Front. Mech. Eng. 2019, 14(1): 21–32
https://doi.org/10.1007/s11465-018-0512-z

RESEARCH ARTICLE

Yimin ZHANG

Principle of maximum entropy for reliability analysis in the design of machine components

Abstract We studied the reliability of machine components with parameters that follow an arbitrary statistical distribution using the principle of maximum entropy (PME). We used PME to select the statistical distribution that best fits the available information. We also established a probability density function (PDF) and a failure probability model for the parameters of mechanical components using the concept of entropy and the PME. We obtained the first four moments of the state function for reliability analysis and design. Furthermore, we attained an estimate of the PDF with the fewest human bias factors using the PME. This function was used to calculate the reliability of the machine components, including a connecting rod, a vehicle half-shaft, a front axle, a rear axle housing, and a leaf spring, which have parameters that typically follow a non-normal distribution. Simulations were conducted for comparison. This study provides a design methodology for the reliability of mechanical components for practical engineering projects.

Keywords machine components, reliability, arbitrary distribution parameter, principle of maximum entropy

1 Introduction

Since humans first began to manufacture tools, several basic principles have been applied. Tools should be reliable, durable, and easily repairable in the event of breakage. This notion is the earliest concept of reliability. Reliability theory was developed during the Second World War as an integrated engineering discipline to address the problem of failures in military electronic components and equipment. Since then, machine reliability has achieved remarkable progress, and a considerable number of monographs, textbooks, and papers [1–14] have been published. Currently, reliability theory is widely used in the full life cycle (design, production, and maintenance) of machines in a variety of industrial sectors, such as aerospace, manufacturing, transportation, metals, petrochemicals, medical devices, and food processing. However, for various reasons, the industrial sectors in China have not adopted modern reliability engineering practices, particularly in the areas of machine design and research. Many sectors and industries in China would benefit from these practices. In machine design, reducing weight can not only save material and reduce costs but also lessen the energy consumed by machines and enhance their performance. In reducing the weight of a machine, the machine should be sufficiently reliable and safe. Thus, designing a machine to be lightweight involves designing for reliability. Improvements in reliability design methods result in the production of lightweight, high-performance, and high-quality machines.

The principle of maximum entropy (PME) is a method for selecting the statistical distribution of a random variable that best fits the available information. If only partial information regarding an unknown probability distribution is available, then the probability distribution with the maximum entropy that conforms to this information should be selected. This distribution is used for reliability analysis and the design of machine components. In other words, when only partial information is available in the PME, the most reasonable inference regarding the unknown distribution is the most uncertain or random inference (i.e., the inference with the highest entropy that conforms to the known information), and this inference is the only unbiased choice. Any other choices introduce other constraints and assumptions that cannot be known based on the available information. The probability distribution of a random variable is difficult to determine. In general, only certain parameters of the distribution, such as the mean and variance, or certain constraints (e.g., kurtosis and skewness) can be measured. More than one

Received October 23, 2017; accepted December 12, 2017

Yimin ZHANG (✉)
Equipment Reliability Institute, Shenyang University of Chemical Technology, Shenyang 110142, China
E-mail: ymzhang@mail.neu.edu.cn

probability distribution may fit these measured values, but only one of these probability distributions has the maximum entropy. The distribution with the maximum entropy has been investigated in previous studies [15–19]. Although this method is subjective to a certain extent, the distributions obtained using this method can be considered the best fit given the known information.

In this study, the statistical properties of various parameters of mechanical components were used to obtain a probability density function (PDF) of the state function and a reliability design model. A solution for the proposed model was derived, and the first four moments of the state function were obtained. An estimate of the PDF with the fewest human bias factors was obtained using the PME, and this estimate was used to calculate the reliability of the components. A procedure to calculate the reliability of components using entropy and the PME when the probability distribution of the state function is unknown is presented. A distribution function with the minimum bias was obtained using the PME, and an approach for reliability analysis and design of machine components was established. The proposed reliability design method was demonstrated with examples of mechanical components including a connecting rod, a vehicle half-shaft, a front axle, a rear axle housing, and a leaf spring. We established a practical, effective method for calculating the reliability of machine components. Calculating the reliability of a component can save large amounts of human, material, and financial resources; improve the design; shorten the design cycle; increase the quality; reduce energy consumption and costs; reduce weight; and improve the reliability.

2 Principle of maximum entropy

The PME in the analysis of machine component reliability is based on the concept of entropy. The maximum entropy most accurately represents the state of knowledge. When calculating the reliability of a component, the probability distribution with the minimum bias for known constraints is the distribution with the maximum entropy. In the fields of statistics, physics, and engineering, problems are often solved based on measured data and certain conditions or assumptions. In essence, the PME is a deduction from a simple axiom: Events with a high probability of occurrence can easily occur. In general, three properties are of concern for the solution to a problem, namely, existence, uniqueness, and stability. If one or more of these three properties does not hold, then the problem is an uncertain one. These uncertain problems result from incomplete data, complete data with noise, or incomplete data with noise. Various methods are available for solving uncertain problems, and the PME is one method. We selected the maximum entropy solution because in the case of incomplete information, the solution must fit the known data with the fewest number of assumptions possible regarding the unknown part; i.e., the most scientifically transcendent detached perspective should be adopted for data extrapolation and interpretation. Finding the solution can be viewed as extracting information from the data, and the extractable information originates from two sources: The known data and the assumptions made on the unknown part because of the limitations of the known data. These assumptions are equivalent to the insertion of additional information. The condition of maximum entropy represents the least amount of known information, and certainty in the known data requires the minimum amount of added information. Therefore, the maximum entropy solution is a transcendent solution. For cases in which only measured data are available, if no justification is available for selecting a particular distribution function, the distribution (the form and the parameters) with the minimum bias is determined based on the PME. The PME states that when selecting the probability distribution of a random event, all of the known conditions should be satisfied, and no subjective assumptions should be made regarding the unknown characteristics. This principle yields the most even probability distribution and the lowest prediction risk because it maximizes the entropy in the result. Therefore, a model based on the PME is referred to as a maximum entropy model.

2.1 Entropy

A model with maximum entropy preserves all uncertainties, and the risk is minimized. Entropy is a fuzzy concept. Fuzzy concepts generally possess a certain arbitrariness, but they are derived in a systematic manner. If the definition of entropy is arbitrary, then entropy is a simple concept that originates from another arbitrary concept, information. In information theory, the term “information” is used in a narrow sense and is defined as follows:

$$I(x) = -\log f(x), \tag{1}$$

where x represents an event with a given value, and $I(x)$ represents a measurement of the information relevant to the event with a given value. In Eq. (1), the base of the logarithm affects the value of the information function. If the base of the logarithm is 2, the units of $I(x)$ are bit, and if the base of the logarithm is e, the units of $I(x)$ are nat.

Entropy is defined as the mean value of the information. For a continuous variable, entropy (S) is expressed in the following form:

$$S[f(x)] = -\int_R f(x)\log[f(x)]\mathrm{d}x. \tag{2}$$

Information is a measurement of the uncertainty of an individual value of x, and entropy is a measurement of the uncertainty of the value of x within the entire range. High uncertainty means large entropy.

2.2 Statistical moment of a sample

If no rationale exists for selecting a particular distribution function, then the form of the distribution with the minimum bias based on the distribution of the variable in question must be selected. The moments of a sample can be calculated using a simple method based on the information of the sample. The following are the moments for a continuous random variable:

$$S = -\int_R f(x)\ln[f(x)]\mathrm{d}x = \text{maximum}, \tag{3}$$

$$\int_R f(x)\mathrm{d}x = 1, \tag{4}$$

$$\int_R x^i f(x)\mathrm{d}x = m_i, \quad i = 1,2,...,n, \tag{5}$$

where n represents the order of the moment, and m_i represents the ith-order moment about the origin, where the numerical values are determined from the sample.

From the perspective of entropy as the measurement of system uncertainty, equiprobable systems have the highest uncertainty. A PDF itself is a special analytical form. The method of Lagrange multipliers is employed to find the optimal solution, and the entropy is maximized by adjusting $f(x)$.

Let J be the Lagrange function, where λ_0, λ_1, ..., λ_n are Lagrange multipliers. Then, we have

$$J = S + (\lambda_0 + 1)\left[\int_R f(x)\mathrm{d}x - 1\right] + \sum_{i=1}^{n}\lambda_i\left[\int_R x^i f(x)\mathrm{d}x - m_i\right]. \tag{6}$$

In practice, to simplify the result, the multiplier (λ_0+ 1) is used rather than λ_0. We set the derivative $\mathrm{d}J/\mathrm{d}f(x)$ equal to 0. Then, we have

$$-\int_R\{\ln[f(x)] + 1\}\mathrm{d}x - (\lambda_0 + 1)\int_R \mathrm{d}x - \sum_{i=1}^{n}\lambda_i\left(\int_R x^i\mathrm{d}x\right) = 0. \tag{7}$$

By merging the terms inside the integral, we have

$$\ln[f(x)] = \lambda_0 + \sum_{i=1}^{n}\lambda_i x^i, \tag{8}$$

or

$$f(x) = \exp\left(\lambda_0 + \sum_{i=1}^{n}\lambda_i x^i\right). \tag{9}$$

Equations (8) and (9) are the analytical forms of the maximum entropy density function. The remaining problem is to determine the value of each λ_i.

To obtain these values, two equations are required. These two equations are obtained as follows. Substituting Eq. (9) into Eq. (4), we have

$$\int_R \exp\left(\lambda_0 + \sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x = 1. \tag{10}$$

Multiplying Eq. (10) by $\mathrm{e}^{-\lambda_0}$, we have

$$\mathrm{e}^{-\lambda_0} = \int_R \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x. \tag{11}$$

Thus, the first required equation is obtained:

$$\lambda_0 = -\ln\int_R \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x. \tag{12}$$

To obtain the second equation, Eq. (11) is differentiated with respect to λ_i

$$\frac{\partial\lambda_0}{\partial\lambda_i} = -\int_R x^i \exp\left(\lambda_0 + \sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x. \tag{13}$$

From Eqs. (5) and (9), the second equation is then obtained:

$$\frac{\partial\lambda_0}{\partial\lambda_i} = -m_i. \tag{14}$$

To obtain the value of each λ_i, a system of simultaneous equations is required. By differentiating Eq. (12) with respect to λ_i, we have

$$\frac{\partial\lambda_0}{\partial\lambda_i} = -\frac{\int_R x^i \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x}{\int_R \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x}. \tag{15}$$

Equation (15) can be solved for λ_0,λ_1,...,λ_n. From the solution, λ_0 can be calculated using Eq. (12). To simplify the calculation, Eq. (15) is rewritten as follows:

$$1 - \frac{\int_R x^i \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x}{m_i\int_R \exp\left(\sum_{i=1}^{n}\lambda_i x^i\right)\mathrm{d}x} = \varepsilon_i, \tag{16}$$

where ε_i represents the residual, which can be reduced to approximately 0. By calculating the minimum of the sum of the squared residuals, we can obtain the solution to the problem:

$$\varepsilon = \sum_{i=1}^{n} \varepsilon_i^2 = \text{minimum}, \tag{17}$$

where the solution has converged if $\varepsilon < \delta$ or $|\varepsilon_i| < \delta$ for all i (where δ is the specified allowable error). Equation (12) is used to calculate λ_0, and the integral in Eq. (16) can be calculated using numerical methods.

3 Calculation of the reliability of mechanical components

In modern manufacturing, product quality and reliability are becoming increasingly important for market competitiveness, and reliability is one of the most important performance metrics for machines. The basic task of machine reliability design is to use mathematical models, methods, and practical guidance for the design of components based on mathematics, physics, materials science, and mechanical engineering in combination with testing and statistical analyses of failure data. Using this approach, the operating performance and the state or service life of a machine under specified operating conditions can be estimated or predicted in the development stage to ensure that the reliability of the machine will meet the requirements. To enhance the reliability of a machine, the reliability of the components must be improved first. Similar to other properties, reliability has to be designed into the machine and ensured through production and maintenance. When designing a machine, the statistical variation of its basic parameters should be considered, and uncertainty analysis must be performed to obtain an accurate assessment of the actual conditions and ensure that the design operating performance is consistent with the actual operating performance. The PME is a practical, effective reliability analysis and design method and is generally applicable to the design of machine components. Thus, its use can be expanded to other industrial sectors.

The following steps provide an algorithm for calculating the reliability of components for design and analysis:

1) Obtain the mean value, the variance, and the third and fourth moments of each random variable;

2) Calculate the mean value, the variance, and the third and fourth moments of the state function;

3) Obtain the first four moments and the boundary values of the state function and select an initial value determination method;

4) Calculate the values of the Lagrange multipliers λ_0, λ_1, ..., λ_n;

5) Obtain the expression of the PDF for the state function;

6) Calculate the reliability of the component;

7) Compare the value obtained in Step 6) with that obtained using the Monte Carlo method.

Figure 1 show the flow chart of the PME algorithm to compute the reliability of a component.

4 Numerical examples

The objective of the study is to illustrate the applications of the proposed method for reliability analysis of machine components. Several examples are considered [20–23].

4.1 Reliability of a connecting rod

A connecting rod (Fig. 2) is generally forged from medium carbon steel or alloy steel and consists of a shank and a hole at each end, one for a pin and the other for a crank. Failure analysis of connecting rods revealed that connecting rods typically fail by tensile fracture. Connecting rods can have a variety of cross sections (e.g., circular, rectangular, and "I"). In this section, a connecting rod with an "I" cross section is analyzed.

The tensile stress (σ) on the connecting rod is

$$\sigma = \frac{F}{a(h-2t)+2bt}, \tag{18}$$

where F represents the maximum tensile force, and a, b, h, and t are the dimensions of the cross section.

In the stress-strength interference method, the state function ($g(\boldsymbol{X})$) for the ultimate state of stress is as follows:

$$g(\boldsymbol{X}) = r - \sigma, \tag{19}$$

where r represents the material strength of the connecting rod. The random variables are $\boldsymbol{X}=(r\ F\ a\ t\ h\ b)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

The material strength r of a connecting rod is $(\mu_r,\sigma_r) = (235, 12.92)$ MPa. The geometry sizes are $(\mu_a,\sigma_a)=(14, 0.23)$ mm, $(\mu_t,\sigma_t)=(27.5, 0.28)$ mm, $(\mu_h,\sigma_h)=(140, 0.53)$ mm, $(\mu_b,\sigma_b)=(96, 0.47)$ mm. The load F is $(\mu_F,\sigma_F,C_3(F), C_4(F))=(4.67\times10^5\ \mathrm{N}, 3.11\times10^4\ \mathrm{N}, 3.34\times10^{13}\ \mathrm{N}^3, 4.86\times10^{18}\ \mathrm{N}^4)$ N. We attempted to determine the reliability of the connecting rod.

We computed the reliability as follows.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 162.8207,\ m_2 = 190.4317,$$

$$m_3 = -123.3200,\ m_4 = 2773.4420.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

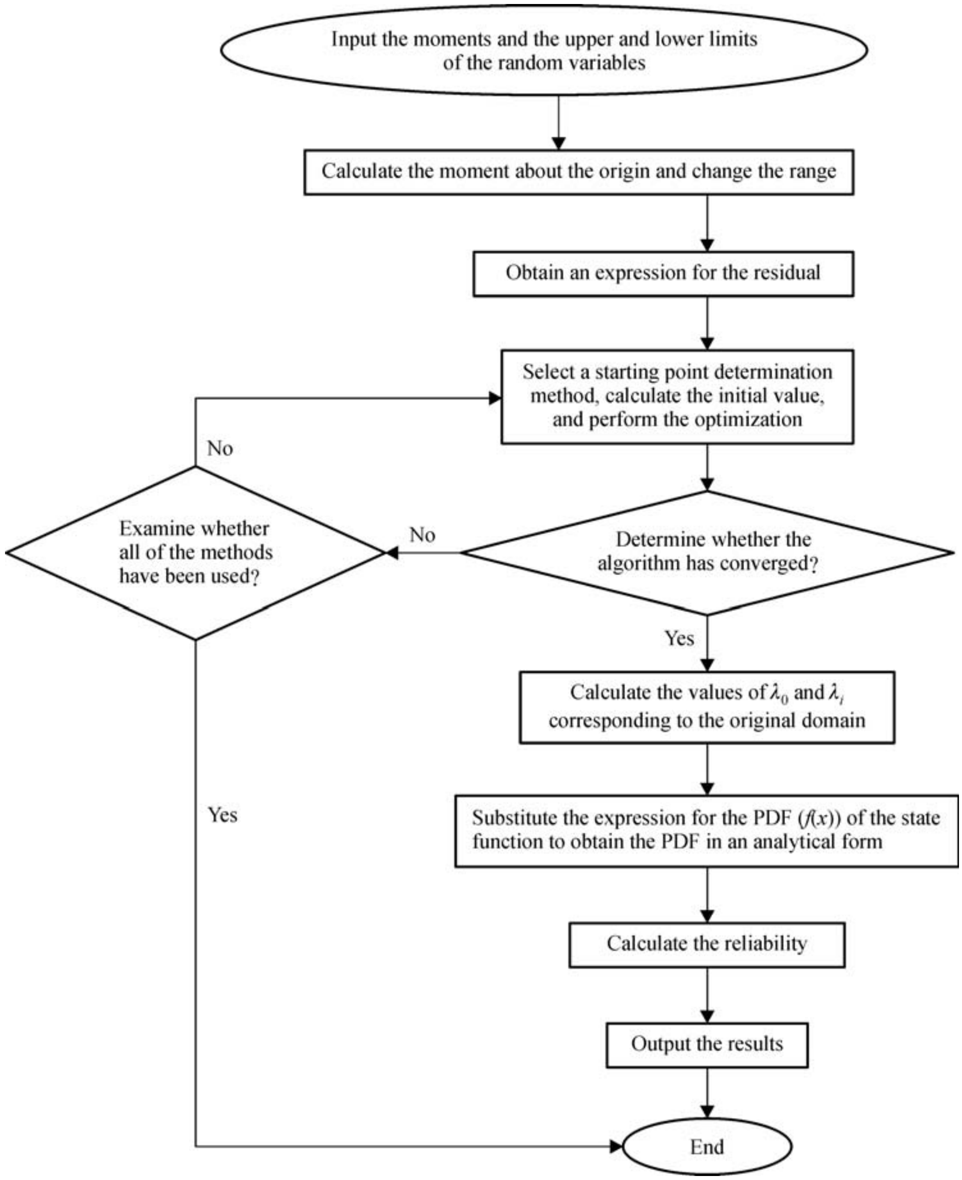

Fig. 1 Computational diagram of the maximum entropy method

$$\lambda_0 = -0.14693 \times 10^2,\ \lambda_1 = 0.21271 \times 10^{-1},$$

$$\lambda_2 = 0.12808 \times 10^{-2},\ \lambda_3 = -0.82169 \times 10^{-5},$$

$$\lambda_4 = 0.12203 \times 10^7.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.14693 \times 10^2 + 0.21271 \times 10^{-1}x + 0.12808 \times 10^{-2}x^2 - 0.82169 \times 10^{-5}x^3 + 0.12203 \times 10^{-7}x^4).$$

4) We calculated the reliability of the connecting rod as

$$R = 0.99999.$$

We approximated the reliability of the connecting rod using the Monte Carlo method $R_{\mathrm{MCS}} = 1.0$.

4.2 Reliability of a half-shaft

A half-shaft, or an axle shaft, is a solid shaft that transmits power from the differential to the drive wheels in vehicles, such as automobiles and trucks. The two half-shafts must be able to rotate independently because the left and right drive wheels should rotate at different speeds (Fig. 3). The primary task of the half-shafts is to transmit torque. The three main configurations of axles that use half-shafts are

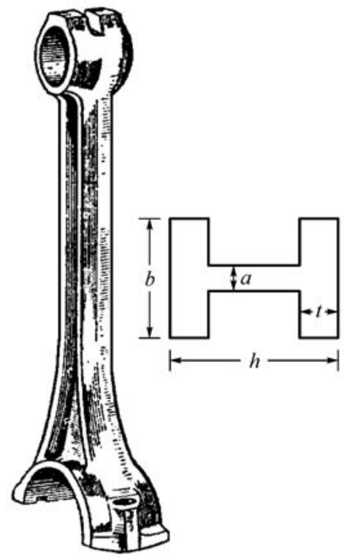

Fig. 2 Connecting rod

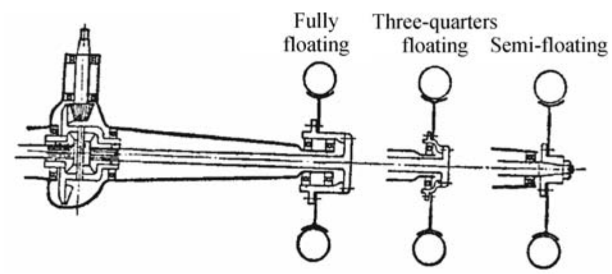

Fig. 3 Half-shaft

fully floating, semifloating, and 3/4 floating. In the second and third types, the half-shafts and axle housing constitute a structural member that bears some portion of the weight and the lateral force in addition to the torque. As a result, in addition to the torque exerted by the engine, the half-shafts are also subjected to bending moments.

In the fully floating type, the half-shafts are subjected only to torsion. The torsional stress on this type of half-shaft is

$$\tau = \frac{16T}{\pi d^3}, \tag{20}$$

where T represents the torque transmitted by the half-shaft, and d represents the diameter of the half-shaft.

From the stress-strength interference method, the state function expressed by the ultimate stress is as follows:

$$g(\boldsymbol{X}) = r - \frac{16T}{\pi d^3}, \tag{21}$$

where r represents the torsional strength of the shaft material. The random variables are $\boldsymbol{X}=(r\ T\ d)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

A half-shaft is effected torsional moment T that is $(T) = (1.1769\times10^7$ N·mm, 9.8227×10^5 N·mm, 1.0722×10^{18} (N·mm)^3, 5.0473×10^{24} (N·mm)^4). The material strength r is $(\mu_r,\sigma_r) = (1050, 40)$ MPa. The current design diameter is $(\mu_d,\sigma_d) = (42, 0.21)$ mm. We attempted to calculate the reliability of the half shaft.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 240.9752,\ m_2 = 6306.6520,$$

$$m_3 = -348291.3000,\ m_4 = 112706300.0000.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

$$\lambda_0 = -0.89630 \times 10,\ \lambda_1 = 0.19034 \times 10^{-1},$$

$$\lambda_2 = 0.40242 \times 10^{-4},\ \lambda_3 = -0.28511 \times 10^{-6},$$

$$\lambda_4 = 0.22379 \times 10^{-9}.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.89630 \times 10 + 0.19034 \times 10^{-1}x$$
$$+ 0.40242 \times 10^{-4}x^2 - 0.28511 \times 10^{-6}x^3$$
$$+ 0.22379 \times 10^{-9}x^4).$$

4) We calculated the reliability of the connecting rod

$$R = 0.99163.$$

We approximated the reliability of the connecting rod using the Monte Carlo method

$$R_{\mathrm{MCS}} = 0.99238.$$

4.3 Reliability of a front axle

The front axle (Fig. 4) of an automobile is mounted underneath the vehicle frame, and the front wheels are mounted at each of the two ends of the front axle. The front axle is connected by the suspension to the frame (or the monocoque body). The front axle is often low in the middle portion to provide clearance from the oil pan of the engine.

To reduce the amount of material and maintain approximately uniform strength at all points, an H-beam structure is often used in the middle portion of the front axle to transmit the forces and moments between the two wheels and the two springs. As a result, the front axle is

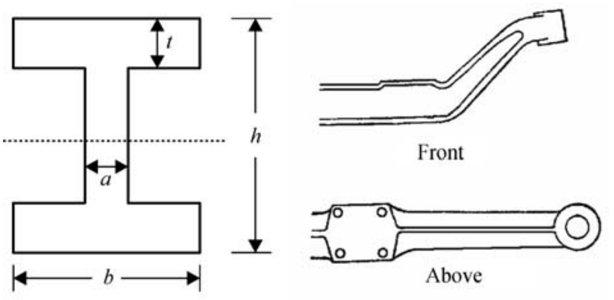

Fig. 4 Fore-axle

subjected to the combined actions of bending and torsion.

The cross-sectional coefficient (W_x) of the front axle is

$$W_x = \frac{a(h-2t)^3}{6h} + \frac{b}{6h}\left[h^3-(h-2t)^3\right], \tag{22}$$

and the polar cross-sectional coefficient is

$$W_\rho = 0.8bt^2 + 0.4\frac{(h-2t)a^3}{t}. \tag{23}$$

The maximum normal stress and the maximum shear stress at the high-stress point are

$$s = \frac{M}{W_x}, \tag{24}$$

$$\tau = \frac{T}{W_\rho}, \tag{25}$$

where M and T represent the bending moment and torque, respectively. In accordance with fourth strength theorem, the resulting stress on the front axle is

$$\sigma = \sqrt{s^2 + 3\tau^2}. \tag{26}$$

The state equation for the ultimate state of stress is

$$g(\boldsymbol{X}) = r-\sigma, \tag{27}$$

where r represents the material strength of the front axle. The random variables are $\boldsymbol{X}=(r\ M\ T\ a\ t\ h\ b)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

The fore-axle of a vehicle affects torsional moment and bending moment. The torsional moment T is $(T)=(3.0834\times10^6$ N·mm, 2.5017×10^5 N·mm, 1.806×10^{16} (N·mm)^3, 2.135×10^{22} (N·mm)^4). The bending moment M is $(M)=(3.5192\times10^6$ N·mm, 3.1961×10^5 N·mm, 3.6947×10^{16} (N·mm)^3, 5.5364×10^{22} (N·mm)^4). The geometry sizes of risk section are $(\mu_a,\sigma_a)=(12,\ 0.06)$ mm, $(\mu_t,\sigma_t)=(14,\ 0.07)$ mm, $(\mu_h,\sigma_h)=(80,\ 0.4)$ mm, and $(\mu_b,\sigma_b)=(60,\ 0.3)$ mm. The material strength is $(\mu_r,\sigma_r)=(667,\ 25.3)$ MPa. We attempted to determine the reliability of the front axle.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 215.4988,\ m_2 = 1931.8930,$$

$$m_3 = -52660.16,\ m_4 = 8893478.00.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

$$\lambda_0 = -0.89004 \times 10,\ \lambda_1 = -0.46183 \times 10^{-1},$$

$$\lambda_2 = 0.82709 \times 10^{-3},\ \lambda_3 = -0.30545 \times 10^{-5},$$

$$\lambda_4 = 0.29611 \times 10^{-8}.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.89004 \times 10 - 0.46183 \times 10^{-1}x + 0.82709 \times 10^{-3}x^2 - 0.30545 \times 10^{-5}x^3 + 0.29611 \times 10^{-8}x^4).$$

4) We calculated the reliability of the connecting rod:

$$R = 0.99963.$$

We approximated the reliability of the connecting rod using the Monte Carlo method:

$$R_{\mathrm{MCS}} = 0.99969.$$

4.4 Reliability of a rear axle housing

The rear axle housing (Fig. 5) of a vehicle supports and protects the final reduction drive and differential. The axle housing is both a power-transmission component and a load-bearing component of the drive axle and ensures that the relative positions of the left and right wheel axles are fixed. In addition, the axle housing bears the weight of the vehicle and transmits various forces on the wheels to the vehicle body or the frame through the suspension. Therefore, the axle housing must provide sufficient strength and stiffness. The weight of the axle housing can be reduced to improve the ride performance of the vehicle. The axle housing can be easily produced, assembled, dissembled, and repaired. The three main types of axle housings are banjo, split, and unitized carrier.

In general, the rear axle housing is subjected to a combination of bending and torsion. Under normal circumstances, the high-stress points of the housing are located at the bases of the leaf springs and the flange fillets. The cross sections of the rear axle housing at these

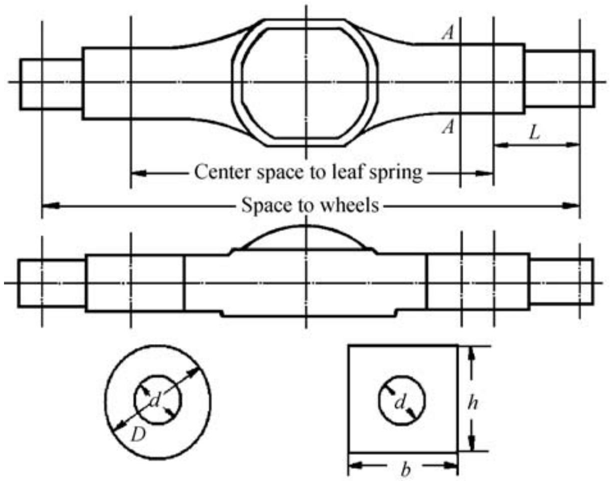

Fig. 5 Rear-axle housing

locations are essentially tubular. Therefore, an equation for the reliability of the rear axle housing is derived for an annular cross section and for a rectangular cross section with a hollow circular center.

The housing with an annular cross section is subjected to bending stress and torsional stress

$$s = \frac{32DM}{\pi(D^4 - d^4)}, \tag{28}$$

$$\tau = \frac{16DT}{\pi(D^4 - d^4)}, \tag{29}$$

where M and T represent the bending moment and the torque, respectively, and D and d represent the outer and inner diameters of the axle tube at the high-stress cross section, respectively. In accordance with fourth strength theorem, the resulting stress at the high-stress points of the housing is

$$\sigma = \sqrt{s^2 + 3\tau^2} = \frac{32D}{\pi(D^4 - d^4)}\sqrt{M^2 + 0.75T^2}. \tag{30}$$

The housing with a rectangular cross section is subjected mainly to bending, and the bending stress is

$$\sigma = \frac{M}{W_n}, \tag{31}$$

where M represents the bending moment, and W_n represents the cross-sectional bending resistance coefficient, which is determined using the following equation:

$$W_n = \frac{bh^2}{6}\left(1 - 0.59\frac{d^4}{bh^3}\right), \tag{32}$$

where d represents the diameter of the inner opening at the high-stress cross section, and b and h represent the dimensions of the outer rectangle at the high-stress cross section.

In the stress-strength interference method, the state equation for the ultimate state of stress is as follows:

$$g(\boldsymbol{X}) = r - \sigma, \tag{33}$$

where r represents the material strength of the housing. The random variables for the housing with an annular cross section are $\boldsymbol{X} = (r\ M\ T\ D\ d)^{\mathrm{T}}$, and the random variables for the housing with a rectangular cross-section center are $\boldsymbol{X} = (r\ M\ b\ h\ d)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

The torsional moment T is $(T) = (4.4868\times10^6$ N·mm, 3.7442×10^5 N·mm, -4.1743×10^{16} (N·mm)^3, 7.9139×10^{22} (N·mm)^4). The bending moment M on risk section is $(M) = (6.3809\times10^6$ N·mm, 6.0319×10^5 N·mm, -1.6181×10^{17} (N·mm)^3, 5.0766×10^{23} (N·mm)^4). The inside diameter d and outside diameter D of the risk section are $(\mu_d,\sigma_d) = (74,\ 0.37)$ and $(\mu_D,\sigma_D) = (84,\ 0.42)$ mm, respectively. The material strength r is $(\mu_r,\sigma_r) = (443,\ 27.5)$ MPa. We attempted to determine the reliability of the rear axle housing.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 120.1691,\ m_2 = 1614.3290,$$

$$m_3 = 8442.4190,\ m_4 = 953281.4000.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

$$\lambda_0 = -0.91056 \times 10,\ \lambda_1 = 0.76910 \times 10^{-1},$$

$$\lambda_2 = -0.35288 \times 10^{-3},\ \lambda_3 = 0.24788 \times 10^{-6},$$

$$\lambda_4 = -0.44315 \times 10^{-9}.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.91056 \times 10 + 0.76910 \times 10^{-1}x$$
$$-0.35288 \times 10^{-3}x^2 + 0.24788 \times 10^{-6}x^3$$
$$-0.44315 \times 10^{-9}x^4).$$

4) We calculated the reliability of the connecting rod:

$$R = 0.99869.$$

We approximated the reliability of the connecting rod using the Monte Carlo method:

$$R_{\mathrm{MCS}} = 0.99863.$$

The bending moment M on risk section is $(M)=(1.0976\times10^8$ N·mm, 1.0367×10^7 N·mm, -8.1367×10^{20} (N·mm)^3, 4.4569×10^{28} (N·mm)^4). The inside diameter d of the risk section is (μ_d,σ_d)=(103, 0.515) mm. The sizes b and h of quadrate section are $(\mu_b,\sigma_b)=(123, 0.615)$ and $(\mu_h,\sigma_h)=(143, 0.715)$ mm, respectively. The material strength r is $(\mu_r,\sigma_r)=(433, 27.5)$ MPa. We attempted to determine the reliability of the rear axle housing.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 111.8853,\ m_2 = 1700.6420,$$

$$m_3 = 20374.9400,\ m_4 = 3265104.0000.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

$$\lambda_0 = -0.85830 \times 10,\ \lambda_1 = 0.81302 \times 10^{-1},$$

$$\lambda_2 = -0.53501 \times 10^{-3},\ \lambda_3 = 0.13620 \times 10^{-5},$$

$$\lambda_4 = -0.249306 \times 10^{-8}.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.85830 \times 10 + 0.81302 \times 10^{-1}x$$

$$-0.53501 \times 10^{-3}x^2 + 0.13620 \times 10^{-5}x^3$$

$$-0.249306 \times 10^{-8}x^4).$$

4) We calculated the reliability of the connecting rod:

$$R = 0.99798.$$

We approximated the reliability of the connecting rod using the Monte Carlo method:

$$R_{\mathrm{MCS}} = 0.99942.$$

4.5 Reliability of a leaf spring

A leaf spring (Fig. 6) is an elastic beam with approximately uniform strength and is composed of one or more spring steel plates of various lengths. The load borne by the rear suspension of a truck often varies over a very wide range. Therefore, the suspension is required to have variable stiffness. To achieve this condition, a small leaf spring (referred to as an auxiliary spring) is often installed above the main leaf spring. With this arrangement, the stiffness of the suspension is greater under the fully loaded condition than under the no-load condition.

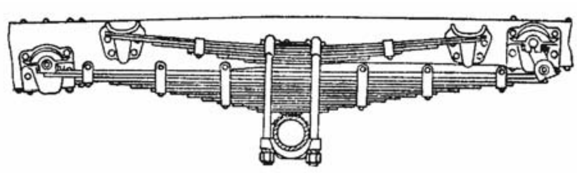

Fig. 6 Multileaf spring

4.5.1 Reliability of a single-plate leaf spring

A single-plate leaf spring is an elastic beam with a longitudinally varying cross section. The advantages of this type of spring are that it requires much less steel to produce, it is light in weight, and it improves the ride performance of the vehicle (because interplate friction, which is a problem in multiplate leaf springs, does not occur, so the single-plate leaf spring has remarkable longevity). A single-plate leaf spring is generally 40%–50% lighter than a multiplate leaf spring with the same service life. Several problems with single-plate leaf springs (e.g., complex production processes, high cost, and safety) must be addressed. However, single-plate leaf springs are expected to become common as the design improves. Another type of leaf spring composed of a small number (2–4) of steel plates is also in use. This type of leaf spring has characteristics similar to those of a single-plate leaf spring and has been used extensively in other countries.

The stress on a single-plate leaf spring is

$$\sigma = \frac{3Pl}{2bh^2}, \tag{34}$$

where P represents the load, b represents the width of the leaf spring, h represents the thickness of the leaf spring in the design area, and l represents the span of the leaf spring.

In the stress-strength interference method, the state equation for the ultimate state of stress is as follows:

$$g(\boldsymbol{X}) = r-\frac{3Pl}{2bh^2}, \tag{35}$$

where r represents the material strength of the leaf spring. The random variables are $\boldsymbol{X}=(r\ P\ l\ b\ h)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

4.5.2 Reliability of a multiplate leaf spring

The leaf springs in various machines are mostly simply supported laminated plates that bear loads in the middle section. Given a certain width, b, a steel plate is cut into several narrow plates, which are then stacked to form a multiplate leaf spring. The stress on a multiplate leaf spring is

$$\sigma = \frac{3Pl}{bnh^2}, \tag{36}$$

where P represents the load, b, h, and l represent the width, thickness, and length of the leaf spring, respectively, and n represents the number of steel plates comprising the leaf spring. Strictly speaking, the effect of friction between the laminated plates on the stress should be considered, but it was ignored in this analysis. This assumption is commonly used to design leaf springs for automobiles and trolley cars.

In the stress-strength interference method, the state equation for the ultimate state of stress is as follows:

$$g(\boldsymbol{X}) = r - \frac{3Pl}{2bnh^2}, \tag{37}$$

where r represents the material strength of the leaf spring. The random variables are $\boldsymbol{X}=(r\ P\ l\ b\ h)^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

4.5.3 Reliability of a multiplate leaf spring composed of steel plates with different thicknesses

Currently, reliability design methods for leaf springs are limited to single-plate leaf springs and multiplate leaf springs composed of steel plates with the same thickness. An equation for calculating the reliability of a multiplate leaf spring composed of steel plates with different thicknesses and a reliability design method for this type of leaf spring are derived. Under the condition that the statistical parameters of the random variables are known, the PME can be used to directly design reliable leaf springs with numerical methods. In this manner, accurate reliability information for leaf springs can be readily obtained.

Multiplate leaf springs are mostly simply supported laminated springs that bear loads in the middle section. Given a certain width, b, a steel plate is cut into several narrow plates that are then stacked to form a multiplate leaf spring. The equal-stress beam equation is often used to calculate the vertical load on a leaf spring. The stress in the vertical direction on a leaf spring is

$$\sigma = \frac{3Pl}{2b}\frac{h_i}{n_1h_1^3+n_2h_2^3+...+n_mh_m^3}. \tag{38}$$

The thickest plate bears the largest stress, which is

$$\sigma_{\max} = \frac{3Pl}{2b}\frac{h_{\max}}{n_1h_1^3+n_2h_2^3+...+n_mh_m^3}. \tag{39}$$

The steel plates comprising the spring are assumed to have the following proportional relationship:

$$h_1=\alpha_1h_{\max},\ h_2=\alpha_2h_{\max},\ ...,\ h_m=\alpha_mh_{\max}. \tag{40}$$

Thus, we have

$$\sigma_{\max} = \frac{3Pl}{2bh_{\max}^2}\frac{1}{n_1\alpha_1^3+n_2\alpha_2^3+...+n_m\alpha_m^3}. \tag{41}$$

where P represents the load, b, h_i, and l represent the width, thickness, and span of the leaf spring, respectively, and n_i represents the number of steel plates with a thickness of h_i.

In the stress-strength interference method, the state equation for the ultimate state of stress is as follows:

$$g(\boldsymbol{X}) = r - \sigma_{\max}, \tag{42}$$

where r represents the material strength of the leaf spring. The random variables are $\boldsymbol{X}=(r\ P\ l\ b\ h_{\max})^{\mathrm{T}}$. The mean value ($E(\boldsymbol{X})$), the variance ($Var(\boldsymbol{X})$), the third moment ($C_3(\boldsymbol{X})$), and the fourth moment ($C_4(\boldsymbol{X})$) of $\boldsymbol{X}$ are known. The random variables are independent of one another, but the distribution of $\boldsymbol{X}$ is unknown.

The geometrical sizes of the composite springs are $(\mu_b,\sigma_b)=(90,\ 0.45)$ mm, $(\mu_l,\sigma_l)=(1475.5,\ 7.375)$ mm, $(\mu_{h_1},\sigma_{h_1})=(11,\ 0.055)$ mm, $(\mu_{h_2},\sigma_{h_2})=(10,\ 0.05)$ mm, and $(\mu_{h_3},\sigma_{h3})=(9,\ 0.045)$ mm. The number of panels is $n_1=2$, $n_2=6$, $n_3=4$. The load P is $(P)=(1.6567\times10^4$ N, 8.2516×10^2 N, 6.7288×10^8 N^3, 2.6608×10^{12} N$^4)$. The material strength r is $(\mu_r,\sigma_r)=(614,\ 45.8)$ MPa. We attempted to determine the reliability of the leaf spring.

1) We obtained the mean value, the variance, and the third and fourth moments of each random variable and the first four moments of the state function:

$$m_1 = 226.9287,\ m_2 = 2502.9840,$$

$$m_3 = -8581.8110,\ m_4 = 792867.4000.$$

2) We used the PME to determine the coefficient of the distribution function of the state function:

$$\lambda_0 = -0.14364\times10^2,\ \lambda_1 = 0.82985\times10^{-1},$$

$$\lambda_2 = -0.17909\times10^{-3},\ \lambda_3 = -0.69550\times10^{-9},$$

$$\lambda_4 = -0.34835\times10^{-10}.$$

3) We obtained an analytical expression for the PDF of the state function:

$$f(x) = \exp(-0.14364\times10^2+0.82985\times10^{-1}x$$

$$-0.17909\times10^{-3}x^2-0.69550\times10^{-9}x^3$$

$$-0.34835\times10^{-10}x^4).$$

4) We calculated the reliability of the connecting rod:

$$R = 0.99999.$$

We approximated the reliability of the connecting rod using the Monte Carlo method:

$$R_{\mathrm{MCS}} = 0.99988.$$

5 Conclusions

Reliability analysis and design theory are receiving increasing attention from all industries and are an important area of research, both in academia and industry. The method developed in this study allows designers to set reasonable margins of safety for machines and evaluate the effects of random parameters on reliability, thereby ensuring that the predicted operating performance of a design will be accurate. In addition, with this method, sufficient safety and reliability and economic efficiency can be obtained. The PME reliability design method for mechanical components proposed in this study uses the PME to obtain an estimate of the PDF of the state function with the least human bias under the condition that the statistical parameters of the random variables are known, and this PDF is used to calculate the reliability of mechanical components. A theoretical derivation and numerical results demonstrate that the proposed method is practical and reliable. The proposed method was used to calculate the reliability of various mechanical components including a connecting rod, a half-shaft, a front axle, a rear axle housing, and several types of leaf springs and quickly obtain accurate reliability data. By calculating and analyzing the reliability of mechanical components based on entropy and the PME, the PME approach for machine reliability demonstrated the following characteristics: 1) The PDF generated from the PME is a theoretical model with an explicit mathematical expression, and a simple algorithm that can be readily implemented on a computer can be derived. 2) The PDF of a random variable determined using the PME has a relatively high goodness of fit when a relatively large number of samples are available but a relatively low goodness of fit when only a relatively small number of samples are available. 3) No special requirements are needed for the distribution of the random variables when using the PME to analyze the reliability of a component. The PME has relatively high accuracy when used to calculate the reliability of a component with random parameters that follow a non-normal distribution. 4) A difference is observed between the results obtained using the PME and Monte Carlo method. This deviation mainly occurs because different upper and lower boundary values are used in the PME program when estimating the PDF of the state function, resulting in a slightly different value of the coefficient from that produced by the Monte Carlo method, which in turn results in a slightly different distribution for the PDF. Thus, the value of the cumulative distribution function at the given value (i.e., the calculated reliability) is different from that obtained using the Monte Carlo method. 5) When constraints are properly selected, and a reasonable randomness is applied, the PDF derived from the PME can be used to find the specific distribution followed by the data, providing an accurate probability model for the state function for the ultimate state of stress used in reliability design for mechanical components. This approach is an improvement over approximate method. In particular, using the PME to calculate reliability is straightforward when the random variables follow a non-normal distribution.

Acknowledgements We would like to express our appreciation to the National Natural Science Foundation of China (Grant No. U1708254) for supporting this research.

References

1. Kapur K C, Lamberson L R. Reliability in Engineering Design. New York: John Wiley & Sons, 1977
2. Dhillon B S, Singh C. Engineering Reliability. New York: John Wiley & Sons, 1981
3. Henley E J, Kumamoto H. Reliability Engineering and Risk Assessment. Englewood Cliffs: Prentice-Hall Inc., 1981
4. O'Connor P D T. Practical Reliability Engineering. New York: John Wiley & Sons, 1984
5. Wasserman G S. Reliability Verification, Testing, and Analysis in Engineering Design. New York: Marcel Dekker, Inc., 2002
6. Ang A H S, Tang W H. Probability Concepts in Engineering Planning and Design. New York: John Wiley & Sons, 2006
7. Elsayed A E. Reliability Engineering. 2nd ed. New York: John Wiley & Sons, 2012
8. Klyatis L M. Accelerated Reliability and Durability Testing Technology. New York: John Wiley & Sons, 2012
9. Zhang Y. Reliability Design of Automobile Components. Beijing: Beijing Institute of Technology Press, 2000 (in Chinese)
10. Zhang Y. Introduction of Mechanical Reliability. Beijing: Science Press, 2012 (in Chinese)
11. Zhang Y. Reliability-based design for automobiles in China. Frontiers of Mechanical Engineering in China, 2008, 3(4): 369–376
12. Zhang Y. Connotation and development of mechanical reliability-based design. Chinese Journal of Mechanical Engineering, 2010, 46 (14): 167–188
13. Zhang Y. Review of theory and technology of mechanical reliability for dynamic and gradual systems. Journal of Mechanical Engineering, 2013, 49(20): 101–114 (in Chinese)
14. Zhang Y, Sun Z. The reliability syllabus of mechanical products. Journal of Mechanical Engineering, 2014, 50(14): 14–20 (in Chinese)
15. Siddall J N. Optimal Engineering Design: Principles and Application. New York: Marcel Dekker, 1982
16. Dai Y S, Xie M, Long Q, et al. Uncertainty analysis in software reliability modeling by Bayesian approach with maximum-entropy principle. IEEE Transactions on Software Engineering, 2007, 33

(11): 781–795

17. Sung Y H, Kwak B M. Reliability bound based on the maximum entropy principle with respect to the first truncated moment. Journal of Mechanical Science and Technology, 2010, 24(9): 1891–1900
18. Shi X, Teixeira A P, Zhang J, et al. Structural reliability analysis based on probabilistic response modelling using the maximum entropy method. Engineering Structures, 2014, 70: 106–116
19. Chatzis S P, Andreou A S. Maximum entropy discrimination Poisson regression for software reliability modeling. IEEE Transactions on Neural Networks and Learning Systems, 2015, 26(11): 2689–2701
20. Zhang Y, Liu Q. Reliability-based design of automobile components. Proceedings of the Institution of Mechanical Engineers. Part D, Journal of Automobile Engineering, 2002, 216(6): 455–471
21. Zhang Y, He X, Liu Q, et al. Reliability sensitivity of automobile components with arbitrary distribution parameters. Proceedings of the Institution of Mechanical Engineers. Part D, Journal of Automobile Engineering, 2005, 219(2): 165–182
22. Zhang Y. Reliability-based robust design optimization of vehicle components, Part I: Theory. Frontiers of Mechanical Engineering, 2015, 10(2): 138–144
23. Zhang Y. Reliability-based robust design optimization of vehicle components, Part II: Case studies. Frontiers of Mechanical Engineering, 2015, 10(2): 145–153