Front. Mech. Eng. 2019, 14(1): 76–84
https://doi.org/10.1007/s11465-018-0522-x

RESEARCH ARTICLE

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An exploratory study for predicting component reliability with new load conditions

Abstract Reliability is important to design innovation. A new product should be not only innovative, but also reliable. For many existing components used in the new product, their reliability will change because the applied loads are different from the ones for which the components are originally designed and manufactured. Then the new reliability must be re-evaluated. The system designers of the new product, however, may not have enough information to perform this task. With a beam problem as a case study, this study explores a feasible way to re-evaluate the component reliability with new loads given the following information: The original reliability of the component with respect to the component loads and the distributions of the new component loads. Physics-based methods are employed to build the equivalent component limit-state function that can predict the component failure under the new loads. Since the information is limited, the re-evaluated component reliability is given by its maximum and minimum values. The case study shows that good accuracy can be obtained even though the new reliability is provided with the aforementioned interval.

Keywords reliability, component, failure mode, prediction, random variable

1 Introduction

Many studies have found that innovation is a leading factor of product success [1–4]. An innovative product has not only new functionalities, a new architecture, new environmental integrations, and a new user interface, but also satisfactory reliability. If the reliability is low with high likelihood of failures, the product will not survive no matter how it is innovative in other aspects.

Reliability is the ability that a component or a system performs its intended function without failures. It is also a quantitative measure of the integrity of the component or the system. It is evaluated quantitatively by the probability that the component or the system works properly without a failure under a specified environment through a desired period of time [5]. It is vital to predict the reliability during the design stage. If the reliability requirement is not met, redesign is needed. This process continues until the reliability requirement is satisfied. There are two major kinds of reliability methods, including statistics-based methods [6] and physics-based methods [7]. The former is a data-driven method, which uses field or test data to estimate the reliability, while the latter focuses on estimating the reliability based on physics models (called limit-state functions) that can predict a failure state.

Physics-based reliability methods are more widely used in the parameter design stage since it does not require physical testing but only limit-state functions [8]. During the parameter design stage, limit-state functions are often available. They are derived from physics theories and principles. For example, many limit-state functions for mechanical components are those design models from domains of statics, dynamics, materials, and heat transfer. Common physics-based reliability methods include the first order reliability method (FORM), the second order reliability method (SORM), Monte Carlo simulation (MCS), and the saddlepoint approximations [9–12].

During the component design stage, the component reliability could be predicted either by a statistics-based method, a physics-based method, or the combination of both. The component is designed for conditions specified by the component designers, and the reliability is also estimated for the same conditions. If the component is used in a new product or system, however, the conditions of the component may change, and the new conditions may be different from the ones for which the component is originally designed. The component reliability may also change. For example, a component will have lower

Received December 18, 2017; accepted April 2, 2018

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reliability when it is put into use in harsher conditions such as higher temperature, pressure, and humidity.

The common condition change is the change in loads to which a component is subjected. The loads are usually random with certain probability distributions. The distributions may be totally different in new conditions if the component is used for a new product or system. During the system design stage, system designers will need to re-evaluate the component reliability based on which system reliability can be therefore predicted.

Many studies have been conducted for system reliability prediction with components applied in new conditions. Cheng and Du [13] presented a physics-based method to achieve a reliability interval of the system with new conditions. This method creates an optimization model based on the stress-strength interference theory (SSIT), and it can produce a narrow reliability interval even if the information about dependent component failures is limited. Similarly, Hu and Du [14,15] proposed a reliability prediction method that re-evaluates complete component reliability in new conditions by reconstructing the component limit-state functions, thereby accurately predicting the system reliability.

The above methods are applicable to systems that are subjected to one system load, which is distributed to the components of the system. In real-world applications, it is common that multiple loads act on components. For example, a cantilever beam may have a load at the end and a moment at the center. Another case is a supporting beam with multiple loads applied to different locations. In these cases, if the reliability data are given in the form of datasets composed of input variables (such as dimensions, loads, and properties of materials) at a certain state (either safety or failure), in the new condition, supervised learning methods [16–18] could be adopted for reliability prediction. The recent work in Refs. [19,20] developed a new approach applying support vector machines to rebuild component limit-state functions, thereby accurately estimating system reliability under multiple loads. However, if the prediction model is trained using the reliability data (training points) collected in the original conditions in component design stage, the method may not work.

The purpose of this work is to explore the feasibility of predicting component reliability with multiple loads in new conditions. In this feasibility study we mainly focus on the following situation: The original component reliability data are obtained by the component designer under working conditions defined by the component designers and are then provided to the users (system designers) of the component. During the new product design stage, the system designers will need to re-evaluate the component reliability due to changes in the new operating environment. Through a case study, we demonstrate the feasibility of estimating component reliability bounds in new load conditions different from those that are used by component designers. This is an extension of the work in Ref. [14] into multiple loads.

The rest of this paper is organized as follows. We first review the physics-based reliability methods in Section 2, and we then briefly discuss a possible way for component reliability re-evaluation for multiple new loads in Section 3. A case study is given in detail in Section 4. The case study deals with a simply supported beam, which is originally designed for a load at the center and is then used in a new condition with two new loads acting on different locations rather than the center. Conclusions and future work are discussed in Section 5.

2 Physics-based reliability methods

Physics-based reliability methods are commonly used to predict reliability by connecting reliability with the physics of failure. The basic principle is to map random input variables into state variables (responses) using a physics model, which is called a limit-state function and is given by

$$Y = g(\boldsymbol{X}), \tag{1}$$

where $\boldsymbol{X}$ is a vector of random input variables, and Y is the state variable. If $Y>0$, we have a safe state; otherwise, we have a failure state.

For components that may fail due to excessive stresses, the SSIT model can be used [21,22]. Specifically, if a component is subjected to a stress (load) which exceeds its strength (resistance), the component will be unable to perform properly as desired, and a failure occurs. The component limit-state function is then given by [22]

$$Y = g(\boldsymbol{X}) = S_{\text{strength}}(\boldsymbol{X}) - S_{\text{stress}}(\boldsymbol{X}), \tag{2}$$

where $S_{\text{strength}}(\boldsymbol{X})$ is the strength (or resistance) of the component, and it could be a yield strength, an allowable stress, or an allowable deformation, $S_{\text{stress}}(\boldsymbol{X})$ is the stress (or load) being experienced by the component, such as a torque, a force, or a deflection. The SSIT model is illustrated in Fig. 1, in which the shadow region represents failures where the stress exceeds the strength.

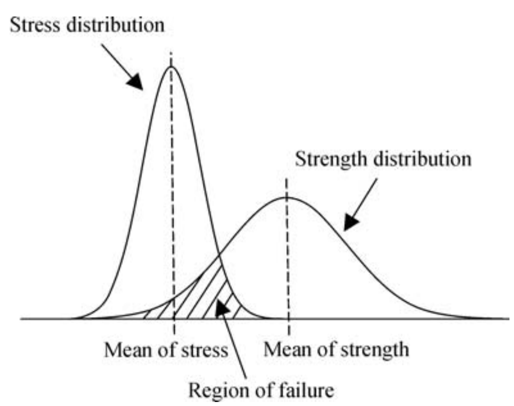

Fig. 1 Stress-strength interference theory (SSIT) model

The reliability of a component is then given by

$$R = \Pr\{Y = g(\boldsymbol{X})>0\}. \tag{3}$$

Likewise, $Y>0$ defines the safe region, and $Y\leqslant 0$ defines the failure region. For a component with n failure modes, the reliability is given by

$$R = \Pr\left\{\bigcap_{i=1}^{n} Y_i>0\right\}, \tag{4}$$

where $Y_i = g_i(\boldsymbol{X})$ is the limit-state function for the ith failure mode.

Since each failure mode may be caused by different types of stresses which are resulted from external Load L, the stress $S_{\text{stress},i}(\boldsymbol{X})$ is a function of L. Y_i in Eq. (4) can be written as

$$Y_i = S_{\text{strength},i} - h_i(L), \tag{5}$$

where $h_i(L)$ is a function of Load L. To seek for a general expression of limit-state function, we rewrite Eq. (5) as

$$Y'_i = S_i - L = h_i^{-1}(S_{\text{strength},i}) - L, \tag{6}$$

where $S_i = h_i^{-1}(S_{\text{strength},i})$, which is the resistance to the Load L. Thus, Eq. (4) is equivalent to

$$R = \Pr\left\{\bigcap_{i=1}^{n} Y'_i>0\right\} = \Pr\left\{\bigcap_{i=1}^{n} S_i>L\right\} = \Pr\{S>L\}, \tag{7}$$

in which $S = \min(S_1, S_2, ..., S_n)$ is the generalized component resistance. The generalization of the limit-state function directly links the component reliability R with the external Load L, making it possible for component designers (suppliers) to provide the reliability data in the form of component reliabilities at different load levels without revealing the design details. Note that, failures may also be caused by excessive deflections, but in this work, we mainly focus on those due to excessive stresses.

3 Component reliability re-evaluation with new loads and multiple failure modes

The current business model in engineering is increasingly calling for manufacturers functioning as system integrators that use numerous outside component suppliers [23–25]. Although component reliability data under specific working conditions may be provided from component suppliers to system designers, the component reliability may change because of the new working conditions of the component in the new system. This requires the re-evaluation of the component reliability.

If all the design details of the component are available, the limit-state functions of the components will also be available. Then the new component reliability can be readily estimated with all the known information. It is, however, difficult to do so because not all the design details of the component are accessible to the component users or system designers. In this work, we assume that system designers have good knowledge about the mechanism of each component failure mode and can build explicit component limit-state functions for new working conditions. During the model building process, the proprietary information of the component is not required because only the forms of limit-state functions are concerned, and model parameters are not needed. As mentioned in Section 1, the previous reliability re-evaluation method is for only one system load [13–15], and this work focuses on exploring the feasibility of re-evaluating component reliability with multiple system loads in new conditions.

We assume that the component supplier evaluates the component reliability with only one component load. To perform the component reliability analysis, they could use a statistics-based method, such as testing, or a physics-based method, such as FORM or SORM. In this work, we propose that system designers use the SSIT model in Eq. (6) for the component reliability re-evaluation no matter what reliability method was used by component designers. To explain it is possible to do this, we use a rectangular beam, as shown in Fig. 2, with a width of b_0 and a height of h_0 of the cross-section as an example. The beam suppliers apply an axial tension L to the beam, and the yield strength of the material is S_{strength}. According to the SSIT model, the reliability is given by

$$R = \Pr\left(Y = g(\boldsymbol{X}) = S_{\text{strength}} - \frac{L}{b_0 h_0}>0\right), \tag{8}$$

where $\frac{L}{b_0 h_0}$ is the normal stress, or function $h(L)$ in Eq. (5). According to Eq. (6), the reliability equation could be written as

$$R = \Pr(Y' = S - L = b_0 h_0 S_{\text{strength}} - L>0) = \Pr(S>L), \tag{9}$$

in which $S = b_0 h_0 S_{\text{strength}}$ is the generalized resistance of the beam. This example confirms that the general limit-state function can be rewritten as the difference between the component resistance and the component load as shown in Eq. (6).

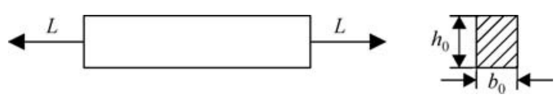

Fig. 2 Rectangular beam with an axial tension L

Component designers then use the limit-state function to calculate the reliability R or the probability of failure $p_f = 1 - R$. During the reliability analysis process, they change the level of the component Load L. The actual values of the load are represented by l. After the reliability analysis, a function of the probability of failure $p_f(l)$ is then obtained. If the limit-state functions are not available, component designers perform reliability testing at different levels of the component load. Using the testing results, they can also obtain the probability of failure $p_f(l)$. If p_f is given with discrete values, a Kriging-based fitting method is applied to obtain a continuous function of p_f with respect to l [14]. Component suppliers then provide $p_f(l)$ to system designers without revealing the design details of the component, such as the dimensions of the component and materials properties, which are proprietary to component suppliers.

When system designers re-evaluate the component reliability for their new product, they do not have sufficient information to do so. But they do have information of the generalized component resistance S. In fact, the function $p_f(l)$ provided by the component supplier is actually the cumulative distribution function (CDF) of S. For a constant l, according to Eq. (3),

$$p_f(l) = 1-R(l) = 1-\Pr(S>l) = \Pr(S<l). \tag{10}$$

Note that the CDF of S is defined by

$$F_S(s) = \Pr(S<s). \tag{11}$$

If we set $l=s$ in Eq. (10), we have $p_f(s) = \Pr(S<s)$, which is equal to $F_S(s)$ in Eq. (11). As a result,

$$F_S(s) = p_f(s) = 1-R(s). \tag{12}$$

This means that the CDF of S is available if $p_f(l)$ is given by the component supplier.

The above conclusion is also true for components with n failure modes, for which $F_S(s)$ could be expressed by

$$F_S(s) = \Pr\left\{\bigcup_{i=1}^{n} S_i \leqslant L\right\} = \Pr\{S \leqslant L\}, \tag{13}$$

in which S_i is the component resistance with respect to the ith failure mode, and S is the generalized component resistance given by $S = \min(S_1, S_2, ..., S_n)$. With the obtained $F_S(s)$ and the known distribution of new Load L, it is possible for system designers to predict the component reliability in the new conditions.

The above discussion is based on the component subjected to only one load. However, in cases that a component is subjected to multiple loads in new conditions, the distribution of its generalized resistance obtained from Eq. (13) may not be usable. To address this issue, in this work we propose a new method which uses the equivalent loads to reproduce the same effects (such as excessive bending or fracture) as those produced by the only load used by component designers. System designers predict the component reliability with an interval defined by the upper and lower bounds.

Assume that for the two failure modes the corresponding component resistances are S_1 and S_2. With m loads $L_1, L_2, ..., L_m$ applied to the component in the new condition, the component reliability is calculated by

$$R_{\text{new}} = \Pr\{S_1>Z_1 \cap S_2>Z_2\}, \tag{14}$$

where $Z_i = c_1^i L_1 + c_2^i L_2 + ... + c_m^i L_m$, $i = 1, 2$, is the equivalent load related to the i-th failure mode, which causes the same effects as those used in the original component design by the component supplier, and c_1^i, c_2^i, ..., c_m^i are load coefficients. Details of how to find the equivalent loads will be discussed in Section 4.

Considering the two circumstances $Z_1>Z_2$ and $Z_1<Z_2$, R_{new} is computed by

$$R_{\text{new}} = P_1 + P_2 + P_3 + P_4, \tag{15}$$

in which

$$P_1 = \Pr(S_1>Z_1 \cap S_2>Z_2 \cap S_1>S_2 \cap Z_1>Z_2), \tag{16}$$

$$P_2 = \Pr(S_1>Z_1 \cap S_2>Z_2 \cap S_1>S_2 \cap Z_1<Z_2), \tag{17}$$

$$P_3 = \Pr(S_1>Z_1 \cap S_2>Z_2 \cap S_1<S_2 \cap Z_1>Z_2), \tag{18}$$

$$P_4 = \Pr(S_1>Z_1 \cap S_2>Z_2 \cap S_1<S_2 \cap Z_1<Z_2). \tag{19}$$

The above probabilities P_1, P_2, P_3, and P_4 are shown in the probability diagrams in Figs. 3 and 4. According to Eqs. (16) and (18), where $Z_1>Z_2$ holds, P_1 and P_3 are related to the two shadow areas in Fig. 3(a). Figure 3(b)

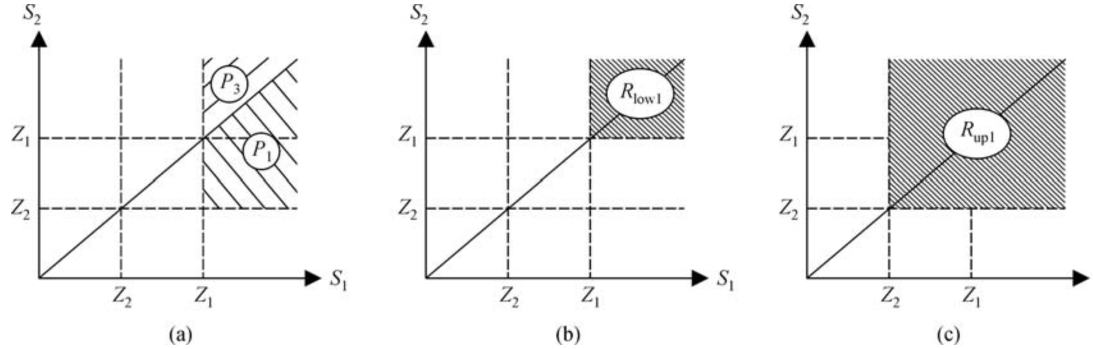

Fig. 3 Probability diagrams under condition $Z_1>Z_2$

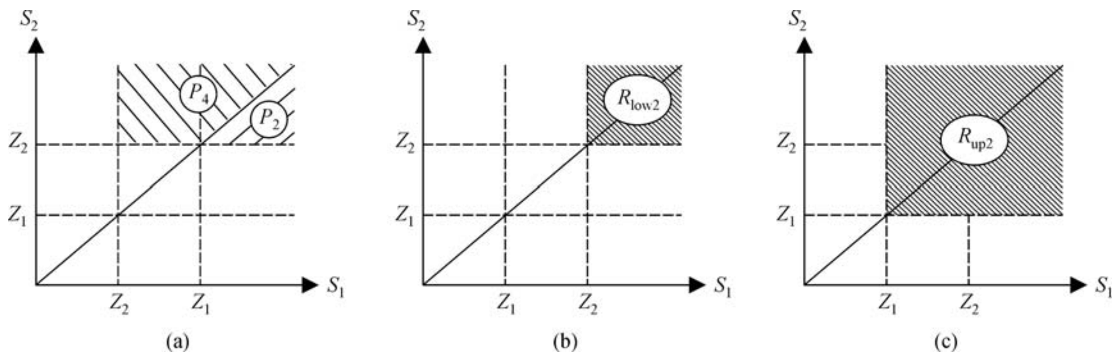

Fig. 4 Probability diagrams under condition $Z_2 > Z_1$

shows the probability of event $\{S>Z_1 \cap Z_1>Z_2\}$ given by

$$R_{\text{low1}} = \Pr\{S>Z_1 \cap Z_1>Z_2\}, \tag{20}$$

in which $S = \min(S_1,S_2)$. Figure 3(c) shows the probability of event $\{S>Z_2 \cap Z_1>Z_2\}$ given by

$$R_{\text{up1}} = \Pr\{S>Z_2 \cap Z_1>Z_2\}. \tag{21}$$

The following inequalities are found:

$$R_{\text{low1}}<P_1 + P_3<R_{\text{up1}}. \tag{22}$$

Similarly, according to Eqs. (17) and (19), where $Z_2>Z_1$ holds, P_2 and P_4 are shown in the shadow areas in Fig. 4(a). Figure 4(b) shows the probability of event $\{S>Z_2 \cap Z_1<Z_2\}$ computed by

$$R_{\text{low2}} = \Pr\{S>Z_2 \cap Z_1<Z_2\}. \tag{23}$$

Figure 4(c) shows the probability of event $\{S>Z_1 \cap Z_1 <Z_2\}$ computed by

$$R_{\text{up2}} = \Pr\{S>Z_1 \cap Z_1<Z_2\}. \tag{24}$$

And the following inequalities are obtained:

$$R_{\text{low2}}<P_2 + P_4<R_{\text{up2}}. \tag{25}$$

By adding Eqs. (22) and (25), the system designers predict the new reliability with the bounds defined by

$$R_{\text{low}}<R_{\text{new}}<R_{\text{up}}, \tag{26}$$

in which

$$R_{\text{low}} = R_{\text{low1}} + R_{\text{low2}}, \tag{27}$$

$$R_{\text{up}} = R_{\text{up1}} + R_{\text{up2}}. \tag{28}$$

Since the system designers have a good knowledge of L_1, L_2, ..., L_m, they could derive the distributions of Z_1 and Z_2. With the distribution of S available, R_{up} and R_{low} can be readily estimated using MCS or any other physics-based reliability methods.

For complicated components with more than two failure modes, the proposed method is also applicable. For example, given a component with three failure modes caused by excessive stress, according to Eq. (14), the component reliability is computed by

$$R_{\text{new}} = \Pr\{S_1>Z_1 \cap S_2>Z_2 \cap S_3>Z_3\}, \tag{29}$$

where S_3 is the component resistance related to the third failure mode, and Z_3 is the corresponding equivalent load. Enumerating all the conditions determined by the mutual relationship between S_i $(i = 1,2,3)$ and between Z_i $(i = 1,2,3)$, R_{new} is then calculated by summing all the probabilities that each condition occurs. Therefore, the lower and upper bounds of R_{new} could be obtained. Note that as the number of failure modes increases, although the proposed method still works, the number of event combinations will grow exponentially, leading to much lower efficiency. One possible way of addressing this problem is to approximate the system reliability with the probabilities of all combinations of two individual failure modes, but additional information may be required from component suppliers. This needs a future investigation.

The other assumption of the proposed method is that system designers could build explicit component limit-state functions so that equivalent loads can be identified. If a component limit-state function is from simulations, such as finite element analysis, the limit-state function will be a black-box model. For this case, system designers should conduct design of experiments [26] and run the simulation a number of times. Then a surrogate model, which is explicit with respect to the new loads, could be established. The proposed method can then be applied.

4 A case study

A component supplier designs and makes a beam as shown in Fig. 5. The component supplier evaluates the reliability of the beam for the situation where the beam is simply supported at Points A and B, and a Load L is applied to the center point as indicated in Fig. 5. Two failure modes are observed due to the excessive bending stress and shear stress developed in the beam. By varying the levels of

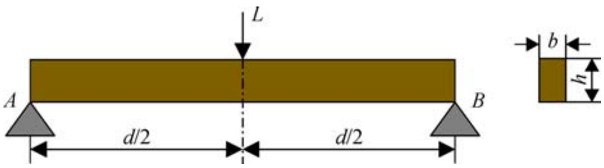

Fig. 5 A beam under testing condition

Load L, expressed by l, the component supplier estimates the probability of failure of the beam $p_f(l)$ and then provides the information to the system designers.

The user of the beam is a company (system designer) that uses the beam in their new system. The new condition of the beam in the system is shown in Fig. 6, which indicates that there are two new random loads acting at two different locations rather than the center of the beam. The task of the system designer is to re-evaluate the component reliability in the new condition without knowing the design details such as the distributions of the properties of materials (the allowable normal stress or shear stress).

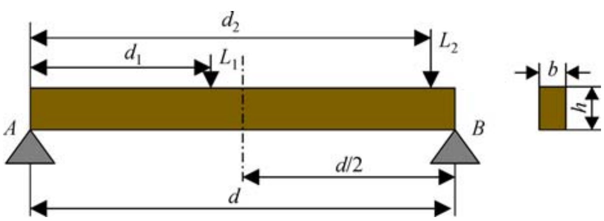

Fig. 6 The beam works in new condition

Next, we show how the component supplier evaluates the original component reliability and then how the system designer predicts the new component reliability in the new condition.

4.1 Initial reliability analysis by the component supplier

The design variables and material properties determined by the component supplier are given in Table 1, in which $S_{\text{strength},1}$ and $S_{\text{strength},2}$ denote the component resistances for bending stress and shear stress, respectively. And variables d, b, and h are deterministic parameters for the beam dimensions as shown in Fig. 5.

If a statistics-based approach is used, the beam designer applies the load to the center of the beam at different levels, expressed by l, and then record testing results. Then the statistics of the testing results are used to estimate the probability of failure of the beam, denoted by $p_f(l)$.

If a physics-based approach is used, the beam designer uses physics models to predict the state of the beam. The supporting forces at Points A and B are obtained using force analysis and given by

$$F_A = F_B = \frac{L}{2}. \tag{30}$$

Then the maximum normal stress T_1 and shear stress T_2 are given by

$$T_1 = \frac{Mh}{2I} = \frac{dh}{8I}L, \tag{31}$$

$$T_2 = \frac{VQ}{Ib} = \frac{h^2}{16I}L, \tag{32}$$

in which $I = \frac{bh^3}{12}$ is the moment of inertia, $M = \frac{Ld}{4}$ is the maximum moment, $V = F_B = \frac{L}{2}$ is the maximum shear stress, and $Q = b\frac{h}{2}\frac{h}{4}$ is the first moment of area.

According to the SSIT model in Eq. (4), the beam reliability is given by

$$R = \Pr\{S_{\text{strength},1} > T_1 \cap S_{\text{strength},2} > T_2\}. \tag{33}$$

After generalization based on Eq. (7), Eq. (33) is written as

$$R = \Pr\{S_1 > L \cap S_2 > L\}, \tag{34}$$

in which $S_1 = \frac{8I}{dh}S_{\text{strength},1}$ and $S_2 = \frac{16I}{h^2}S_{\text{strength},2}$. Thus, the generalized beam resistance is $S = \min(S_1, S_2)$, and the reliability is calculated by $R = \Pr\{S > L\}$, and then the probability of failure of the beam is $p_f = 1 - R$.

Using the above reliability analysis approaches, the beam designer obtains the reliability dataset in the form of limited values of $p_f(l)$ with respect to different load level l, denoted by $\left(l, p_f(l)\right)$ as shown in Table 2. Then the dataset is provided to the system designers.

4.2 New reliability prediction by system designers

Assume that the beam will be put into a new system under a new condition and will be subjected to two independent Loads L_1 and L_2 as shown in Fig. 6. The system designer has good knowledge of the new condition, such as the

Table 1 Beam design details

Variable	$S_{\text{strength},1}$ /MPa (normal)	$S_{\text{strength},2}$ /MPa (normal)	d/m (deterministic)	b/m (deterministic)	h/m (deterministic)
Mean	220	65	1.0	0.075	0.18
Standard deviation	30	10	–	–	–

assembly dimensions, the distributions of the new loads, and the locations to which the loads are applied. Details of the new condition are given in Table 3. Note that d_{AB} is the assembly dimension of the beam, which is equal to the length of the beam d shown in Table 1. The system designer does not have resources to perform reliability testing, and she or he chooses only a physics-based approach. She or he does not have the design details in Table 1 either.

The system designer derives the forces developed at Points A and B as follows:

$$F_1 = \frac{1}{d_{AB}}[L_1(d_{AB}-d_1)+L_2(d_{AB}-d_2)], \tag{35}$$

$$F_2 = \frac{1}{d_{AB}}(L_1 d_1 + L_2 d_2). \tag{36}$$

Since the original reliability is derived when Load L is applied to the center point, the system designer needs to find the equivalent Loads Z_1 and Z_2 with respect to the two failure modes. First, the maximum normal stress and maximum shear stress are calculated by

$$\sigma_{\max} = \frac{F_1 d_1 h}{2I}, \tag{37}$$

$$\tau_{\max} = \frac{F_2 Q}{Ib}. \tag{38}$$

Substituting Eqs. (35) and (36) into Eqs. (37) and (38), respectively, and replacing d_{AB} with d, they then have

$$\sigma_{\max} = \frac{dh}{8I}\frac{4d_1}{d^2}[L_1(d-d_1)+L_2(d-d_2)], \tag{39}$$

$$\tau_{\max} = \frac{h^2}{16I}\frac{2}{d}(L_1 d_1 + L_2 d_2). \tag{40}$$

After generalization based on Eqs. (39) and (40), the equivalent Loads Z_1 and Z_2 are given by

$$Z_1 = \frac{4d_1}{d^2}[L_1(d-d_1)+L_2(d-d_2)] = c_1^1 L_1 + c_2^1 L_2, \tag{41}$$

$$Z_2 = \frac{2}{d}(L_1 d_1 + L_2 d_2) = c_1^2 L_1 + c_2^2 L_2, \tag{42}$$

where $c_1^1 = \frac{4d_1}{d^2}(d-d_1)$, $c_2^1 = \frac{4d_1}{d^2}(d-d_2)$, $c_1^2 = \frac{2d_1}{d}$, and $c_2^2 = \frac{2d_2}{d}$. The generalized resistances S_1 and S_2 for the two failure modes maintain the same as that in Eq. (34), thus the reliability of the beam in new condition is calculated by

$$R = \Pr\{S_1 > Z_1 \cap S_2 > Z_2\}. \tag{43}$$

Comparing Eq. (34) with Eq. (43), we can find that Z_1 and Z_2 have the same effects on the component resistances S_1 and S_2 as Load L in the original condition. This is the reason that Z_1 and Z_2 are called equivalent loads in new conditions.

Then, using the reliability data in Table 2, the system designer finds the complete CDF of the component resistance S, which indicates the distribution of S, as shown in Fig. 7.

Table 2 Probabilities of failure of the beam at different load levels

Load/kN	p_f
100	2.000×10^{-7}
150	1.310×10^{-5}
200	6.328×10^{-4}
250	1.429×10^{-2}
300	0.123
350	0.448
400	0.815
450	0.973
500	0.998
550	1.000

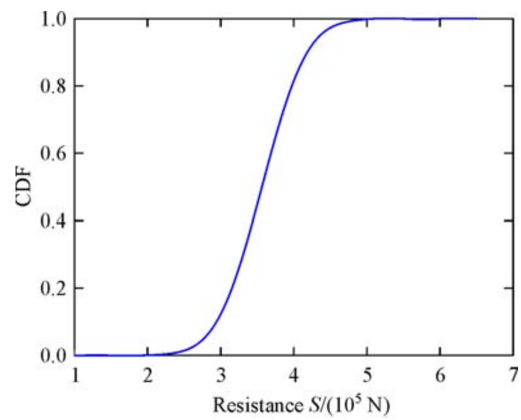

Fig. 7 The cumulative distribution function (CDF) of S

Since system designer also knows the distributions of L_1 and L_2, according to Eq. (20), $R_{\text{low}1}$ could be easily calculated using Monte Carlo simulation. Similarly, $R_{\text{up}1}$, $R_{\text{low}2}$ and $R_{\text{up}2}$ are obtained according to Eqs. (21), (23) and (24), respectively. Note that, L_1 and L_2 could be random variables of any distributions. For the special example discussed above, in which both L_1 and L_2 follow normal distributions, it is easy to find the mean values and standard deviations of Z_1 and Z_2, which are given by

$$\begin{cases} \mu_{Z_1} = c_1^1 \mu_{L_1} + c_2^1 \mu_{L_2} \\ \sigma_{Z_1} = \sqrt{(c_1^1 \sigma_{L_1})^2 + (c_2^1 \sigma_{L_2})^2} \end{cases}, \tag{44}$$

Table 3 Details of the new condition

Variable	L_1/kN (normal)	L_2/kN (normal)	d_1/m (deterministic)	d_2/m (deterministic)	d_{AB}/m (deterministic)
Mean	60	250	0.35	0.95	1.0
Standard deviation	6	20	–	–	–

$$\begin{cases} \mu_{Z_2} = c_1^2 \mu_{L_1} + c_2^2 \mu_{L_2} \\ \sigma_{Z_2} = \sqrt{(c_1^2 \sigma_{L_1})^2 + (c_2^2 \sigma_{L_2})^2} \end{cases}. \quad (45)$$

With all the detailed distributions of S, Z_1 and Z_2 available, the following results are obtained:

$$R_{\mathrm{low1}} = \Pr\{S > Z_1 \cap Z_1 > Z_2\} = 0.973425, \quad (46)$$

$$R_{\mathrm{up1}} = \Pr\{S > Z_2 \cap Z_1 > Z_2\} = 0.9997491, \quad (47)$$

$$R_{\mathrm{low2}} = \Pr\{S > Z_2 \cap Z_1 < Z_2\} = 5.27 \times 10^{-5}, \quad (48)$$

$$R_{\mathrm{up2}} = \Pr\{S > Z_1 \cap Z_1 < Z_2\} = 5.27 \times 10^{-5}. \quad (49)$$

Then, using Eqs. (27) and (28), the estimated lower bound and upper bound are given by

$$R_{\mathrm{low}} = R_{\mathrm{low1}} + R_{\mathrm{low2}} = 0.9734777, \quad (50)$$

$$R_{\mathrm{up}} = R_{\mathrm{up1}} + R_{\mathrm{up2}} = 0.9998018. \quad (51)$$

4.3 Final results

The predicted upper bound of the beam reliability is $R_{\mathrm{up}} = 0.9998018$, and the lower bound is $R_{\mathrm{low}} = 0.9734777$. We also calculate the true reliability value $R_{\mathrm{true}} = 0.9998018$, which is obtained using MCS method as if all the information of the beam in Table 1 and the details of loads in new conditions in Table 2 were known. R_{true} resides in the predicted reliability bounds [0.9734777, 0.9998018]. Compared with the true reliability value, R_{low} has an error of 2.63%, while R_{up} has the same value as R_{true}. The results indicate that it is possible to predict new component reliability in new working conditions with multiple new loads. In this example, the system designer does not need to know component details, such as the yield strength, the allowable shear stress, the length, and the dimensions of the cross-section.

5 Conclusions

This work presents a preliminary study on the re-evaluation of the reliability of a component that works in a new system with conditions different from the ones for which the component is originally designed. The new conditions are multiple new loads acting on the component. The re-evaluation is performed by system designers of the new product that uses the existing component. The proposed method helps system designers predict the new component reliability without knowing component design details, which are normally proprietary to the component supplier.

The case study involved in this work considers two new component loads. It demonstrates that it is possible to re-evaluate component reliability under a different environment without requiring detailed information from the component supplier. This is especially important for innovative product design. The proposed method can certainly help engineers manage and reduce risk during the innovative product design.

The full development of the methodology is our future work, including the extension from two component failure modes to more failure modes. More complicated working conditions with multiple types of loads such as moments or torques will also be investigated.

Acknowledgements This study is based upon work supported by the National Science Foundation (Grant No. CMMI 1562593) and Intelligent Systems Center at Missouri University of Science and Technology.

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