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Pilot pattern design scheme with branch and bound in PSA-OFDM system

Shuchen Wang[1] · Suixiang Gao[1,2] · Wenguo Yang[1]

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Abstract

Pilot symbol assisted (PSA) channel estimation is an important means to improve the communication quality of orthogonal frequency division multiplexing (OFDM) systems. The insertion position of the pilot in the frequency domain and time domain of the OFDM symbol is called the pilot pattern. The appropriate pilot pattern can greatly reduce channel estimation error and enhance communication quality. In this paper, the branch and bound (BnB) method is adopted to design the pilot pattern BnB-PP for the first time. Specifically, the result of the linear minimum mean square error method is taken as the target value of channel estimation in PSA-OFDM systems. For branching, pilot positions are randomly selected one by one in the form of the binary tree. For the boundary, after the pilots are filled randomly, a correction term is subtracted from the result of channel estimation at this time to present the expectation boundary. The results show that BnB-PP is better than the common pilot pattern. When signal-to-noise ratio is 36, the average MSE of channel estimation for 32 and 64 pilots in 1344 data signals is reduced by 93.24% and 62.33% respectively compared with the lattice-type pilot pattern.

Keywords Pilot pattern · Branch and bound · PSA-OFDM · Channel estimation · LMMSE

✉ Suixiang Gao
sxgao@ucas.ac.cn

Shuchen Wang
wangshuchen191@mails.ucas.ac.cn

Wenguo Yang
yangwg@ucas.ac.cn

1 School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

2 Zhongguancun Laboratory, Beijing, China

1 Introduction

OFDM modulation realizes the parallel transmission of serial data through frequency division multiplexing, which makes it able to resist multipath fading (Stuber et al. 2004). In 3G and 4G, OFDM is gradually mature (Wang 2005), and it is still one of the most important technologies in 5G wireless communication network (Andrews et al. 2014).

When OFDM data block is transmitted in the wireless channel, it will fade in both frequency domain (Panta and Armstrong 2004) and time domain (Fu et al. 2003). And the wireless channel is random, which affects the communication quality. Therefore, it is necessary to estimate the channel state information (CSI). Channel estimation can get the channel impulse response to improve the performance of OFDM system (Colieri et al. 2002). And PSA-OFDM system (Yu and Li 2013) provides a priori information for channel estimation by inserting pilot data at a specific location of the transmitted data (Fig. 1).

The three most important problems in the PSA-OFDM system are where to insert pilots (He et al. 2021), what data to insert as pilots (Ma et al. 2020) and channel estimation methods (Liu et al. 2014; Soltani et al. 2019; Morelli and Mengali 2001). In this paper, we only focus on the first problem, that is, the pilot pattern design problem. The channel response at the pilot position can be directly estimated from the pilot signal. And because the channel fading is continuous in the time and frequency domain, the channel response at the non pilot position can be estimated according to the changes of the pilot signal in the time and frequency domain. In general, a good pilot pattern should arrange the pilot signal in the different time and frequency positions as dispersedly as possible to reduce the channel estimation error and improve the reliability of the PSA-OFDM system.

Most of the pilot patterns are designed according to the channel characteristics, while the pilot pattern design scheme in this paper is based on data-driven, which is more in line with the complex channel in the actual situation. It is essentially an NP-hard combinatorial optimization problem. And it can be modeled as $0-1$ mixed integer linear programming (MILP), but due to the poor structure and large scale, it is still difficult to solve by solver of MILP directly (So et al. 2014; Mallach 2018). We adopt the framework of BnB with random binary tree branches. It is difficult to obtain the boundary on each node, so this paper presents an expected boundary method with a correction term. The simulation results confirm the superiority of BnB-PP.

The main contributions of this paper are summarized as follows:

- **Method:** BnB framework is adopted to solve the pilot pattern design problem for the first time.
- **Bound:** A heuristic calculation method of boundary in BnB is given.
- **Accuracy and generalization:** The results show that BnB-PP is better than the common pilot pattern under most SNR, especially when the number of pilots is small.

The rest of this paper is outlined as follows: Sect. 2 summarizes the related work of predecessors. Section 3 provides the principle of channel estimation in the PSA-OFDM system, while Sect. 4 discusses the method of designing the pilot pattern with

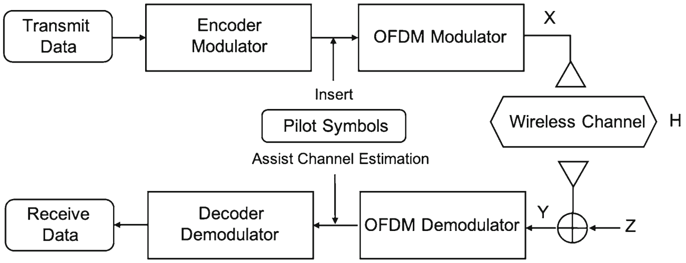


Fig. 1 PSA-OFDM system

BnB. The simulation results are presented and explained in Sect. 5. The conclusion and future works have been provided in Sect. 6.

2 Related work

Different from studying channel characteristics from the electromagnetic wave level like ray tracing (Mbugua et al. 2021), channel estimation is generally at the link level or system level (Wang et al. 2020). And Pilot signals are in the service of channel estimation.

Channel estimation methods can be divided into the blind, semi-blind and unblinded estimation. Blind and semi-blind channel estimation, such as subspace decomposition (Muquet et al. 2002), can effectively improve the system capacity, but it is difficult to deal with fast fading wireless channels due to its poor flexibility. And PSA-OFDM is the most common communication system for nonblind estimation. The pilot signal is the known specific signal data, which is placed at the specific position of the transmission symbol data of the PSA-OFDM system (Coleri et al. 2002; Kewen et al. 2010). The channel condition is estimated by comparing the difference between the pilot signal at the receiving end and the transmitting end. Least Squares (LS) (Lin 2008) can estimate the channel impulse response of the pilot position, and then it needs to estimate other positions by interpolation (Athaudage and Jayalath 2003; Adegbite et al. 2013). MMSE method is based on LS (Sutar and Patil 2017). MMSE can obtain the impulse response of the whole channel with higher accuracy by incorporating the influence of noise into the calculation, but it needs to obtain or estimate some other characteristics of the channel in advance (Soman et al. 2021). LMMSE is an improved version of MMSE with lower computational complexity (Wu 2021). Some scholars have studied the estimation of channel fading from amplitude and phase by a two-part neural network in frequency domain (Sun and Yuan 2006).

The pilot signal usually adopts the Zadoff–Chu Sequence (Mi et al. 2022). The pilot will occupy valuable data transmission position resources and cause additional overhead. Rearranging the pilot patterns can reduce the number of pilot symbols required by even 10 times, and still maintain the same performance of channel estimation (Tufvesson and Maseng 1997). There are three common pilot patterns in Fig. 2. These are mainly designed by experience and proved feasible in practice (Tong et al. 2004). Intuitively, the block-type pilot pattern is more suitable for channels with large

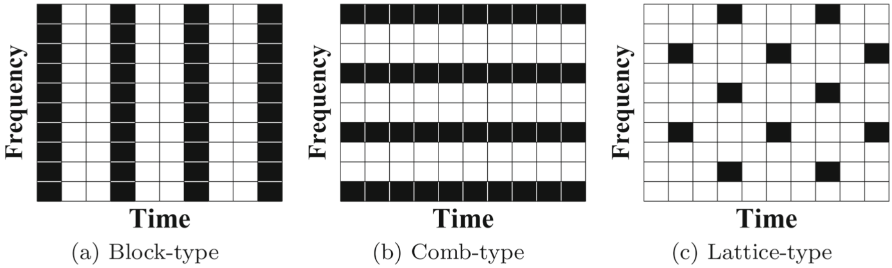


Fig. 2 Common pilot patterns. The whole is a PSA-OFDM symbol, in which the black squares are the position of the pilot signal, and the others are the position of the data signal

frequency selective fading (Shi et al. 2009), while the comb-type pilot pattern is for large time selective fading (Hsieh and Wei 1998). And the lattice-type pilot pattern has better stability (Zhang et al. 2019).

Some researchers have tried some advanced methods to design non-uniform pilot patterns, such as analyzing the relationship between bit error rate and time-frequency fading and then giving the pilot pattern (Zhang et al. 2006). Since most of these studies are data-driven, some researchers apply the deep neural network, including the super-resolution methods in computer vision and the concrete layer (Soltani et al. 2020), to estimate channel and design pilot pattern. Of course, some researchers have abandoned the traditional PSA-OFDM system and proposed a new pilot insertion technology that makes use of the idea of index modulation to perform channel estimation while transmitting additional bits (Acar and Aldırmaz-Çolak 2021). BnB is a classical algorithm for solving integer programming problems (Quadri et al. 2009; Morrison et al. 2016; Tian et al. 2017) and we apply the BnB method to design pilot pattern for the first time.

3 Channel estimation in PSA-OFDM system

This section introduces channel estimation methods in the PSA-OFDM system. The purpose of designing the pilot pattern is to reduce channel estimation error.

Figure 3 shows the transmission process of the OFDM subframe in the channel. Consider the subframe k with n subcarriers and m time slots in the PSA-OFDM system. Channel estimation is performed separately on each subframe and the channel response is different at different frequencies and time slots. We generally assume that there is a linear relationship between the received and transmitted signals. So the received signal in ith subcarrier and jth time slot can be expressed as:

$$y_{i,j}^k = h_{i,j}^k x_{i,j}^k + z_{i,j}^k \tag{1}$$

where $x_{i,j}^k, y_{i,j}^k, h_{i,j}^k, z_{i,j}^k \in \mathbb{C}$ are the received signal, transmitted signal, channel response, and white Gaussian noise respectively.

Set $N = \{(i, j)|i = 1, 2, \ldots, n, j = 1, 2, \ldots, m\}$. $P \subseteq N$ is the pilot pattern. $X_{P,k}, Y_{P,k}, H_{P,k}, Z_{P,k} \in \mathbb{C}^{|P|\times 1}$ is the value in the pilot position while in particular,

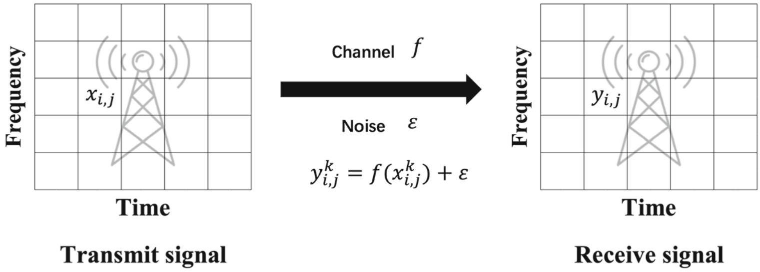


Fig. 3 OFDM subframe transmission in wireless channel

$X_{N,k}, Y_{N,k}, H_{N,k}, Z_{N,k} \in \mathbb{C}^{|N|\times 1}$ is the value at all symbols. Channel estimation in the PSA-OFDM system is estimating the value of $H_{N,k}$ by $X_{N,k}$ and $Y_{N,k}$ and pilot pattern design scheme is how to construct the pilot pattern P.

3.1 Least squares estimation

LS estimates the channel at the pilot position by minimizing (2) where $\|\cdot\|_2$ is ℓ_2 norm and $D_{P,k} \in \mathbb{C}^{|P|\times|P|}$ is the diagonalized matrix of vector $X_{P,k}$.

$$\min_{H} \|Y_{P,k} - D_{P,k}H\|_2^2 \tag{2}$$

The result of the optimization is (3). And when $D_{P,k}$ is a non-singular matrix, $\hat{H}_{P,k}^{LS} = D_{P,k}^{-1}Y_{P,k} \in \mathbb{C}^{|P|\times 1}$. LS estimation is simple in the calculation but has its shortcomings. The mean square error of LS estimation is $\frac{\sigma_z^2}{\sigma_x^2}$, where σ_x^2 and σ_z^2 are the variances of the transmitted signal and the channel noise respectively.

$$\hat{H}_{P,k}^{LS} = (D_{P,k}^{\dagger}D_{P,k})^{-1}D_{P,k}^{\dagger}Y_{P,k} \tag{3}$$

LS estimation only estimates the value at the pilot position. And in order to obtain the channel response at the non pilot positions, the most common method is interpolation, such as the bilinear interpolation algorithm, the nearest neighbor interpolation algorithm, the cubic spline interpolation algorithm, etc. This is based on the fact that the channel changes are continuous and small between adjacent symbols in both the time domain and frequency domain. For this reason, the pilot pattern should be as decentralized as possible.

3.2 Minimum mean square error estimation

LS estimation is looking for the H that can best represent the received signal $Y_{P,k}$ with $D_{P,k}H$. Nevertheless, a more direct method of estimating H is minimizing $\|H_{N,k} - H\|_2^2$. MMSE method obtains the channel estimation $\hat{H}_{P,k}^{MMSE} \in \mathbb{C}^{|N|\times 1}$ of the whole subframe k by multiplying $\hat{H}_{P,k}^{LS}$ by a coefficient matrix $\hat{W}_{P,k}^{MMSE} \in \mathbb{C}^{|N|\times|P|}$. That is

$$\hat{H}_{P,k}^{MMSE} = \hat{W}_{P,k}^{MMSE} \hat{H}_{P,k}^{LS} \tag{4}$$

$\hat{W}_{P,k}^{MMSE}$ is determined by minimizing the estimation error:

$$\min_{W} \| H_{N,k} - W \hat{H}_{P,k}^{LS} \|_2^2 \tag{5}$$

The result of $\hat{W}_{P,k}^{MMSE}$ by solving optimization problem (5) is

$$\hat{W}_{P,k}^{MMSE} = R_{NP}(R_{PP} + \frac{\sigma_z^2}{\sigma_x^2} I_P)^{-1} \tag{6}$$

where $I_P \in \mathbb{R}^{|P|\times|P|}$ is the identity matrix, $R_{PP} = \mathbb{E}\{H_P H_P^{\dagger}\} \in \mathbb{C}^{|P|\times|P|}$ is the auto-correlation matrix of the pilot and $R_{NP} = \mathbb{E}\{H_N H_P^{\dagger}\} \in \mathbb{C}^{|N|\times|P|}$ is the cross-correlation matrix between the pilot and all symbols (Edfors et al. 1998; Athaudage and Jayalath 2004).

In fact, R_{NP} and R_{PP} are unknown while sometimes the correlation between position (i_1, j_1) and (i_2, j_2) can be approximated as follows:

$$r_{(i_1,j_1),(i_2,j_2)} = \frac{\sigma_H^2 J_0(2\pi f_{max} T(j_1 - j_2))}{1 + j2\pi \tau_{max} \Delta f (i_1 - i_2)} \tag{7}$$

where σ_H^2 is the total average power of the channel response, J_0 is the first kind of zero order Bessel function, f_{max} is the maximum Doppler frequency, T is the symbol block time, τ_{max} is the maximum delay spread, and Δf is the subcarrier spacing (Athaudage and Jayalath 2004).

From a certain point of view, $\hat{H}_{P,k}^{LS}$ multiplied by $(R_{PP} + \frac{\sigma_z^2}{\sigma_x^2} I_P)^{-1}$ can be regarded as the noise reduction process of $\hat{H}_{P,k}^{LS}$ according to the auto-correlation matrix and noise. Then, the process of multiplying R_{NP} is to reallocate the noise-reduced $\hat{H}_{P,k}^{LS}$ to this subframe by weighted summation according to the cross-correlation matrix.

3.3 Linear minimum mean square error estimation

In LMMSE, $\frac{\sigma_z^2}{\sigma_x^2}$ is replaced by expectation $\frac{\beta}{SNR}$ (Edfors et al. 1998). β is a channel modulation type parameter. For QPSK modulation, $\beta = 1$ and for 16QAM modulation, $\beta = 17/9$.

$$\beta = E(X^2)E\left(\frac{1}{X^2}\right) \tag{8}$$

$$\hat{H}_{P,k}^{LMMSE} = \hat{W}_{P,k}^{LMMSE}\hat{H}_{P,k}^{LS} \tag{9}$$

$$= R_{NP}\left(R_{PP} + \frac{\beta}{SNR}I_P\right)^{-1}(D_{P,k}^{\dagger}D_{P,k})^{-1}D_{P,k}^{\dagger}Y_{P,k} \tag{10}$$

In this method, $\hat{W}_{P,k}^{LMMSE}$ is uniquely determined by the channel response, modulation mode and SNR. $\hat{H}_{P,k}^{LMMSE}$ is the linear transformation of $Y_{P,k}$. In practical application, the coefficient matrix of $Y_{P,k}$ only needs to be calculated once, which saves a lot of calculation time of matrix multiplication and inversion. Therefore, the channel estimation methods in Sects. 4 and 5 are LMMSE.

4 Pilot pattern design scheme with branch and bound

Let's first describe the pilot pattern design problem. Set $S_P = diag\{s_{(i,j)}^P\} \in \{0, 1\}^{|N|\times|N|}$ and $s_{(i,j)}^P = 1$ when $(i, j) \in P$. The pilot pattern is designed with S_P as the variable to minimize the channel estimation result.

The calculation of $\hat{H}_{P,k}^{LMMSE}$ mentioned earlier is actually a linear weighted sum of $Y_{P,k}$ or $\hat{H}_{P,k}^{LS}$, which can be transformed into a linear weighted sum of $Y_{N,k}$ or $\hat{H}_{N,k}^{LS}$ and the weight of non pilot position is 0. Then the result of LMMSE for subframe k with pilot pattern P can be expressed as:

$$\hat{H}_{P,k}^{LMMSE} = R_{NP}\left(R_{PP} + \frac{\beta}{SNR}I_P\right)^{-1}\hat{H}_{P,k}^{LS} \tag{11}$$

$$= R_{NN}\left[S_P\left(R_{NN} + \frac{\beta}{SNR}I_N\right)S_P\right]^{-1}\hat{H}_{N,k}^{LS} \tag{12}$$

$$= W(S_P)\hat{H}_{N,k}^{LS} \tag{13}$$

The inverse operation here is the pseudo inverse operation.

The pilot pattern is designed to determine no more than p pilot insertion positions to make the channel estimation results on all subframes more accurate:

$$Obj(P) = \tfrac{1}{K}\textstyle\sum_k^K \|H_{N,k} - \hat{H}_{P,k}^{LMMSE}\|_2^2 \tag{14}$$

$$= \tfrac{1}{K}\textstyle\sum_k^K \|H_{N,k} - W(S_P)\hat{H}_{N,k}^{LS}\|_2^2 \tag{15}$$

$$\begin{aligned} &\min \quad Obj(P) \\ &s.t. \quad \sum_{i=1}^{n}\sum_{j=1}^{m} s_{(i,j)}^P = p \\ &\qquad s_{(i,j)}^P \in \{0, 1\} \end{aligned} \tag{16}$$

where K is the number of PSA-OFDM symbols used to design the pilot pattern.

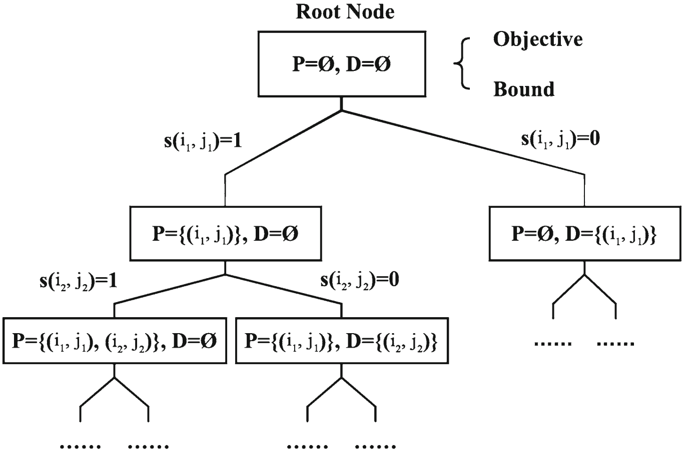


Fig. 4 Branch and bound diagram

Even if we can forcibly transform (16) into a 0-1 integer linear programming problem, it will be very difficult to achieve the purpose of matrix inversion through constraints and this makes it difficult to get a feasible solution. Therefore, we do not continue to simplify and BnB is adopted to solve (16). The diagram of BnB is shown in Fig. 4.

4.1 Branch

Branching repeatedly divides the solution space into smaller and smaller subsets. By analogy to the 0-1 knapsack problem, each position is regarded as an item with a weight of 1, and the selected pilot set is a knapsack with a maximum load of p. Each node determines not only a feasible solution P_{Node}, but also a set of positions that are discarded D_{Node}. The difference is that the value of the item is not certain, and the gain brought to the objective function by putting it into the selected set is not linear. Intuitively, the more pilots, the smaller the error of channel estimation, but the smaller the gain brought by increasing pilot points, that is, the law of diminishing marginal returns. How to prove this property and how to utilize it to design approximation algorithms will be one of the directions of our next work.

In this work, we make full use of binary tree branching and random strategy. When branching each node, randomly select a position from $U_{Node} = N - (P_{Node} \cup D_{Node})$ to generate two new nodes based on whether to select the location as the pilot location. At the leaf node, $P_{Node} = p$ or $N = (P_{Node} \cup D_{Node})$.

Similarly, each branch can randomly select several nodes to add to P_{Node} and D_{Node}. Compared with deciding only one node at a time, this involves several random pruning by default. But it also greatly reduces the number of calculations in the case of a huge computing scale, which may lead to unexpected results.

4.2 Bound

The boundary is an important feature of the node, which refers to the lower boundary here. The boundary is the optimal value of the objective function under the premise of current P_{Node} and D_{Node}. It is usually replaced by the expected value. The boundary can contribute to speeding up the branch and bound. On the one hand, after each branch, no further branch will be made for any child node whose boundary exceeds the known feasible solution value. In this way, many nodes can be ignored, thus narrowing the search scope. On the other hand, the boundary is also an important basis to determine the node branch priority. And the upper bound, which shows the worst result of continuing branching, is the objective function value corresponding to the selected set P_{Node}. Here, it defaults that adding the number of pilots will not make the channel estimation result worse.

However, the boundary cannot be estimated accurately, so we give a heuristic method. Take $P_r \subseteq U_{Node}$ at random such that $|P_{Node} \cup P_r| = p$, which means that the pilot number is supplemented to p at random in the alternative set. The boundary B_{Node} is defined as the following algorithm 1, where P_{Norm} is a traditional standard pilot pattern and γ is the correction rate parameter.

Algorithm 1 Boundary

Require: $P_{Node}, D_{Node}, P_{Norm}, \gamma, p, N$
Ensure: B_{Node}
1: $B_{Node} = 0$
2: **for** $i = 1, 2, \cdots, t$ **do**
3: Take $P_r \subseteq N - (P_{Node} \cup D_{Node})$ at random such that $|P_{Node} \cup P_r| = p$
4: $B_{Node} = B_{Node} + Obj(P_{Node} \cup P_r)$
5: **end for**
6: $B_{Node} = \frac{1}{t} B_{Node} - \alpha \cdot Obj(P_{Norm})$
7: where $\alpha = \gamma(1 - \frac{|P_{Node}|}{p})(1 - \frac{|P_{Node}|+|D_{Node}|}{|N|})$

Instead of directly taking the approximate expectations of the objective value corresponding to the randomly supplemented pilot mode as the boundary, we subtract a correction term. The meaning of the correction term is the gap between the random and real boundaries. The smaller the number of pilot positions selected and determined not to be selected, the more inaccurate the random term and the larger the correction term. Here, we default that the number of pilots is positively related to the accuracy of channel estimation. And according to this calculation, the B_{Node} of the child node may be smaller than that of the parent node, so sometimes max operation is performed on the boundary of the node and its parent node.

4.3 Priority

In such a large-scale branch and bound problem, the priority strategy can effectively accelerate the speed of finding a better solution. In this work, the worst boundary strategy and the optimal target value strategy are adopted alternately.

Branch and Bound is the process of gradually reducing the gap between the upper and lower bounds. A node with a low boundary has great potential for continuing branches, but it generally has experienced fewer branches. The strategy of choosing such nodes preferentially is called the worst boundary strategy. It can improve the minimum lower boundary and is also very similar to the breadth-first strategy. The upper bound is determined by the target value. The optimal target value strategy is selecting a node with a small upper bound, which generally has experienced more branches. It can quickly find a good solution and is also very similar to the depth-first strategy.

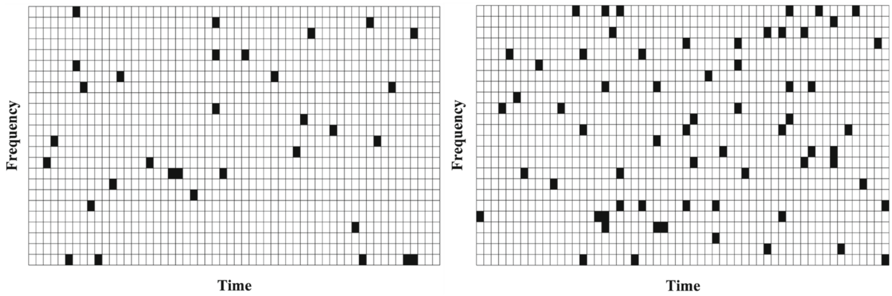
4.4 Complexity

The total number of leaf nodes is $\sum_{i=0}^{p} \binom{n\times m}{i}$. For each different P on the node, $W_{LMMSE}(S_P)$ needs to be calculated. Since there are many 0 in S_P, the calculation of the pseudo inverse in $W(S_P)$ is actually equivalent to the calculation of the pseudo inverse of a $|P| \times |P|$ matrix. This still requires a lot of computation and generally, the optimal solution cannot be obtained by this algorithm. However, due to the consideration of node computing priority and pruning operation, BnB is still an efficient solution finding strategy in this problem and has achieved good results in simulation.

The whole process can be expressed as the following algorithm 2.

Algorithm 2 Branch and Bound

Require: n, m, p, ε
Ensure: $BestNode$
$N = \{(i,j)|i = 1, 2, \cdots, n, j = 1, 2, \cdots, m\}$
$Obj = Root.obj = +\infty$, $Root.Bound = -\infty$,
$Root.P = Root.D = \emptyset$
$Node = \{Root\}$, $BsetNode = Root$
while $Node \neq \emptyset$ and $Obj > \varepsilon$ **do**
  $node = \arg\max_{node \in Node} Priority(Node)$
  $Node = Node - \{node\}$
  if $|node.P| < p$ and $|node.P| + |node.D| < |N|$ **then**
    $nodeLeft = nodeRight = node$
    Random select $(i, j) \in N - node.P - node.D$
    $nodeLeft.P = nodeLeft.P \cup \{(i, j)\}$
    $nodeRight.D = nodeRight.D \cup \{(i, j)\}$
    update $nodeLeft.Bound$ and $nodeRight.Bound$
    if $nodeLeft.Bound < Obj$ **then**
      $Node = Node \cup \{nodeLeft\}$
    end if
    if $nodeRight.Bound < Obj$ **then**
      $Node = Node \cup \{nodeRight\}$
    end if
    $nodeLeft.obj = \frac{1}{K} \sum_k^K \|H_{N,k} - \hat{H}(k, nodeLeft.P)\|_2^2$
    if $nodeLeft.obj < Obj$ **then**
      $Obj = nodeLeft.Obj$, $BestNode = nodeLeft$
    end if
  end if
end while



(a) The number of pilots is 32 (b) The number of pilots is 64

Fig. 5 Pilot pattern obtained by branch and bound

5 Simulation results

In this section, we get BnB-PP with different pilot numbers on the simulation data and compare the channel estimation results of the pilot pattern we designed with the common pilot pattern under different SNR.

In the simulation, Vienna 5G Link Level Simulator (Pratschner et al. 2018) is introduced to simulate wireless signal transmission under the 5G NR standard. Only one antenna is set at both the transmitter and receiver. The subcarrier interval is 60kHz. Each frame consists of $m = 56$ time slot and $n = 24$ subcarriers. Vehicle-A (VehA) wireless channel model is adopted, and the center frequency is 2.1GHz. The modulation mode is 64 Quadrature Amplitude Modulation (64QAM) and $\beta = 2.6854$ at this time. The speed of user equipment (UE) is 20m/s. And the total average power of the channel response σ_H^2 was assumed to be 1.

This paper mainly studies the cases when the number of pilots is 32 and 64. As shown in Fig. 5, the pilot pattern is designed by BnB under the data of $SNR = 36$. Here, just take a relatively small data scale $K = 5$, so that the result can achieve a certain generalization performance. Set correction factor $\gamma = 1$. It can be seen that the pilot patterns are uneven and relatively scattered, which is consistent with the design idea of the conventional pilot pattern. Taking the result when the number of pilots is 64 as an example. From the frequency domain, there is only one subcarrier frequency without the pilot. Most of the subcarriers are equipped with 2 or 3 pilots. From the time domain, there must be pilots on every three consecutive time slots, and the number of pilots in each time slot does not exceed 4. And it is rare for two pilots to be adjacent.

Figure 6 shows the impact of different pilot numbers on channel estimation when $SNR = 36$, and compares BnB-PP with the lattice-type pilot pattern. The channel estimation method here is LMMSE and for visualization, MSE is converted to dB. The effect is better than the lattice type under each pilot number, and the smaller the pilot number, the better the effect. A smaller number of pilots means a larger channel capacity, and OFDM symbols can transmit more information, which fully demonstrates the value of this method.

Figures 7 and 8 respectively show the comparison of MSE estimated by the channel under different SNR when the number of pilots is 32 and 64. Due to the frequency

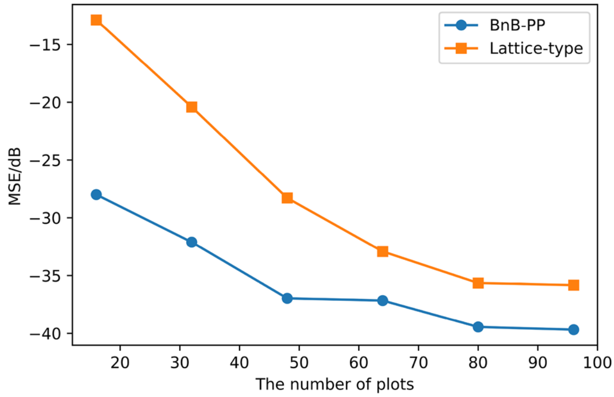


Fig. 6 MSE of channel estimation with different pilot numbers

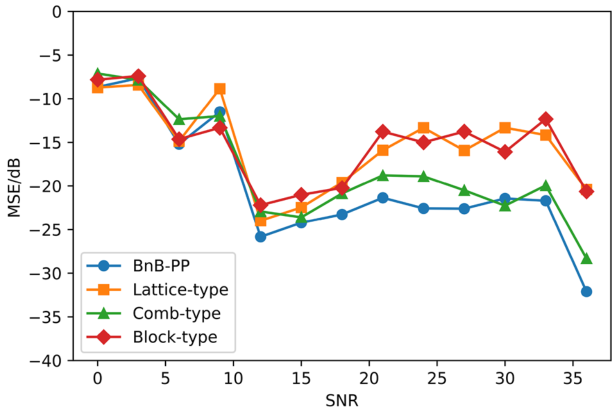


Fig. 7 MSE of channel estimation when the number of pilots is 32

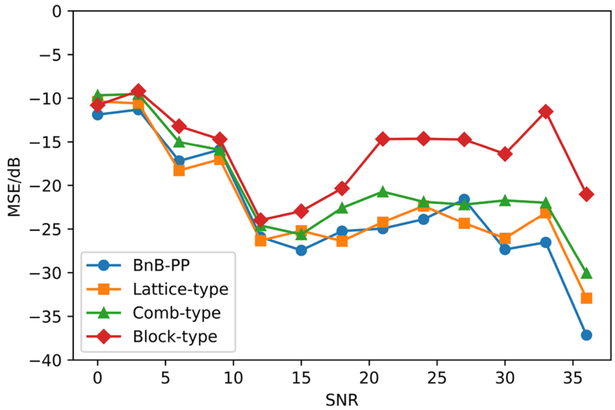


Fig. 8 MSE of channel estimation when the number of pilots is 64

Table 1 Average decline rate of MSE when SNR is 36

Methods	The number of pilots			
	32		64	
Block-type	$8.63e-3$	92.88%	$7.91e-3$	97.57%
Comb-type	$1.47e-3$	58.29%	$0.98e-3$	80.51%
Lattice-type	$9.09e-3$	93.24%	$0.51e-3$	62.33%
BnB-PP	$0.61e-3$		$0.19e-3$	

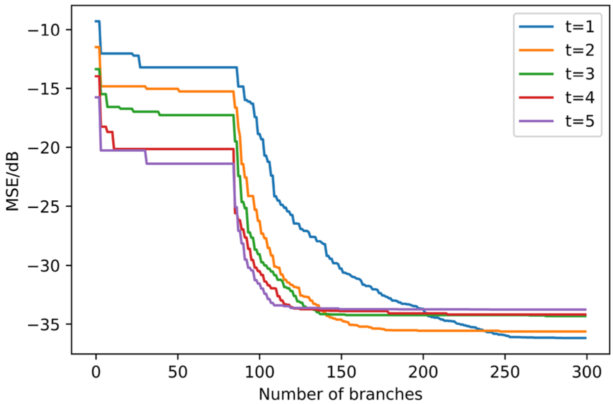


Fig. 9 Calculation process when t positions are selected for each branch

selective fading of this channel, the comb pilot mode also shows good performance. It can be seen that when the pilot number is 32 and the SNR is greater than 10, BnB-PP is better than other pilot modes. And in other cases, the performance of BnB-PP is stable. Table 1 shows the MSE of each method when SNR is 36 and the MSE reduction rate compared with the BnB-PP method. For example, when there are 32 and 64 pilots, the average MSE of channel estimation of the Lattice-type pilot pattern is $9.09e-3$ and $0.51e-3$ respectively, and that of BnB-PP is $0.61e-3$ and $0.19e-3$. The BnB-PP reduces 93.24% and 62.33% respectively compared with the lattice-type pilot pattern.

In the above experiment, only one position point shall be determined for each branch. Similarly, it can be expanded to discuss t location points for each branch, which requires deciding whether to select these t position points at the same time. This is also equivalent to a lot of pruning operations on the original method. The idea of considering position points in batches is based on the fact that the number of position points to be selected is far more than the number of pilot points. Even if a large number of positions are discarded, a good solution can be obtained. Figure 9 shows the calculation process when t is from 1 to 5 to obtain the pilot pattern of 64 position points. In the calculation process, with the increase of the number of branches, a smaller MSE solution is gradually obtained. Here is the average MSE corresponding to SNR=36. As we can see, the greater t at the beginning, the faster MSE decreases. After a large number of iterations, $t = 1$ gives the most accurate results. And in a large number of subsequent calculations, MSE decreases more slowly. In some cases, it is possible to select an appropriately large t to speed up the branching process. The introduction of t provides a method to speed up the branching process.

6 Conclusion

In this work, we propose a pilot pattern design scheme for BnB based LMMSE channel estimation in PSA-OFDM systems. And a large number of random strategies are adopted in branching and boundary calculation. The simulation results show that the pilot pattern BnB-PP we designed is better than the common pilot pattern under most SNR, especially when the number of pilots is small, which is of great significance to improve the system capacity and communication quality of the PSA-OFDM system. However, this method is not fully integrated with LMMSE. The computing speed on each node is slow, and there may be a lot of computational redundancy. In future work, we will try to reduce the computing time by matrix operation. And we will consider the MIMO system and analyze the impact of different channel models on pilot pattern design scheme.

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Data availability Enquiries about data availability should be directed to the authors.

Declarations

Competing interests The authors have not disclosed any competing interests.

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