h^-$ phase space subregions, respectively. A technique exploiting the same concept can be used to perform a time-dependent, model-independent D–$\bar{D}$ mixing measurement with $D \to K_S^0h^+h^-$ decays [24, 25], as opposed to the model-dependent approach adopted by the CLEO [26], Belle [27], and BaBar [28] experiments. As in the case of γ measurement, the model-independent method has a statistical power that is slightly inferior to that of the model-dependent analysis ($\sim$ 10%), but that is unaffected by the systematic uncertainty associated with the Dalitz model. Preliminary results of the first model-independent measurement with flavor-tagged $D^0 \to K_S^0\pi^+\pi^-$ decays have been reported by the LHCb experiment, based on data collected in 2011 [29]. That analysis uses the same set of CLEO-c C_i and S_i parameters [7] used in the LHCb measurement of γ. In Section 6, we report estimates of the quantity of quantum-

1) If direct CP violation is allowed, $y'^\pm$ and $x'^{2\pm}$ are measured instead of y' and x'^2, respectively. Their definition can be found in Ref. [1].

correlated $D\bar{D}$ data necessary to prevent restrictions to the precision of the model-independent charm mixing measurements conducted at Belle II and at the upgraded LHCb.

5 Strong-phase constraints from D–$\bar{D}$ mixing

In the previous section, we showed examples where measurements of the strong phases at the charm threshold can improve the precision of the D–$\bar{D}$ mixing parameters. In turn, the D–$\bar{D}$ mixing measurements can make a significant contribution to the strong-phase constraints. In this section, we briefly discuss two cases involving current measurements and two studies related to possible future datasets.

The first example concerns the global fit performed by HFAG [1] to constrain the D–$\bar{D}$ mixing parameters through a combination of measurements from different experiments. One of the quantities taken as input is $y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$, which has been measured at BaBar, Belle, CDF, and LHCb from the D–$\bar{D}$ mixing analysis of $D^0 \to K^{\pm}\pi^{\mp}$ decays. Here, y' is used to constrain the mixing parameters, in conjunction with the measurement of $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ provided by the CLEO-c collaboration [Eqs. (17) and (18)]. The resultant value of $\delta_{K\pi}$ obtained from the HFAG global combination is $\delta_{K\pi} = 7.3^{+9.8}_{-11.5}$ [1], which is significantly more precise than the constraints provided by both the CLEO-c [Eq. (19)] and BESIII [Eq. (21)] experiments.

In Section 3.3, we discussed the measurement of R_f and the average strong-phase difference of the D^0 and $\bar{D}^0$ amplitudes for the $K^-\pi^+\pi^+\pi^-$ final state using the CLEO-c charm threshold data [15]. A substantial error reduction can be achieved for these parameters by exploiting the D–$\bar{D}$ interference in the mixing analysis, as shown by a simulation study based on the expected number of selected WS ($D^0 \to K^+\pi^+\pi^-\pi^-$) and RS ($D^0 \to K^-\pi^+\pi^+\pi^-$) decays at the LHCb for a dataset corresponding to 3 fb^{-1} [30]. From the ratio of the WS to RS decays as a function of the measured D proper time t,

$$r(t) = r_D^2 + r_D R_D(y\cos\delta_D - x\sin\delta_D)\Gamma t + \frac{x^2+y^2}{4}(\Gamma t)^2, \tag{31}$$

the linear coefficient can be extracted. Next, exploiting the existing knowledge of x, y, and r_D, a constraint in the (R_D, δ_D) space is obtained. The left plot in Fig. 4 shows the constraint on the $(R_{K3\pi}, \delta_{K3\pi})$ plane obtained by the CLEO-c experiment [31][2)]. The right plot shows the reduction to the selected region when the CLEO-c measurement is combined with the $(R_{K3\pi}, \delta_{K3\pi})$ information obtained from the fit to the simulated data. This proves that the charm mixing input from 3 fb^{-1} of LHCb data can substantially improve the uncertainty of R_f and the average strong-phase difference in $D \to K^+\pi^+\pi^-\pi^-$.

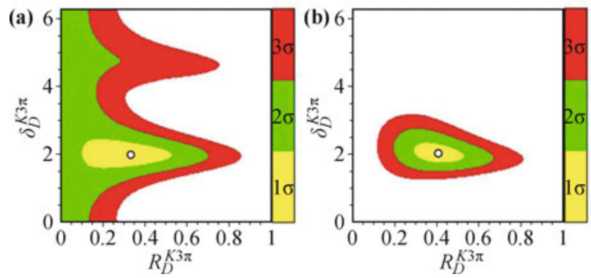

Fig. 4 **(a)** Constraints on (R_D, δ_D) obtained by CLEO-c [31][2)]. **(b)** Constraints obtained after combining the CLEO-c results with the input from simulated $D \to K^+\pi^+\pi^-\pi^-$ charm mixing data [30]. The simulated signal sample is similar in size to that expected in the dataset collected by LHCb in 2011 and 2012.

The authors of Ref. [30] have also studied the performance of a binned, model-independent method for γ extraction using $B \to DK$, $D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$ decays, in which the charm parameters used as input in each bin are measured not only at the charm threshold, but also from charm mixing [32]. The performance is significantly enhanced compared to methods in which a single bin is used, especially once the shape and number of the bins are optimized. The $D^0 \to K^{\pm}\pi^{\mp}\pi^+\pi^-$ phase spaces are divided into N bins, and the technique described in Ref. [30] is applied to each bin in order to constrain the charm parameters. The left plot in Fig. 5 is an example of the constraint on γ obtained using a simulated $B \to DK$ sample corresponding to 8 fb^{-1} of LHCb data, in conjunction with 2.9 fb^{-1} of simulated BESIII-like charm threshold data. The result is compared with the uncertainty on γ obtained when input from the LHCb $D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$ mixing measurement is used to further constrain the charm parameters [Fig. 5(b)]. The precision is improved by approximately a factor of four. In Ref. [32], several scenarios featuring different LHCb and charm threshold dataset sizes are considered. The primary conclusion is that, for the LHCb upgrade dataset (50 fb^{-1}), a γ uncertainty of 4° can be obtained using $B \to DK$ and $D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$ decays, provided that both the LHCb charm mixing sample and the BESIII charm threshold dataset (2.9 fb^{-1}) are used to constrain the input charm parameters. It is also evident that the combination of the two charm datasets results in a far

[2)] Following the results reported in Ref. [30], this measurement has been updated [15]. The new result is shown in Fig. 3.

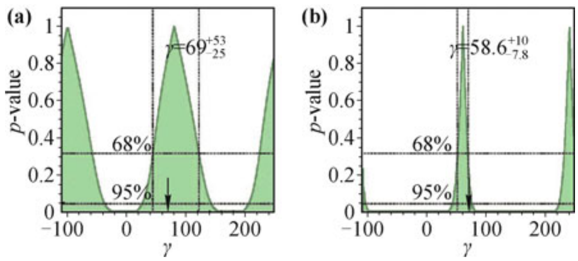

Fig. 5 **(a)** Constraint on γ from a simulated $B \to DK$, $D \to K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ sample selected at LHCb on a dataset of 8 fb^{-1} in conjunction with 2.9 fb^{-1} of simulated charm threshold data. **(b)** Constraint on γ when the $D \to K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ mixing measurement at LHCb is added. The plots are taken from Ref. [32].

superior performance than that obtained using these datasets individually. If the CLEO-c sample (0.8 fb^{-1}) is used instead of the BESIII sample, the error of γ increases to 8°.

In Section 3.1, we introduced the C_i and S_i parameters, which describe the average strong-phase difference in the $D^0 \to K^0_S h^+ h^-$ $(h, = \pi, K)$ phase space regions. These parameters have been measured by the CLEO-c experiment, exploiting the D–$\bar{D}$ quantum correlation at the charm threshold [7]. In Ref. [25], a simulation study was performed in order to evaluate the sensitivity of the direct extraction of C_i and S_i from D–$\bar{D}$ mixing measurements. It was found that, if external constraints are placed on x and y, the mixing fit can be used to determine $|q/p|$, $\arg(q/p)$, C_i, and S_i. Assuming a selection of 100 million flavor-tagged $D^0 \to K^0_S\pi^+\pi^-$ events (which is not far from what can be expected for 50 fb^{-1} at LHCb or 50 ab^{-1} at Belle II), along with a knowledge of x and y that is approximately four times more accurate than the current data, the uncertainties on C_i and S_i can be reduced by approximately 60% on average, compared to the values currently yielded by the CLEO-c experiment. This level of reduction is similar to that expected from the analysis of the data collected to date by the BESIII experiment at $\Psi(3770)$, which amounts to an integrated luminosity of approximately 2.9 fb^{-1}.

6 $D\bar{D}$ dataset sizes for future γ and charm mixing measurements

The precise measurement of γ is one of the main goals of the LHCb experiment, its planned upgrade [35], and the Belle II experiment [36]. Figure 6 summarizes the precision achieved by the BaBar and Belle experiments [33, 34], the current precision obtained at LHCb [17], and the projected uncertainty of LHCb and Belle II over the next decade [35–37]. The combined error assumes no correlations between the LHCb and Belle II measurements. By the year 2025, the overall uncertainty is expected to decrease to 1° or lower. To achieve this level of precision, it is not sufficient to increase the size of the B samples. Instead, it is necessary to take a number of subleading effects that have been safely neglected to date into account, such as those arising from D–$\bar{D}$ mixing, possible CP violation in D decays, and CP violation in K^0 decays. In addition, it is necessary to reduce the uncertainty on the external parameters entering the γ measurement, including those measured at the charm threshold by exploiting the $D^0\bar{D}^0$ quantum correlation.

In the following, we give estimates of the size of the $e^+e^- \to \Psi(3770) \to D\bar{D}$ dataset required in order to prevent restrictions to the precision of γ over the next decade. The upgraded LHCb experiment is expected to achieve a precision of approximately 1.9° using $B^{\pm} \to DK^{\pm}$ and $D \to K^0_S\pi^+\pi^-$ decays on a dataset of 50 fb^{-1} [38]. The expected precision of Belle II on a dataset of 50 ab^{-1} is 2.0° [36], which would yield an approximate uncertainty of 1.4° when combined with the LHCb sample [39]. The systematic error on γ related to the CLEO-c measurement of the C_i and S_i parameters is approximately 3° (ranging between 2° and 4° depending on the binning scheme [7]). If we somewhat arbitrarily require that the γ error associated with the C_i and S_i uncertainty be approximately one third of the total (i.e., $1.4°/3 \approx 0.5°$) and assume a CLEO-c-like performance

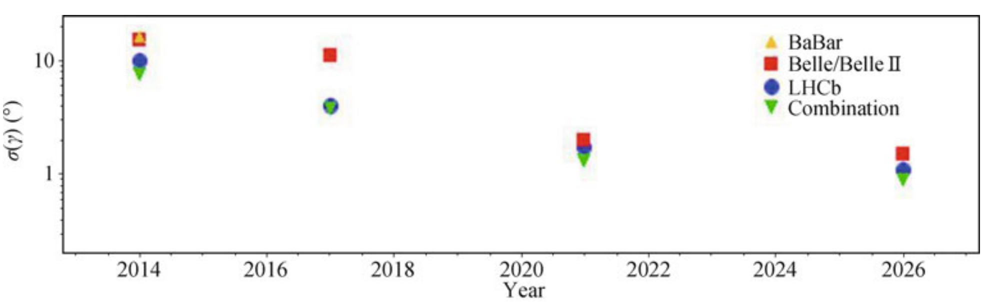

Fig. 6 Uncertainty of the angle γ as measured by the BaBar [33], Belle [34] and LHCb [17] experiments, and projected uncertainty from LHCb and Belle II over the next decade [35–37].

that is not saturated by systematic uncertainty, we find $\int L\,\mathrm{d}t \approx 30$ fb^{-1} from the relation $3°/\sqrt{\int L\,\mathrm{d}t/0.82\ \mathrm{fb}^{-1}} = 0.5°$.

In a similar manner, we can examine the required precision of the strong-phase difference $\delta_{K\pi}$ between the $D^0 \to K^+\pi^-$ and $\bar{D}^0 \to K^+\pi^-$ amplitudes to prevent limitation of the γ measurement for the ADS method, using the full LHCb and Belle II datasets. In this case, additional data at the charm threshold are probably unnecessary, for two main reasons. First, knowledge of $\delta_{K\pi}$ primarily affects the uncertainty of δ_B rather than γ [40]. This is likely related to the fact that δ_B, γ, and $\delta_{K\pi}$ enter the ADS observables $R^{\pm}$ [see Eq. (16)] as $\epsilon_{\pm} = \delta_B + \delta_{K\pi} \pm \gamma$, from which it follows that $\gamma = (\epsilon_+ - \epsilon_-)/2$ is not directly related to the value of $\delta_{K\pi}$. Second, $\delta_{K\pi}$ can be constrained effectively using the D–$\bar{D}$ mixing measurements [1], as noted in Section 5.

In principle, a large quantity of quantum-correlated $D^0\bar{D}^0$ data may facilitate improvement of the modeling of the Dalitz plot distribution of three-body D decays, in particular that of $D^0 \to K^0_S\pi^+\pi^-$, and constitute progress in controlling the corresponding systematic uncertainty in the model-dependent analysis of γ (see Section 3.1). However, it is difficult to predict what advantage could be achieved.

The simulation study presented in Ref. [25] has evaluated the extent of the charm threshold data required in order to measure the C_i and S_i of $D^0 \to K^0_S\pi^+\pi^-$ with a precision that does not limit the extraction of the D–$\bar{D}$ mixing and CP violating parameters in the model-independent analysis. The outcome of the study is that, for $|q/p|$ and $\arg(q/p)$, the CLEO-c uncertainties of C_i and S_i become dominant in mixing measurements comprising 25 million and 75 million selected events, respectively. With 100 million decays, which is the approximate extent of the data that the Belle II and upgraded LHCb experiments are expected to have collected by the termination of their nominal programs, 13 fb^{-1} of CLEO-c-like data is sufficient to reduce the systematic uncertainty associated with the C_i and S_i of both $|q/p|$ and $\arg(q/p)$ to less than half the statistical uncertainty. A CLEO-c-like dataset of 70 fb^{-1} is required in order to satisfy the same condition for x and y.

7 Time-independent D–$\bar{D}$ mixing measurement near the $\Psi(4040)$ resonance

In the $e^+e^- \to \Psi(3770) \to D\bar{D}$ process, we allow one of the two D mesons to decay to f_1 and the other to f_2. Using the notation for the model-independent analysis introduced in Section 3.1, we indicate with k_i and c_i/s_i the flavor-tagged rate and the cosine/sine of the strong-phase difference integrated over the i^{th} phase space bin in the $D^0 \to f_1$ final state. K_i, C_i, and S_i indicate the analogous parameters for the $D^0 \to f_2$ decay. We follow the discussion in Ref. [24]. The rate M_{ij} of the $\Psi(3770) \to D^0\bar{D}^0 \to f_1f_2$ process in the phase space bin ij, with the condition that the decay to f_1 occurs before the decay to f_2, is given by

$$\begin{aligned} M'_{ij}(t_2>t_1) &= k_iK_{-j} + k_{-i}K_j \\ &-2\sqrt{k_ik_{-i}K_jK_{-j}}(c_iC_j+s_iS_j) \\ &-K_j\sqrt{k_ik_{-i}}(y\,c_i - x\,s_i) \\ &-K_{-j}\sqrt{k_ik_{-i}}(y\,c_i + x\,s_i) \\ &+k_i\sqrt{K_jK_{-j}}(y\,C_j - x\,S_j) \\ &+k_{-i}\sqrt{K_jK_{-j}}(y\,C_j + x\,S_j) + \mathcal{O}(x^2,y^2)\,. \end{aligned} \tag{32}$$

In practice, the measurement of the decay time of the D mesons at a symmetric machine is very difficult because they are produced almost at rest. Instead, the rate averaged over the decay order of the two decays is measured, which is given by summing Eq. (32) with the same equation after the $k \leftrightarrow K$, $c \leftrightarrow C$, $s \leftrightarrow S$, $i \leftrightarrow j$ (all divided by two) exchange. Thus,

$$\begin{aligned} M'_{ij} &= k_iK_{-j} + k_{-i}K_j - 2\sqrt{k_ik_{-i}K_jK_{-j}}(c_iC_i+s_iS_i) \\ &+\mathcal{O}(x^2,y^2)\,. \end{aligned} \tag{33}$$

As a result, the time-integrated decay rate of $\Psi(3770) \to D^0\bar{D}^0 \to f_1f_2$ does not depend on the D–$\bar{D}$ mixing parameters at first order, in agreement with the findings of Section 2 and in particular Eq. (9). It has been observed [41] that it is possible to produce a $D^0\bar{D}^0$ pair with both quantum numbers $C = \pm 1$ in the $e^+e^- \to \Psi(4040) \to D^0\bar{D}^{*0}$ process, depending on the D^{*0} final state. $C = -1$ (antisymmetric, as in $\Psi(3770)$) for $D^0\pi^0$ and $C = 1$ (symmetric) for $D^0\gamma$. When $C = 1$, the terms that are linear in x and y do not cancel in the time-integrated rate expression, which takes the form

$$\begin{aligned} M'_{ij} &= k_iK_{-j} + k_{-i}K_j - 2\sqrt{k_ik_{-i}K_jK_{-j}}(c_iC_j+s_iS_j) \\ &+2K_j\sqrt{k_ik_{-i}}(y\,c_i - x\,s_i) \\ &+2K_{-j}\sqrt{k_ik_{-i}}(y\,c_i + x\,s_i) \\ &+2k_i\sqrt{K_jK_{-j}}(y\,C_j - x\,S_j) \\ &+2k_{-i}\sqrt{K_jK_{-j}}(y\,C_j + x\,S_j) + \mathcal{O}(x^2,y^2)\,. \end{aligned} \tag{34}$$

In the case of flavor-tagged events, where $k_{-i} = 1$ and $k_i = 0$, Eq. (34) becomes

$$M'_i = K_i + 2\sqrt{K_iK_{-i}}(y\,C_i - x\,S_i) + \mathcal{O}(x^2,y^2)\,. \tag{35}$$

This can be compared with the rate of incoherent flavor-tagged decays selected from $D^{*+} \to D^0\pi^+$

$$M_i = K_i + 2\sqrt{K_i K_{-i}}(y\, C_i + x\, S_i) + \mathcal{O}(x^2, y^2)\,. \quad (36)$$

Equations (32)–(36) have been written by neglecting CP violation in mixing. When CP is permitted, q/p factors appear in the terms containing x and y (see, e.g., [42]).

The statistical sensitivity in the extraction of the mixing parameters x, y, and q/p from the $e^+e^- \to \Psi(4040)$ process was examined in Ref. [24]. In that study, a sample corresponding to an integrated luminosity of 1 ab^{-1} was simulated. Then, the yields of four event categories were estimated using double-tagged event efficiencies measured by the CLEO-c detector at the $\Psi(3770)$ resonance as input. The following four categories were examined:

(a,b) Flavor-tagged and double-tagged $D \to K^0_S\pi^+\pi^-$ decays in the coherent $C = -1$ state (from which K_i and C_i, S_i, respectively, were extracted using Eq. (33));

(c,d) Flavor-tagged $D \to K^0_S\pi^+\pi^-$ decays in the coherent $C = 1$ state and incoherent flavor-tagged $D \to K^0_S\pi^+\pi^-$ decays (from which x, y, and q/p were extracted using Eqs. (35) and (36)). The incoherent events could be selected from the $\Psi(4040) \to D^\pm D^{*\mp}$ decays.

The mixing parameters were extracted using a combined maximum likelihood fit to the generated yields [24]. Further, an analogous study was performed with the $D \to K^\pm\pi^\mp\pi^0$ decays, which are characterized by relatively large branching fractions and a large interference between the Cabibbo-suppressed and Cabibbo-favored modes, followed by mixing. The resulting statistical uncertainties for both the $D \to K^0_S\pi^+\pi^-$ and $D \to K^\pm\pi^\mp\pi^0$ decays are reported in Table 2, together with their combined errors.

Table 2 Statistical sensitivity to the D–$\bar{D}$ mixing and CP violation parameters using $D \to K^0_S\pi^+\pi^-$ and $D^0 \to K^\pm\pi^\mp\pi^0$ decays in a time-integrated, model-independent Dalitz plot analysis based on a simulated sample of $e^+e^- \to \Psi(4040)$ events corresponding to an integrated luminosity of 1 ab^{-1} [24].

Parameter	$D \to K^0_S\pi^+\pi^-$	$D \to K^\pm\pi^\mp\pi^0$	Combined
$x\,(10^{-4})$	11.8	6.0	5.3
$y\,(10^{-4})$	8.5	6.1	5.0
$\lvert q/p\rvert\,(10^{-2})$	3.8	1.7	1.6
$\arg(q/p)$	2.5°	1.7°	1.4°

8 Time-dependent D–$\bar{D}$ mixing measurement at the $\Psi(3770)$ resonance

In the previous section, we discussed the extraction of the D–$\bar{D}$ mixing and CP violation parameters at a charm factory running near the $\Psi(4040)$ resonance using time-integrated measurements. It was noted that the same approach cannot be applied to $\Psi(3770)$ because, in this case, the rates depend on the mixing parameters x and y only at second order. On the other hand, the time-dependent rate of the $\Psi(3770) \to D^0\bar{D}^0 \to f_1 f_2$ process does depend linearly on x and y, but measurement using a symmetric machine is difficult because of the small average flight length of the D mesons (~ 18 μm), which is difficult to resolve experimentally. A possible solution involves utilizing an asymmetric machine to enhance the time-separation of the two D decays. This scenario has been studied within the SuperB project [43] and we summarize the preliminary results here [44].

The Super$B$ detector was equipped with a six-layer silicon vertex detector, in which the innermost layer was positioned 1.5 cm from the interaction point. This provided a very good vertex resolution [43]. In the study, the center-of-mass (CM) was boosted along the $e^\pm$ beam direction with a $\beta\gamma$ varying in the 0.15–0.90 range. Figure 7 shows a schematic of the interaction region.

The asymmetric-machine study included all double-tag combinations, with the D and $\bar{D}$ mesons decaying to CP-even ($K^+K^-, \pi^+\pi^-$), CP-odd ($K^0_S\pi^0$), $K^\pm\pi^\mp$, or semileptonic final states. The three-body channels were not considered in this preliminary study. The yield of each double-tag combination was extrapolated from CLEO-c data and then rescaled according to the dependence of the reconstruction efficiency as a function of the CM boost. This was measured using a fast simulation of the SuperB detector. The t resolution of the D decays as a function of the CM boost was also computed. Finally, the charm mixing and CP violation parameters (x, y, $|q/p|$, and $\arg(q/p)$) were extracted from a maximum likelihood fit to the time-dependent rates. Figure 8(a) shows the average error on the t difference of the two D decays as a function of the CM boost. As expected, the precision improves for higher boost values. On the other hand, Fig. 8(b) shows that the geometric efficiency decreases as the CM boost increases. As a consequence, the best performance is obtained from the balance between the time separation and the reconstruction efficiency.

In this simplified study, the flavor mistag rate was set

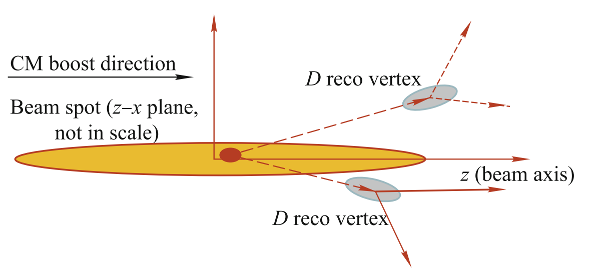

Fig. 7 Schematic view of an event $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ in the SuperB interaction region [43]. The CM boost along the z direction increases the spatial separation of the D decay vertices.

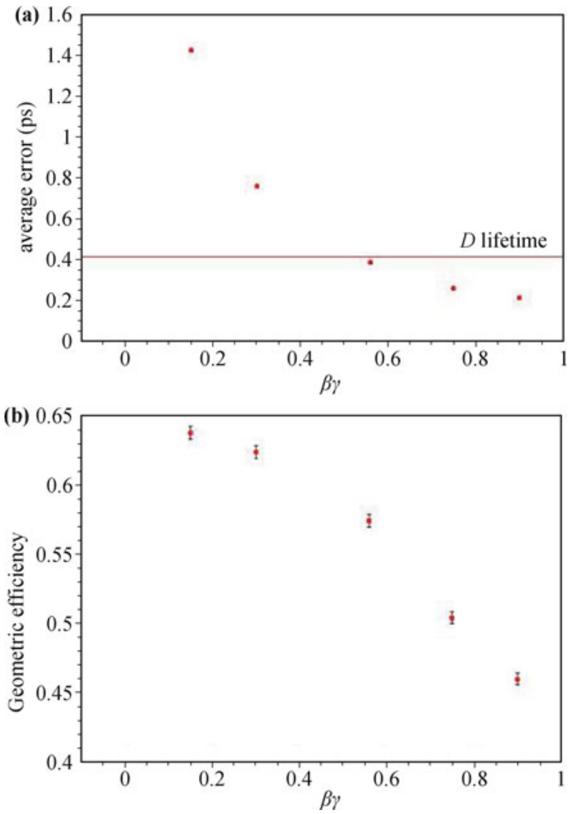

Fig. 8 **(a)** Average error on the proper time difference of the D and $\bar{D}$ decays as a function of the CM boost in $e^+e^- \to \Psi(3770) \to D\bar{D}$ events; **(b)** geometric efficiency of $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0 \to (K^-\pi^+)(K^-\pi^+)$ as a function of the CM boost.

to zero and the backgrounds were neglected. The optimum performance at $\Psi(3770)$ was achieved for CM boost values ranging from 0.3 to 0.6. The results for a dataset corresponding to an integrated luminosity of 3 ab^{-1} and a boost of $\beta\gamma = 0.56$ are summarized in Table 3. These estimates should be regarded as approximate indications to be used as the starting point for further, more accurate studies.

Table 3 Statistical sensitivity to charm mixing and CP violation parameters in a time-dependent analysis of double-tagged $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ events with the D mesons reconstructed in two-body and semileptonic final states. The sample of simulated events corresponds to an integrated luminosity of 3 ab^{-1}. The assumptions used in the study are discussed in Section 8.

Parameter	Stat. error at $\Psi(3770)$, no mistag, 3 ab^{-1}
x (%)	0.04–0.06
y (%)	0.02–0.04
$\|q/p\|$ (%)	2–5
$\arg(q/p)$ (°)	1–3

9 Time-independent measurement of y_{CP} at $\Psi(3770)$

In Section 7, it was noted that the time-integrated double-tag rates of the $\Psi(3770)$ decay do not depend on the x and y parameters at first order. However, it is possible to use a time-independent method to extract y_{CP}, which is defined in Eq. (29). This method is based on the reconstruction of CP-tagged semileptonic D decays and single-tagged (ST) D decays to CP eigenstates. Let us consider a neutral D meson "prepared" as a CP eigenstate. The partial decay width of the semileptonic decay is sensitive to the flavor content only and, therefore, it does not depend on the CP eigenvalue of the parent D. On the other hand, the total decay width of the D does depend on its CP eigenvalue, $\Gamma_{CP\pm} = \Gamma(1 \pm y_{CP})$. As a result, the ratio of the semileptonic branching fraction of the CP-even and CP-odd states is

$$\mathcal{B}_{D_{CP+}\to l}/\mathcal{B}_{D_{CP-}\to l} = (1 - y_{CP})(1 + y_{CP}) \,. \tag{37}$$

Hence, y_{CP} can be written, for example, as [45–55]

$$y_{CP} \approx \frac{1}{4}\left(\frac{\mathcal{B}_{D_{CP-}\to l}}{\mathcal{B}_{D_{CP+}\to l}} - \frac{\mathcal{B}_{D_{CP+}\to l}}{\mathcal{B}_{D_{CP-}\to l}}\right) . \tag{38}$$

Equation (38) has been used by the BESIII experiment to measure y_{CP} on a dataset of 2.9 fb^{-1} [47]. The measurement of $\mathcal{B}_{D_{CP\pm}\to l}$ exploits the quantum-correlation of the $D^0\bar{D}^0$ system. D mesons are reconstructed into a CP eigenstate, providing the number $N_{CP\pm}$ of ST events. For a subset of the ST events, the tagged partner D meson is also observed in a semileptonic decay channel, providing the number $N_{CP\pm;l}$ of double-tagged (DT) events. Next, $\mathcal{B}_{D_{CP\pm}\to l}$ is obtained as

$$\mathcal{B}_{D_{CP\pm}\to l} = \frac{N_{CP\pm;l}}{N_{CP\pm}}\frac{\epsilon_{CP\pm}}{\epsilon_{CP\pm;l}} \,, \tag{39}$$

where $\epsilon_{CP\pm}$ and $\epsilon_{CP\pm;l}$ are the detection efficiencies of the ST and DT decays, respectively. The CP-tagged events have been selected by the BESIII detector using the CP-even modes (K^+K^-, $\pi^+\pi^-$, and $K^0_S\pi^0\pi^0$) and the CP-odd modes ($K^0_S\pi^0$, $K^0_S\omega$, and $K^0_S\eta$). The extremely small CP-odd component in the K^0_S state has been found to have a negligible effect on the measurement, and the CP-impurities in $K^0_S\pi^0\pi^0$, $K^0_S\omega$, and $K^0_S\eta$ have been quantified and taken into account as a systematic uncertainty. The result is then

$$y_{CP} = (-2.0 \pm 1.3 \pm 0.7)\% \,. \tag{40}$$

The extrapolated error on a dataset corresponding to an integrated luminosity of 3 ab^{-1} is 0.05%, under the hypothesis that the systematic uncertainty can be scaled as the statistical uncertainty. Although many sources of systematic uncertainties cancel in the ratio that determines y_{CP} in Eq. (38), the assumption that the systematic uncertainty can be pushed to this level of precision requires confirmation.

10 Comparison of charm mixing sensitivities

We summarize a selection of sensitivity studies for the measurement of charm mixing and CP violation parameters for five scenarios: Datasets from the upgraded LHCb (50 fb^{-1}), Belle II (50 ab^{-1}), and three hypothetical $e^+e^- \to \Psi(3770)$ and $e^+e^- \to \Psi(4040)$ runs with integrated luminosity of 3 ab^{-1} each. The decay channels analyzed in each case are given below:

Ψ(3770): Time-independent measurement of y_{CP} using $e^+e^- \to \Psi(3770) \to D\bar{D}$ data corresponding to an integrated luminosity of 3 ab^{-1}. The estimate is an extrapolation of the BESIII measurement reported in Section 9.

Asymmetric Ψ(3770): Time-dependent D–$\bar{D}$ mixing analysis using $e^+e^- \to \Psi(3770) \to D\bar{D}$ data corresponding to an integrated luminosity of 3 ab^{-1}. The center-of-mass is boosted with $\beta\gamma = 0.56$. The $D\bar{D}$ pairs are reconstructed in several combinations of two-body decays using a SuperB-like detector [43]. Further details are given in Section 8.

Ψ(4040): Time-integrated D–$\bar{D}$ mixing analysis using $e^+e^- \to \Psi(4040) \to D^*\bar{D}$ data corresponding to an integrated luminosity of 3 ab^{-1}. The estimates are obtained from the sensitivity study discussed in Ref. [24], which was based on $D \to K^0_S\pi^+\pi^-$ and $D \to K^+\pi^-\pi^0$ decays reconstructed assuming a CLEO-c detector performance. More details are given in Section 7.

LHCb: Model-independent D–$\bar{D}$ mixing analysis with flavor-tagged $D \to K^0_S\pi^+\pi^-$ decays based on a dataset corresponding to an integrated luminosity of 50 fb^{-1} [48]. See also Ref. [38] for older, official estimates.

Belle II: Model-dependent D–$\bar{D}$ mixing analysis with flavor-tagged $D \to K^0_S\pi^+\pi^-$ decays based on a dataset corresponding to an integrated luminosity of 50 ab^{-1} [48, 49].

The sensitivity estimates are reported in Table 4. Note that the contents of this table do not provide a quantitative comparison of the sensitivity of each experiment. To accomplish such a comparison, it would be necessary to consider the main decay channels in each scenario and to evaluate their combined errors, including the systematic contributions. Such a level of information is not available at present (although the interested reader can find predictions for other charm mixing measurements at LHCb and Belle II in Refs. [35, 38, 48–50]). Nonetheless, Table 4 gives some indications of these sensitivities. In particular, the sensitivities of the upgraded LHCb and Belle II in their nominal samples will likely be similar. While the LHCb may collect larger samples of D decays to final states containing charged tracks only, Belle II will benefit from a significantly better reconstruction of final states with neutrals. A number of measurements may be limited by systematics and, from the available estimates, it seems that the differences will not usually be very large. The sensitivity estimates at $\Psi(4040)$ were conducted using toy simulations and, as such, they may be optimistic. However, it should be possible to control the systematic effects [24]. Overall, it seems reasonable to state that a dataset of a few ab^{-1} is required in order to achieve a sensitivity close to that expected at LHCb and Belle II. Similar conclusions may be drawn for the asymmetric $\Psi(3770)$ scenario. In this case, it is worth noting that the CM boost introduces a degradation of the reconstruction efficiency that affects all measurements [see Fig. 8(b)], including the searches for rare or Standard Model (SM)-forbidden decays. We do not comment on the technical issues related to the construction of an asymmetric e^+e^- machine operating at the charm threshold. For a symmetric e^+e^- machine operating at the $\Psi(3770)$ resonance, the measurement of y_{CP} may attain a precision of 0.05 with a dataset of 3 ab^{-1}. This is similar to the uncertainty expected at Belle II [49], but it is perhaps larger than that achieved at the upgraded LHCb [48]. Here, the statistical precision is expected to be one order of magnitude smaller, but a robust estimate of the systematic uncertainty is currently unavailable.

11 Time-integrated CP asymmetries

In this section, we compare the expected experimental sensitivity of direct CP asymmetries for a selection of

Table 4 Uncertainties on charm mixing and CP violation parameters for a selection of decay channels in different scenarios: Symmetric $e^+e^- \to \Psi(3770)$ run (single-tagged $D \to f_{CP}$ and CP-tagged $D \to Kl\nu$), asymmetric $e^+e^- \to \Psi(3770)$ run (combination of two-body D final states), $e^+e^- \to \Psi(4040)$ run ($D \to K^0_S\pi^+\pi^-$ and $D \to K^\pm\pi^\mp\pi^0$), upgraded LHCb ($D \to K^0_S\pi^+\pi^-$) and Belle II ($D \to K^0_S\pi^+\pi^-$). The errors in the $\Psi(3770)$ and $\Psi(4040)$ scenarios are statistical. Details are given in Sections 7–10.

Parameter	$\Psi(3770)$ 3 ab^{-1}	$\Psi(3770)$ asymmetric, 3 ab^{-1}	$\Psi(4040)$ 3 ab^{-1}	LHCb 50 fb^{-1}	Belle II 50 ab^{-1}
x (%)	–	0.04–0.06	0.03	0.04	0.08
y (%)	0.05 [y_{CP}]	0.02–0.04	0.03	0.04	0.05
$\lvert q/p\rvert$ (%)	–	2–5	0.9	4	6
$\arg(q/p)$ (°)	–	1–3	0.8	3	4

neutral and charged D decays. Even though this topic is not related to quantum correlation, it is coherent with the physics discussed in this review. Table 5 summarizes the estimated uncertainty for a selection of decays at the upgraded LHCb (50 fb^{-1}), Belle II (50 ab^{-1}), and at a hypothetical CLEO-c-like charm factory with an integrated luminosity of 3 ab^{-1}. The estimates of the LHCb errors for $D^0 \to h^+h^-$ ($h = \pi, K$), $D^0 \to K^0_S K^+$ and $D^+ \to K^+K^-\pi^+$ are taken from Ref. [48] and are a revised version of the estimates in Ref. [38]. The errors for $D^+ \to \pi^+\pi^0$ and $D^+ \to \pi^+\eta$ are taken from Ref. [51]. If a number is not given, it means that the corresponding estimate is not available at present. In the hadronic environment of LHCb, the selection of some channels is expected to be very challenging or practically impossible, as in the case of $D^0 \to K^0_S\pi^0$ or $D^0 \to \pi^0\pi^0$. The Belle II errors are taken from Ref. [52]. The sensitivities at the $\Psi(3770)$ resonance are extrapolated from the CLEO-c measurements [53, 54], as described below. The D^0 yields are rescaled to the integrated luminosity of 3 ab^{-1} and multiplied by 10% in order to simulate the semileptonic tag efficiency. The asymmetry errors are computed from the statistical fluctuation of the D^0 and $\bar{D}^0$ yields, i.e., it is assumed that the systematic uncertainties can be kept to negligible levels. The errors of the charged D modes are scaled from the asymmetry measurements of CLEO-c [53], assuming that the systematic contribution scales as the statistical error.

The expected sensitivity of Belle II for a dataset of 50 ab^{-1} is similar to the estimated sensitivity that a charm factory with a CLEO-c-like detector performance would achieve with a dataset of $\mathcal{O}(1)$ ab^{-1}.

Table 5 Estimated precision of direct CP-violating asymmetries at a charm factory operating on the $\Psi(3770)$ resonance (3 ab^{-1}), compared with the expectations at LHCb (50 fb^{-1}) [48, 51] and Belle II (50 ab^{-1}) [52]. The assumptions used to estimate the sensitivity at the $\Psi(3770)$ are discussed in Section 11.

	$\sigma(A_{CP}) \times 10^4$		
Decay	$\Psi(3770)$ 3 ab^{-1}	LHCb 50 fb^{-1}	Belle II 50 ab^{-1}
$D^0 \to K^+K^-$	5	1	3
$D^0 \to \pi^+\pi^-$	7	1.5	5
$D^0 \to \pi^0\pi^0$	15		9
$D^0 \to K^0_S K^0_S$	37		
$D^0 \to \pi^+\pi^-\pi^0$	2.3		13
$D^0 \to K^0_S\pi^0$	4		3
$D^0 \to K^0_S\eta$	10		7
$D^+ \to K^0_S K^+$	3	1	5
$D^+ \to \pi^+\pi^0$	5	25	
$D^+ \to \pi^+\eta$	4	25	14
$D^+ \to K^+K^-\pi^+$	1.5	0.8	3
$D^+ \to K^+\pi^0$	18		

12 Summary

The D strong-phase parameters provided by the CLEO-c and BESIII experiments are important inputs for many methods that constrain the CKM angle γ, and in the extraction of the D–$\bar{D}$ mixing and CP violation parameters. The majority of the available measurements are based on the 0.8 fb^{-1} CLEO-c dataset, but BESIII is expected to contribute to this area soon, with updates based on a dataset of 2.9 fb^{-1}. This may increase up to approximately 10 fb^{-1} in the coming years. The precision that BESIII is expected to achieve for the strong-phase parameters is generally sufficient to avoid limiting the measurement of γ and D–$\bar{D}$ mixing over the next few years. However, once the Belle II and the upgraded LHCb nominal datasets of 50 ab^{-1} and 50 fb^{-1}, respectively, have been analyzed, this precision may not be sufficient. For the model-independent analysis of γ with $D \to K^0_S\pi^+\pi^-$ decays, a dataset of approximately 30 fb^{-1} is required. For the extraction of the D–$\bar{D}$ mixing and CP violation parameters using a similar model-independent technique, data samples of approximately 70 and 15 fb^{-1} are required, respectively. Obviously, in the hypothetical scenario of an "extreme" flavor experiment [48] capable of exploiting the extremely large amount of heavy-flavor particles produced in the high-luminosity phase of the LHC, significantly larger samples of quantum-correlated $D\bar{D}$ decays would be required in order to support the enhanced precision in the γ, D–$\bar{D}$ mixing, and CP violation measurements.

D–$\bar{D}$ mixing and CP violation parameters can also be measured at the charm threshold, either by selecting $D^0\bar{D}^0$ pairs in a $C = 1$ state from $e^+e^- \to \gamma^* \to D^0\bar{D}^0\gamma$ events, or through the time-dependent analysis of D decays in $e^+e^- \to \Psi(3770) \to D^0\bar{D}^0$ events. The latter scenario requires an asymmetric machine to facilitate measurement of the time separation of the two D decays. For a symmetric machine operating at the $\Psi(3770)$ resonance only, measurement of y_{CP} is possible. In all cases, integrated luminosities of $\mathcal{O}(1)$ ab^{-1} are required in order to achieve statistical sensitivities similar to the uncertainties expected at Belle II and LHCb. Because of the cleanliness of the experimental environment at the charm threshold, and because the analysis methods do not, in general, rely on the knowledge of absolute efficiencies or branching fractions, it may be possible to restrict the systematic uncertainties to a level comparable with the statistical value. However, this topic requires further investigation.

Concerning the measurement of direct CP-violating asymmetries in neutral and charged D decays, the sensitivity attainable at the charm threshold with a dataset of 3 ab^{-1} should be similar overall to that expected at Belle II and superior to that foreseen at LHCb in the final states containing neutrals.

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