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ORIGINAL ARTICLE

GNSS PPP-RTK: integrity monitoring method considering wrong ambiguity fixing

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Abstract

In previous studies on multiple hypothesis solution separations (MHSS) advanced autonomous integrity monitoring (ARAIM) in phase-based GNSS precise point positioning (PPP)-real-time kinematic (RTK), the risk of wrong (or incorrect) ambiguity fixing is generally considered to be negligible after ambiguity validation. However, this unrealistic assumption can introduce risks to PPP-RTK positioning solutions that have strict requirements for accuracy and integrity. In this study, a new integrity monitoring method considering the risks of wrong ambiguity fixing is proposed. The proposed method extends the scope of ARAIM to accommodate a type of pseudo-measurements for integer ambiguity fixing. Then, wrong ambiguity fixing can be treated as a type of fault in the MHSS ARAIM scheme. We test the proposed integrity monitoring method with both simulated and real-world PPP-RTK data sets, demonstrating that this method is conservative and can properly bound the positioning errors. Additionally, we analyze the effect of the probability of wrong ambiguity fixing on PPP-RTK positioning protection levels (PLs). The numerical results show that when the probability of wrong ambiguity fixing is smaller than the magnitude of the integrity budget, the effects on PLs are usually very small. On the contrary, due to the impact of wrong integer ambiguity fixing risks, it can increase the PLs levels of ambiguity-fixed solutions, which however will still be smaller than the PLs of ambiguity-float solutions.

Keywords Global navigation satellite systems (GNSS) · PPP-RTK · Integrity monitoring · Wrong ambiguity fixing · Multiple hypothesis solution separations

Introduction

In the applications of global navigation satellite systems (GNSS), especially for safety–critical applications, integrity risk monitoring including protection level (PL) estimation is mandatory information. Only if the PL does not exceed the predefined alarm limit (AL), the positioning result can be considered reliable and trustworthy for use in navigation. The multiple hypothesis solution separations (MHSS) advanced autonomous integrity monitoring (ARAIM) algorithms are proposed (Blanch et al. 2011; Blanch et al. 2012, 2015) and applied in aviation (Working Group C 2016). However, such integrity monitoring methods were originally designed for single point positioning applications.

However, with the rapid development of highly intelligent transport systems including autonomous driving, the accuracy of single point positioning (SPP) based on pseudorange or smoothed pseudorange measurements cannot meet the positioning requirement due to their low accuracy. Since carrier phase measurements are significantly less noisy than pseudoranges, it becomes essential to use them to achieve centimeter-level positioning performance. However, because the carrier phase measurements are ambiguous which are caused by the unknown integer ambiguities, the positioning methods using carrier phase measurements, such as Precise Point Positioning (PPP) and Real-Time Kinematic (RTK), will be very different from SPP. Therefore, the integrity monitoring scheme for these positioning methods is also critical. There are many studies on the MHSS ARAIM methods and processing schemes of using the carrier phase for positioning that have been proposed based on the ambiguity

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float solution (Gunning et al. 2018, 2019; Blanch et al. 2020).

Nevertheless, compared with the ambiguity-float solutions, studies have shown that the integer ambiguity-fixed solutions for PPP can reduce the time of convergence significantly (Teunissen et al. 2010; Teunissen et al. 2015) and improve the positioning accuracy to a certain extent (Liu et al. 2020). Therefore, it is essential to investigate the integrity monitoring methods for the ambiguity-fixed PPP-RTK solutions. In terms of positioning integrity risk, the main difference between ambiguity-float and ambiguity-fixed solutions is that the procedure of ambiguity fixing may be incorrect in ambiguity-fixed solutions and this may introduce a new type of faults named as wrong (or incorrect) integer ambiguity fixing. The least-squares ambiguity decorrelation adjustment (LAMBDA) method which is the most popular method for ambiguity resolution can be used to search the integer candidates (Teunissen et al. 1995). It is essential to validate the integer ambiguities obtained by the LAMBDA method before accepting them. Faulty measurements in the float solution may contaminate the integer ambiguity fixing process, leading to wrong integer fixing, then unexpected accuracy loss in the ambiguity fixed solutions even with the optimal integer least squares method (Teunissen et al. 2002). Some ambiguity validation methods have been developed over the years to improve the reliability of integer ambiguity-fixing, such as the R ratio test (Verhagen et al. 2013), W-ratio (Wang et al. 1998; Li and Wang 2014) as well as the data-driven approach (Green and Humphreys 2018). Even though these methods are very effective in validating integer ambiguity fixing; nevertheless, incorrect ambiguity fixing can still occur because of the various random errors or faults in the measurements. In addition, a significant limitation of these methods is the lack of connections between wrong ambiguity fixing and its effect on position errors. Moreover, integer least squares estimation may be biased with unmodeled errors, which is not considered in these methods (Verhagen et al. 2013). Furthermore, integer ambiguities may not be directly fixed using the LAMBDA method all the time. In some positioning modes, other methods, for example, the fix-and-hold method, are also applicable. Often, wrong ambiguity fixing could result in large position errors which could reach a few decimeters in horizontal directions or one meter in vertical directions. This will exceed the estimated positioning accuracy which is usually at the centimeter level in PPP-RTK (Wang et al. 2022; Zhang et al. 2023; Zhang and Wang 2023); thus, it is critical to monitor the wrong ambiguity-fixing in the PL estimation in PPP-RTK ambiguity-fixed solutions.

Wang et al. (2023) have proposed a method for integrity monitoring of ambiguity-fixed solutions based on the modified MHSS ARAIM method. This approach takes the probability distribution of the integer ambiguity resolution into account using the bootstrapping method, although this is not the only ambiguity resolution method. However, in PPP-RTK positioning, the evaluation of the probability of wrong ambiguity fixing may be unreliable due to measurement faults. Additionally, in some positioning modes that use the ambiguity fix-and-hold method, it is difficult to evaluate the distribution of wrong fixing vectors. In this paper, a more flexible integrity monitoring method is developed to consider the risks of wrong ambiguity fixing under the proven MHSS ARAIM framework, but without using additional information on the probability distribution of the wrong integer ambiguities, which is in part due to simplicity in implementation but also to its conservative nature in the protection level computations.

In this study, the problems of integrity monitoring considering the risks of wrong ambiguity fixing and limitations of other existing methods are discussed. Then, based on the MHSS ARAIM framework, an integrity monitoring method considering the risks of wrong ambiguity fixing is proposed. Furthermore, the performances of the proposed integrity monitoring method are illustrated based on the simulation and real data sets.

Methodology

Integer ambiguities are resolved using the LAMBDA method with the covariance matrix of the float ambiguities estimated in the float solutions. Therefore, it is possible to have many ambiguities fixed incorrectly at the same time. Even though the measurements and error models are treated differently in PPP-RTK and classic RTK, the process of ambiguity resolution is identical. Thus, the methods for monitoring the integrity risks caused by wrong ambiguity fixing in both the PPP-RTK model and the RTK model are therefore interoperable. In existing studies, there are different methods proposed to consider integrity risks caused by wrong ambiguity fixing, which are reviewed here before the new method based on the MHSS ARAIM framework to consider wrong ambiguity fixing is proposed.

Due to the complexity of ambiguity resolution using the LAMBDA method, protecting such wrong ambiguity fixing is very challenging. In the existing studies, there are mainly two models for integrity monitoring, including fault-free methods and fault-tolerant methods.

Mixed integer least squares

In the mixed integer least squares, both integer and real-valued parameters are present, and also it is assumed that there are no measurement faults, the observation model with unknown parameter vectors including integer and non-integer parts could be written as:

$$y = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}$$

where y is the measurement vector; a is the vector of integer parameters and b is the vector of non-integer parameters; A is the design matrix to link integer unknown vector to measurements and B is the design matrix to link non-integer unknown vector to measurements. The stochastic model of the measurement vector is Q_y. Once the integer ambiguities are fixed as vector $\check{a}$, a straightforward method for updating the ambiguity-fixing integers is to add pseudo-measurements in the estimator, which could be written as:

$$\begin{bmatrix} y \\ \check{a} \end{bmatrix} = \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{2}$$

with the stochastic model as $\begin{bmatrix} Q_y & 0 \\ 0 & Q_{\check{a}} \end{bmatrix}$, where $Q_{\check{a}}$ is a zero-matrix and the STD could be set as a small value (e.g., 10^{-8}) in applications to facilitate the computer to perform matrix operations. It can be also processed as incorporating linear state constraints in a Kalman filter to avoid numerical instability (Simon and Chia 2002).

It is noted that in fixing the integer ambiguities to constant values there is an inherent condition of treating the optimal integer least squares (ILS) results—the vector of optimal combination of integers, as correct values without being attached to the probability of correct ambiguity fixing.

While it may be possible to evaluate the probability of correct ambiguity fixing with float ambiguity rounding or bootstrapping methods, it is difficult to evaluate the probability of the correct ambiguities from the ILS, even for the fault-free model. Therefore, some integrity risk analyzes for wrong ambiguity fixing are simplified.

Integrity monitoring for wrong ambiguity fixing in the fault-free model

For the fault-free model, it is assumed that there are no unmodelled errors existing in the measurement model. Methods have been proposed in fault-free models assuming that all faults are excluded in measurements and only noise and bias in the nominal error model for integrity monitoring considering the risks of ambiguity resolution (Pervan and Chan 2001; Khanafseh and Pervan 2010; Khanafseh and Langel 2011; Khanafseh et al. 2012).

To illustrate the method for the fault-free model, the PL in the vertical direction is taken as an example. PL in horizontal directions could be estimated as the same method. For the ambiguity-float solution based on the fault-free model, the vertical PL (VPL) could be written as

$$P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}\right) = \mathrm{PHMI}_v \tag{3}$$

where $P(*)$ is the symbol of probability; $\hat{x}_v$ and x_v are the estimated and the true value of the position in the vertical direction; PHMI_v is the budget of the probability of hazardous misleading information (PHMI) in the vertical direction.

When considering ambiguity float solutions are fault-free, many studies ignored the risks of wrong ambiguity-fixing in the integrity monitoring process for ambiguity-fixed solutions (Jokinen et al. 2013a, b; Wang et al. 2022). It is clear that if ambiguities are guaranteed to be fixed correctly, the PL of ambiguity-fixed solutions can also be estimated by the above equation; thus, the protection level could be written as the following formula or its variants based on a combination of noise and bias.

$$\mathrm{PL} = K\sigma + F\mathrm{bias} \tag{4}$$

where K is the coefficient of inverse probability of the integrity budget and σ is the estimated STD of the unknown parameter. F is the projection matrix from measurement to the estimated unknown and bias is the bias in the nominal error model. Nevertheless, it is still possible to fix ambiguity incorrectly in real situations. Therefore, it could be written as two mutually exclusive and exhaustive events, which yields

$$\begin{aligned} &P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_0\right)P\left(H_0\right) \\ &\quad + P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_1\right)P\left(H_1\right) = \mathrm{PHMI}_v \end{aligned} \tag{5}$$

where H_0 represents the null hypothesis and H_1 is the alternative hypothesis which means wrong ambiguity fixing in this formula. Considering $P(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_1)$ is difficult to estimate, the equation could be further derived with conservative assumption $P(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_1) \le 1$ which could be written as:

$$\begin{aligned} &P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_0\right)P\left(H_0\right) + P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_1\right)P\left(H_1\right) \\ &\quad < P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_0\right)P\left(H_0\right) + P\left(H_1\right) \end{aligned} \tag{6}$$

$$P\left(\left|\hat{x}_v - x_v\right| > \mathrm{VPL}|H_0\right)P\left(H_0\right) < \mathrm{PHMI}_v - P\left(H_1\right) \tag{7}$$

Then the VPL is estimable based on Eq. (7).

In addition, since the model of ambiguity-float solutions is considered fault-free, the probability of each incorrect integer combination is estimable if ambiguities are fixed using the bootstrapping method based on the ambiguity-float solution (Teunissen 2002). Different from the conservative assumption, a tighter bound could be estimated for integrity monitoring by analyzing the impact of a part of these incorrect combinations on the positioning results could be written as (Khanafseh and Pervan 2010; Khanafseh and Langel 2011; Khanafseh et al. 2012):

$$P(|\hat{x}_v - x_v| > \mathrm{VPL}|H_0)P(H_0) + \sum_{n=1}^{k} P(|\hat{x}_v - x_v| > \mathrm{VPL}|H_n)P(H_n) + \sum_{n=k+1}^{\infty} P(|\hat{x}_v - x_v| > \mathrm{VPL}|H_n)P(H_n) = \mathrm{PHMI}_v \quad (8)$$

$$P(|\hat{x}_v - x_v| > VPL|H_0)P(H_0) + \sum_{n=1}^{k} P(|\hat{x}_v - x_v| > VPL|H_n)P(H_n) < PHMI_v - \sum_{n=k+1}^{\infty} P(H_n) \quad (9)$$

where H_n is the alternative hypothesis of nth incorrect combinations and the 1st to kth combination is evaluated in Eqs. (8–9). Then, VPL is also estimable based on Eq. (9).

Integrity monitoring for the fault-tolerant model

Different from the fault-free model, the PL estimation of the fault-tolerant model is based on detection statistics and it is assumed that there may be undetected errors in the model. The detection test results could be various (e.g.,, multiple hypothesis solution separation, chi-square). The MHSS ARAIM scheme is a fault-tolerant solution. Different from the fault-free model-based solution, the fault-tolerant solution can perform properly in scenarios with or without fault because it is already considered in the model. This is the reason why the integrity monitoring process for aviation applications is usually based on a fault-tolerant solution (Brown 1996). The algorithm of the fault-tolerant model could be expressed as

$$\sum P(\text{position error} > \mathrm{PL} \,\&\, \text{test passed}) \; P(\text{fault occurance}) < \mathrm{PHMI} \quad (10)$$

which can be further represented as:

$$\sum_{n=0}^{\infty} P(|\hat{x}_v - x_v| > \mathrm{VPL}\& y \in \Omega|H_n)P(H_n) < \mathrm{PHMI} \quad (11)$$

where y is the measurement and Ω is the measurement region based on the test statistics.

In the existing studies, wrong ambiguity fixing after validation is not externally considered using the fault-tolerant model when estimating the PLs no matter in the solution separation-based RAIM method (Wang et al. 2020; Wang and El-Mowafy 2021) or chi-square-based RAIM method (Feng et al. 2009, 2012; Gao et al. 2021). Attributed to the computation of the slope for chi-square-based algorithms and the covariance of the solution separation-based algorithms for excluding each satellite, if only one ambiguity is fixed incorrectly, it can still be protected properly if the probability is included in a single satellite fault. This is because the wrong ambiguity fixing for a satellite could be excluded by excluding that specific satellite in this scenario. Nevertheless, the existing methods did not in principle consider the situations where many ambiguities could be fixed incorrectly at the same time, which is also noted by Wang et al. (2022).

Traditional MHSS ARAIM method

In the MHSS ARAIM framework, a fault detection procedure should be conducted before estimating the PL. The principle of MHSS detection is to compare the all-in-view solution and other subset solutions excluding the monitored measurements. The solutions of subsets could be estimated by Least squares of subsets in the least squares estimation or parallel filters of subsets in the Kalman filter. The threshold of fault detection using the solution separation test (SST) could be written as (Blanch et al. 2012):

$$T_{k,q} = K_{\mathrm{fa},q}\sigma_{\mathrm{ss},q}^{(k)} \quad (12)$$

$$\sigma_{\mathrm{ss},q}^{(k)2} = \sigma_q^{(k)2} - \sigma_q^{(0)2} \quad (13)$$

$$K_{\mathrm{fa},1} = K_{\mathrm{fa},2} = Q^{-1}\left(\frac{P_{\mathrm{FA}_{\mathrm{Horizontal}}}}{4N}\right) \lhd \quad (14)$$

$$K_{\mathrm{fa},3} = Q^{-1}\left(\frac{P_{\mathrm{FA}_{\mathrm{Vertical}}}}{2N}\right) \quad (15)$$

where k is the index of subsets (0 represents the all-in-view solution); $\sigma_q^{(k)}$ is the standard deviation of the qth unknown for the kth subsets; $\sigma_{\mathrm{ss},q}^{(k)}$ is the standard deviation of the solution separation test of the qth unknown for the kth subsets versus the all-in-view subsets;

q is the index of coordinates (1, 2, 3 represent the east, north, and vertical coordinate components); $Q^{-1}(p)$ is the quantile of the standard normal distribution for $(1-p)$; $P_{\mathrm{FA\_Vertical}}$ is the false alarm probabilities in the vertical direction; $P_{\mathrm{FA\_Horizontal}}$ is the false alarm probabilities in the horizontal direction; N is the number of monitored subsets; $T_{k,q}$ is the statistics of the solution separation test of the qth unknown for the kth subsets.

The solution separation test could be conducted based on (12–15). If any $\left|x_q^{(k)} - x_q^{(0)}\right| > T_{k,q}$ where $x_q^{(k)}$ is the qth unknown for the kth subsets, it is considered that the fault is detected.

When there is no fault detected, the PL could be estimated as

$$2Q\left(\frac{\mathrm{PL}_q + b_q^{(0)}}{\sigma_q^{(0)}}\right) + \sum_{i=0}^{N} Q\left(\frac{\mathrm{PL}_q - T_{i,q} - b_q^{(i)}}{\sigma_q^{(i)}}\right)P(H_i) = \mathrm{PHMI}_q\left(1 - \frac{P_{\mathrm{unmonitored}}}{\mathrm{PHMI}_{\mathrm{overall}}}\right) \quad (16)$$

where $PHMI_{overall}$ is the overall budget for the PHMI; $PHMI_q$ is the allocated budget of PHMI for qth component; $P_{unmonitored}$ is the probability of unmonitored faults and $b_q^{(i)}$ is the effect caused by the nominal bias at qth components for the ith subset.

New integrity monitoring method considering wrong ambiguity fixing

As shown in Eq. (2), the mixed integer least-squares could be considered as real-valued least squares. Then the ambiguity fixing is considered as a process of adding the pseudo-measurements $\check{a}$, which could be monitored similarly as other measurements in vector y under the MHSS ARAIM framework.

When the ambiguities are fixed into an incorrect integer vector, the positioning errors will have the same distribution of correctly fixed solution but with a bias (Khanafseh and Pervan 2010; Khanafseh and Langel 2011; Khanafseh et al. 2012), which can be verified with the following simulation results presented in Fig. 1, with the separately added four incorrect ambiguity fixing cases where wrong ambiguity fixings are simulated to $\check{a}$ in Eq. (2).

In this study, we will treat wrong ambiguity fixing as a fault, similar to other types of faults, without assigning any probability distributions. Figure 2 presents a schematic diagram illustrating the different assumptions between treating wrong ambiguity fixing as faults and incorporating it into the MHSS ARAIM framework.

As depicted in the schematic diagram, only some of the incorrect integer combinations may be fixed, and after the validation process, some of them may already be protected in certain fault events. Additionally, some combinations may

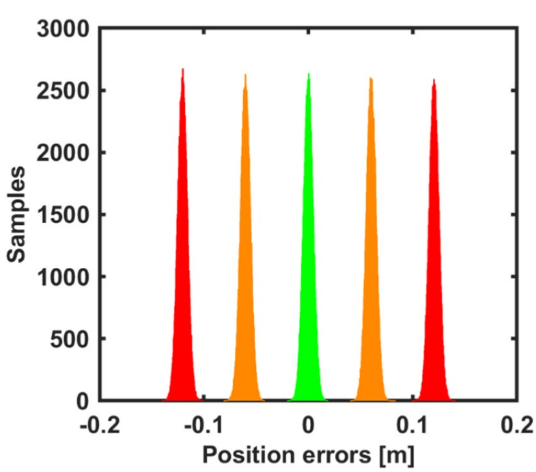

Fig. 1 Positioning error distributions in the east direction for the correct ambiguity fixing (green) and incorrect ambiguity fixing with simulated ± 1 cycle (orange) and ± 2 cycles (red)

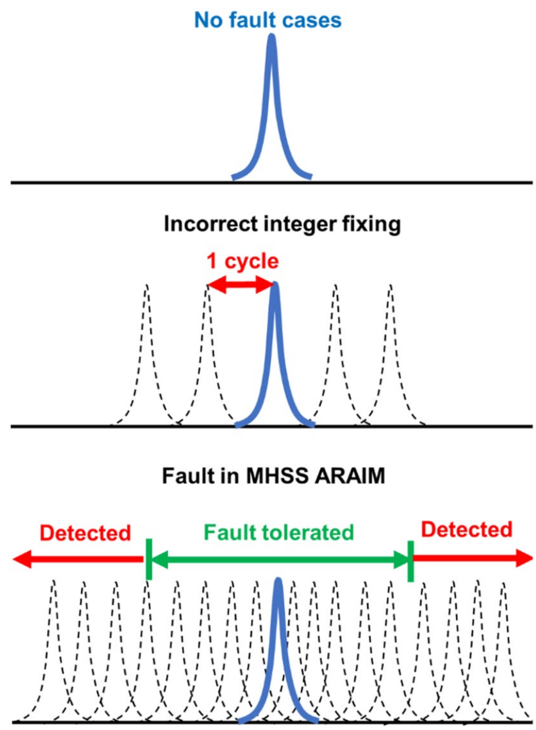

Fig. 2 All the sizes of wrong ambiguity fixings treated as faults protected within the MHSS ARAIM framework

not lead to significant positioning errors. However, in the MHSS ARAIM framework, this information is not taken into account, but on the conservative side, the integrity of the positioning results can be protected adequately for any magnitude of faults, without relying on any assumptions about the overall distribution of faults. Therefore, this approach can safeguard against wrong ambiguity fixings and measurement faults of any size similarly.

The probability of wrong ambiguity fixing can be considered as part of unmonitored faults. This method was originally proposed under the fault-free framework by Pervan and Chan (2001) and was later briefly introduced and analyzed under the MHSS ARAIM framework by Zhang et al. (2023). The limitation of this method is that the probability of wrong ambiguity fixing should be smaller than the overall PHMI budget.

In this study, the proposed new method is to monitor the wrong ambiguity fixing as an individual type of fault. Specifically, in this new integrity monitoring method for PPP-RTK, the wrong ambiguity fixing is monitored as the fifth type of fault event, together with satellite fault, constellation fault, tropospheric correction fault, and ionospheric correction fault (Zhang et al. 2023; Zhang and Wang 2023).

The probability of wrong ambiguity fixing in a fault-free scenario could be estimated by the success rate without other validation methods; nevertheless, this is not always valid. The proposed new method has the flexibility to change the defined probability of wrong ambiguity fixing based on the success rate as a reference, because the ambiguities may not be fixed at the current epoch by the LAMBDA method, or it has been validated based on other strategies.

Considering that the ambiguities are usually fixed by the LAMBDA method, correlations may exist in the ambiguity-fixing results. Therefore, the subset of monitoring wrong ambiguity fixing will exclude all ambiguity-fixing pseudo-measurements.

For the new method, different from the subsets in PPP ambiguity float solutions, the subsets for monitoring wrong ambiguity fixing by excluding all ambiguity-fixing pseudo-measurements are also implemented. A summary of the subset filter for the monitored fault and the correspondingly excluded measurements is presented in Table 1. The scheme of monitoring multiple simultaneous faults will be the same which can exclude multiple corresponding measurements in Table 1.

In the traditional MHSS ARAIM framework, it is considered faults are independent so that it can easily estimate the probability of multiple simultaneous faults and unmonitored faults. Therefore, it is also assumed that the incorrect ambiguity fixing is independent of other faults, and the same method could be used in this study. However, correlations may exist between wrong ambiguity fixing and other faults. In other words, wrong ambiguity fixing may be caused by other faults. Therefore, the process to mitigate the correlation is critical in applications under this assumption. For example, checking the ambiguity fixing results in each subset and rejecting the suspicious ambiguity fixing results when they are different in subsets. This could significantly mitigate the wrong ambiguity fixing caused by other faults. Otherwise, the probability of multiple simultaneous faults cannot be estimated based on the independent event assumptions and the probabilities of multiple simultaneous faults need to be externally determined and this will not change the other part of the MHSS ARAIM based integrity monitoring scheme for PPP-RTK.

Therefore, with the extension of the special pseudo-measurements for integer ambiguity fixing to the current MHSS ARAIM framework, the procedure of the new integrity monitoring method considering wrong ambiguity fixing in PPP-RTK will be identical to PPP float solutions under the MHSS ARAIM framework, but yet with different subset results in the SST and PL estimation process. In addition, the proposed new integrity monitoring method can also be used for cases of continuous fixing or the fix-and-hold method for ambiguity-fixing.

In order to illustrate the fault detection operations with SST, case studies are conducted with simulated PPP-RTK positioning with a total of 13 satellites, under the assumptions that only two types of faults may happen: one satellite fault, or wrong ambiguity fixing fault, but two simultaneous faults including both satellite fault and wrong ambiguity fixing may happen. Therefore, there are a total of 27 SSTs to be calculated in each fault detection case study. Here two special cases are considered. In both cases, faults are simulated on the first satellite along with incorrect ambiguity fixing, albeit with different fault magnitudes. Table 2 displays the SST results for the vertical direction in these two cases. It is evident that faults are correctly detected in Case 1, while they remain undetected in Case 2. This is a normal outcome as the detection of faults may vary depending on their sizes and the geometry of the positioning system. In this particular scenario, it is acceptable for the PL estimation to tolerate undetected faults. Hence, Monte Carlo simulation is necessary to verify the proposed method, and the results of these simulations will be presented in the subsequent sections.

Simulation experiment

Before testing the proposed new integrity monitoring method based on the MHSS ARAIM framework, simulation experiments are presented to illustrate the limitations of the existing methods. For the real dataset simulation, the static dataset results of using PPP-RTK are used in this simulation experiment to avoid undetected outliers in the dataset and mixed with the simulated wrong ambiguity fixing to preserve the valid assumption.

Monte Carlo simulation analysis under the unchanged geometry

In this section, 10^8 samples based on the errors following the Gaussian distribution are simulated for each result.

Table 1 Monitored types of faults and the corresponding subset of ambiguity-fixing results

Monitored fault	Subset filter
Satellite fault	Remove all the measurements of the satellite
Constellation fault	Remove all the measurements of the constellation
Ionospheric correction fault	Remove all the pseudo-measurements of ionospheric corrections
Tropospheric correction fault	Remove all the pseudo-measurements of tropospheric corrections
Incorrect ambiguity fixing	Remove all the pseudo-measurements of ambiguity fixing (return to the ambiguity-float solutions)

Table 2 Two cases of SST when having two simultaneous faults (one satellite fault and wrong ambiguity fixing) (unit: mm)

Excluded subset	Threshold	Case 1		Case 2	
		SST	Detection status	SST	Detection status
Sat1	7.9	0.9	No	1.4	No
Sat2	2.2	0.2	No	1.1	No
Sat3	12.3	5.7	No	0.7	No
Sat4	4.2	0.9	No	1.2	No
Sat5	7.2	5.2	No	0.8	No
Sat6	3.3	0.4	No	0.6	No
Sat7	9.3	7.5	No	1.4	No
Sat8	9.6	1.9	No	0.3	No
Sat9	3.3	0.3	No	0.3	No
Sat10	9.0	2.9	No	2.8	No
Sat11	8.2	3.4	No	2.1	No
Sat12	8.0	2.4	No	1.6	No
Sat13	3.4	0.4	No	2.7	No
Fixed ambiguities	2241.8	1504.9	No	347.4	No
Fixed ambiguities and Sat1	2377.2	2574.3	Detected	435.4	No
Fixed ambiguities and Sat2	2253.0	1485.6	No	68.9	No
Fixed ambiguities and Sat3	2558.6	1546.6	No	303.2	No
Fixed ambiguities and Sat4	2280.3	1400.8	No	439.6	No
Fixed ambiguities and Sat5	2354.0	1272.5	No	36.4	No
Fixed ambiguities and Sat6	2266.0	1235.4	No	440.5	No
Fixed ambiguities and Sat7	2425.2	1401.4	No	195.2	No
Fixed ambiguities and Sat8	2437.1	1378.6	No	527.6	No
Fixed ambiguities and Sat9	2266.6	1341.4	No	460.2	No
Fixed ambiguities and Sat10	2414.2	1350.3	No	358.4	No
Fixed ambiguities and Sat11	2385.8	1728.1	No	464.4	No
Fixed ambiguities and Sat12	2380.6	1545.4	No	613.0	No
Fixed ambiguities and Sat13	2268.2	1421.4	No	307.4	No

In order to facilitate the simulation of the noise of the measurements and the correlation between the results, the results are simulated based on the real geometry of single-epoch measurements. Figures will be used to evaluate the relations between the actual integrity risks and the defined budget of integrity risks. The actual integrity is calculated based on the Monte Carlo method to evaluate the samples exceeding the PLs estimated based on the integrity budget. Lines above the equivalent line represent the situations that the method can properly over-bound integrity risks based on the defined budget. On the contrary, lines below the equivalent line represent the situations that the method underestimates the integrity risks.

The first simulation aims to present the effects of unmonitored wrong ambiguity fixing. In this simulation, 616,864 out of 10^8 samples are found that ambiguities are fixed incorrectly. If all the results are fixed correctly, the actual integrity risks will equal the defined budgets because the stochastic model is perfectly matched in this simulation. Figure 3 presented the result of the integrity risks of these samples with wrong ambiguity fixing versus the integrity risks of the same samples when their ambiguity fixing is correct. The incorrect fix line counts the probabilities of these 616,864 samples exceeding the estimated PLs. Meanwhile, the correct fix line counts the probabilities of these 616,864 samples exceeding the estimated PLs when their ambiguities are fixed correctly.

It can be noticed that the actual integrity risks for incorrect fixing will significantly exceed the defined budget. This showed that the risks caused by the incorrect ambiguity fixing are not ignorable in integrity monitoring.

The second simulation aims to illustrate the limitations of the method based on the fault-free model. In this simulation, the fault is randomly simulated to one measurement and a chi-square test with a false alarm rate of 1% is used to detect faults. Figure 4 showed the results of actual integrity

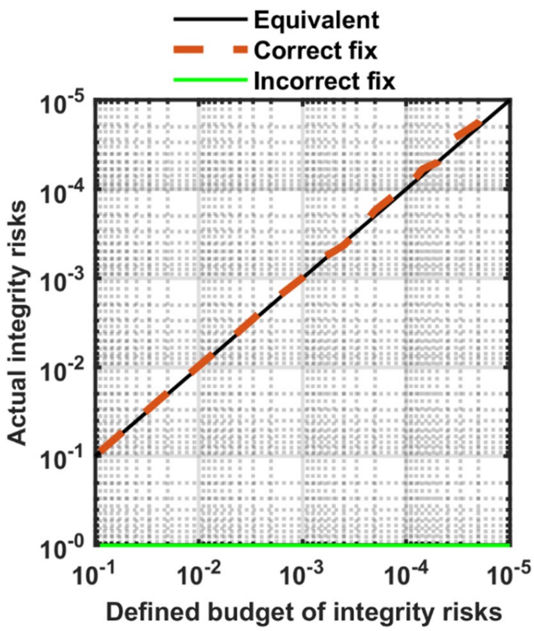

Fig. 3 Actual integrity risks for those samples with ambiguity fixed incorrectly and the results of these same samples with ambiguity fixed correctly based on the defined budget of integrity risks

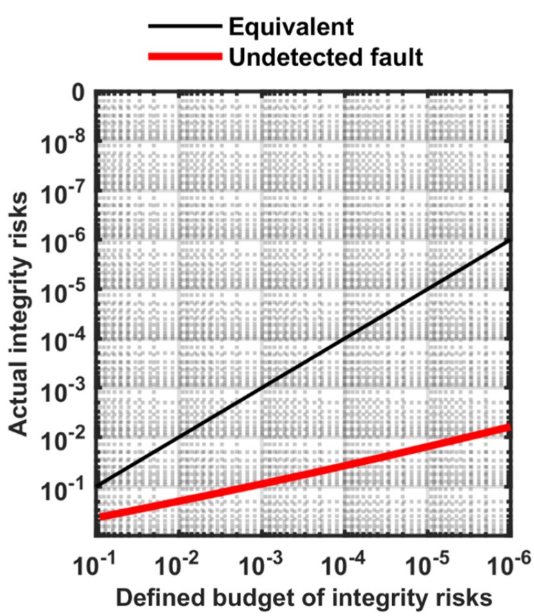

Fig. 4 Actual integrity risks with undetected fault

risks with undetected faults, and the actual integrity risks of results with undetected faults are larger than the defined budget. Therefore, if the undetected faults in measurements are not considered, even though ambiguities are all fixed correctly, the actual integrity risks may be also larger than the budget.

In addition, the proposed method for monitoring wrong ambiguity fixing based on the MHSS framework is validated based on a Monte Carlo simulation with an unchanged positioning geometry and properly determined stochastic model. Considering other faults in the fault model may enlarge the PL and different sizes of faults may also significantly affect the results. In this simulation, the fault model only considers the fault of wrong ambiguity fixing, but this will not change the applicability of the new method based on the MHSS ARAIM algorithms.

The false alarm rate is defined as 10^{-4} in this simulation. The probability of wrong ambiguity fixing is determined based on the pre-evaluated Monte Carlo simulation. Since in this simulation, no faults are simulated to the measurements, the probability of wrong ambiguity fixing using the LAMBDA method could also be calculated based on the success rate. The simulation result is presented in Fig. 5

The results show that the actual integrity risks are very close to the defined budget when the defined budget of integrity risks is larger than the probability of wrong ambiguity fixing. When the defined budget of integrity risks is smaller than the probability of wrong ambiguity fixing, the actual integrity risks of the proposed method will significantly decrease.

Similar results are obtained with different geometry, for example, the results in Fig. 6. It can be noticed that this new method is available in protecting the integrity risks of wrong ambiguity fixing under the ARAIM framework. Meanwhile, the actual integrity risks using this method are

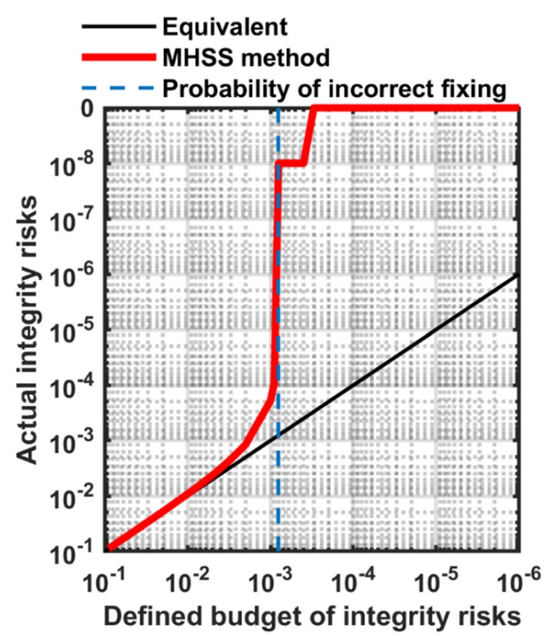

Fig. 5 Monte-Carlo simulation result based on single-epoch PPP-RTK of the defined budget of integrity risks and actual integrity risks using the proposed method

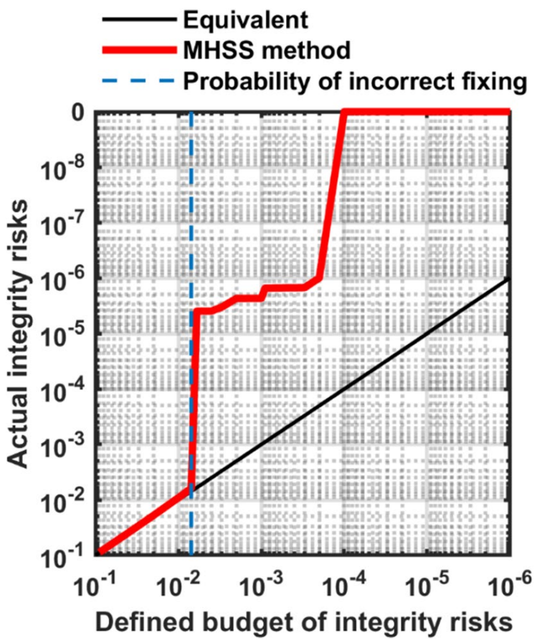

Fig. 6 Another Monte-Carlo simulation based on the defined budget of integrity risks and actual integrity risks using the proposed method

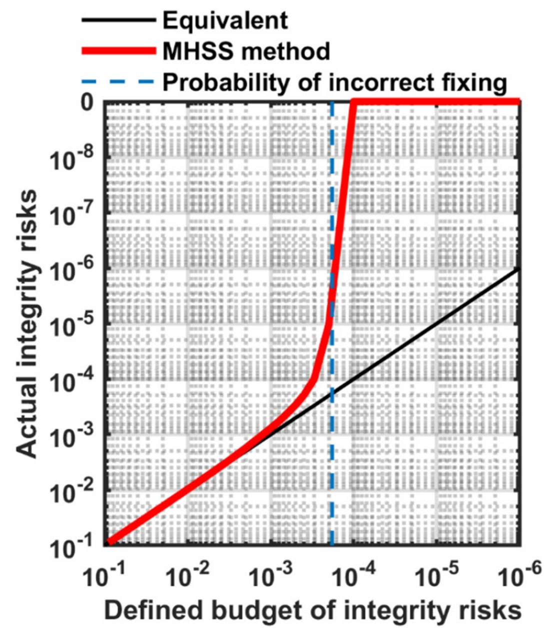

Fig. 7 The Monte-Carlo simulation result with faults of the defined budget of integrity risks and actual integrity risks using the proposed method

significantly smaller than the budget. The numerical results matched with the previous analysis that the assumption of this method to consider wrong ambiguity fixing as a fault is very conservative.

The new method is also tested with simulated measurement faults and considering wrong ambiguity fixing faults. Except for the wrong ambiguity fixing, the probability of satellite fault with 10^{-4} are simulated randomly between 3σ and 10σ to the code or phase measurements. The numerical results are presented in Fig. 7, and it can be noticed that the proposed new method also performs properly in this simulation.

Moreover, Fig. 8 also presents the results of a case with a larger probability of wrong ambiguity fixing and satellite fault with 10^{-3} so that subsets to monitor two simultaneous faults including both satellite faults and incorrect ambiguity fixing are considered. The results show that this method can also properly bound the errors based on the integrity budgets.

Fault simulation analysis in every epoch

In addition, to simulate the scenarios of wrong ambiguity-fixing at different epochs of real datasets, the wrong integer ambiguity-fixings are randomly added to every fixed narrow-lane ambiguity in the static dataset ambiguity-fixing PPP-RTK results. The probability of wrong ambiguity fixing is defined as $P_{\text{wrong\_fixing}} = 10^{-8}$/approach, and the overall PHMI is defined as 10^{-8}/approach. Details of settings could refer to Zhang et al. (2023). It is noted that the unit "per approach" is used to represent the probability of each estimation process (Pervan et al. 1998). In real applications, it can be used as "per sample" and transferred to "per hour" based on the sampling rate (Blanch et al. 2020).

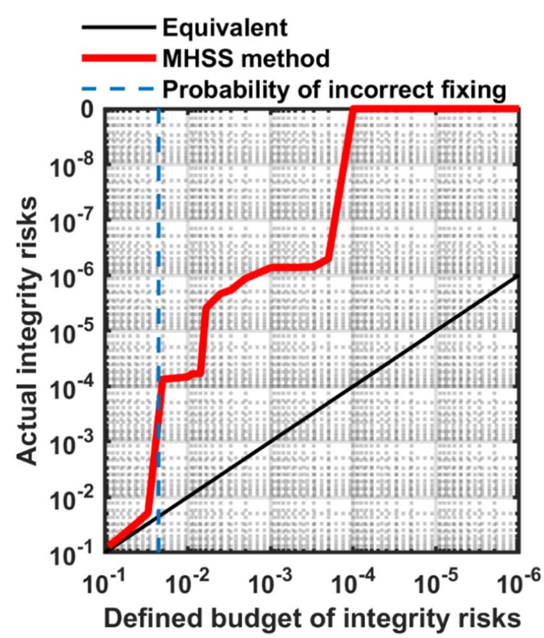

Fig. 8 The Monte-Carlo simulation result considering two simultaneous faults of the defined budget of integrity risks and actual integrity risks using the proposed method

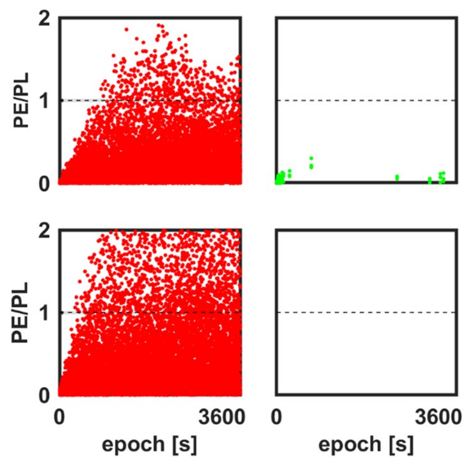

Fig. 9 PPP-RTK ambiguity-fixed results for the ratio of PEs divided by PLs with the simulated wrong ambiguity-fixing: (top) no more than 1 cycle and (bottom) no more than 2 cycles. Each epoch contains three points representing the results of three directions. The red point (3562 out of 3600 epochs no more than 1 cycle and 3600 out of 3600 epochs no more than 2 cycles) is the rejected results (left), and the green point (38 out of 3600 epochs no more than 1 cycle and 0 out of 3600 epochs no more than 2 cycles) is the accepted results (right)

Figure 9 presented the ratio of positioning errors (PEs) divided by PLs, and the ratios larger than 1 represent that the PEs are larger than PLs. Many ratios of PE/PL are small, but they are still detected as containing fault, this is because even though they are not detected in the ambiguity-fixing integer pseudo-measurements excluded subset, they are detected in the satellite excluded subsets. It can be noticed that all the PEs of the accepted results are smaller than the PLs. Considering $P_{\text{wrong\_fixing}} = 10^{-8}$/approach and the overall PHMI is defined as 10^{-8}/approach, it is allowed a part of PEs exceeding PLs but the results are accepted at some epochs. However, none of the results with PEs exceeding PLs are accepted in this simulation, which shows that this method for monitoring wrong ambiguity-fixing is very conservative and can protect the integrity properly.

Real dataset experiment

Except for the performance evaluation based on the simulation results, the effect of wrong ambiguity fixing on the estimated PL in the real kinematic dataset also needs to be investigated. In this section, the ARAIM-related parameters were defined as PHMI = 10^{-8}/approach and $P_{FA} = 10^{-6}$; the fault probability of satellite $P_{\text{sat\_rate}} = 10^{-5}$/hr; the fault probability of constellation $P_{\text{const\_rate}} = 10^{-7}$/hr, and the fault probability of atmospheric corrections $P_{\text{atm}} = 10^{-8}$/approach. Three real-kinematic vehicle-borne datasets A, B, and C collected in Australia and processed with the PPP-RTK method are used in this experiment. A brief introduction to these three datasets is presented in Table 3. More details on these datasets are shown in Zhang et al. (2023).

First, we investigated the results with different settings defined as follows:

- *PL1* PLs for the ambiguity-fixed solution ignoring wrong ambiguity fixing
- *PL2* PLs for the ambiguity-fixed solution with $P_{\text{wrong\_fixing}} = 10^{-5}$/approach
- *PL3* PLs for PPP float solutions

The estimated PLs for three kinematic datasets are presented in Figs. 10, 11, and 12, respectively. It can be noticed

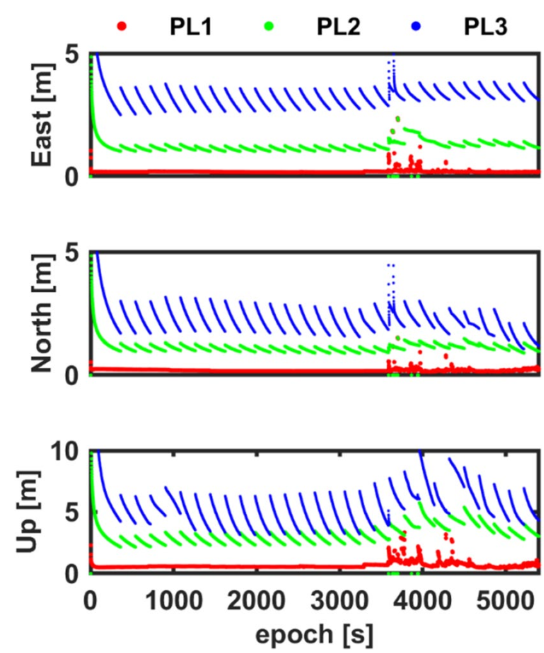

Fig. 10 Estimated PLs for different settings of Dataset A in three directions

Table 3 Information of three datasets used in the kinematic experiment

Dataset	Date	Length	Location	Environment
Dataset A	2021/11/16	34 min	Sydney, Australia	Parking area
Dataset B	2021/11/16	34 min	Goulburn, Australia	Highway
Dataset C	2021/04/03	1h30min	Yass, Australia	Town and highway

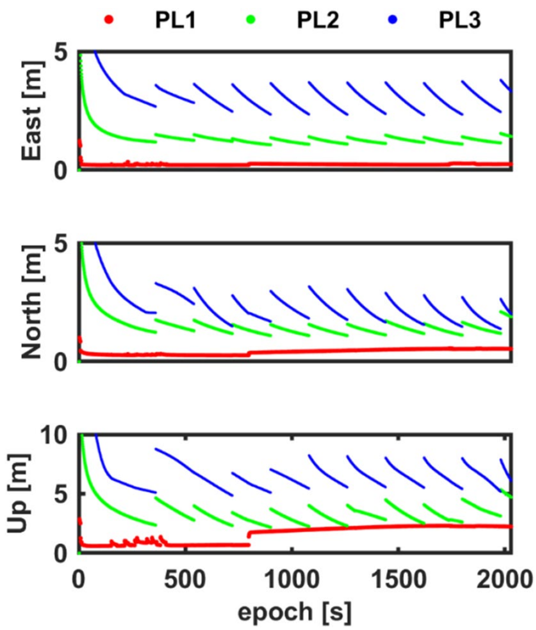

Fig. 11 Estimated PLs for different settings of Dataset B in three directions

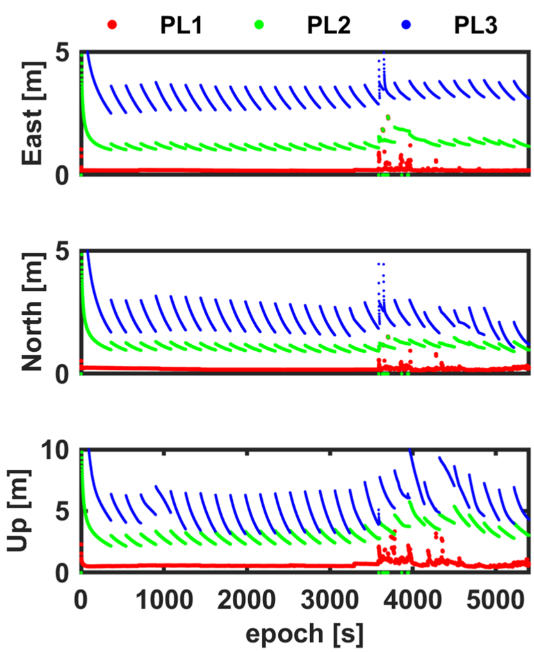

Fig. 12 Estimated PLs for different settings of Dataset C in three directions

that the estimated PL of the proposed integrity monitoring method is between the estimated PL of PPP float solutions and the ambiguity-fixed solutions ignoring the wrong ambiguity fixing.

Moreover, the proposed method is also tested with different probabilities of wrong ambiguity fixing. In addition, the median values of estimated PLs with different probabilities of incorrect ambiguity fixing are summarized in Table 4.

Table 4 Median values of different estimated PLs (cm)

Probability of incorrect fixing	Dataset	E	N	U
10^{-9}	A	34	19	62
10^{-8}	A	97	85	224
10^{-7}	A	108	95	246
10^{-6}	A	116	102	261
10^{-5}	A	122	108	274
10^{-9}	B	24	44	204
10^{-8}	B	100	110	260
10^{-7}	B	112	123	288
10^{-6}	B	120	132	308
10^{-5}	B	127	140	324
10^{-9}	C	19	18	61
10^{-8}	C	94	88	247
10^{-7}	C	105	98	271
10^{-6}	C	113	104	288
10^{-5}	C	118	109	302

The Stanford integrity diagrams of PPP-RTK ambiguity-fixed solutions for these three datasets with different probabilities of incorrect fixing are illustrated in Fig. 13. Since the ambiguity validation process is strict in this study, it can be noticed that these two methods all properly protected the position errors. From the diagrams, it can also be noticed that more results exceed the alert limit (3 m) when the probabilities of incorrect fixing increase.

It can be noticed that there is a significant increase between 10^{-9} and 10^{-8}. In Eq. (16), if $P(H_i)$ larger than $\mathrm{PHMI}_q\left(1-\frac{P_{\mathrm{unmonitored}}}{\mathrm{PHMI}_{\mathrm{overall}}}\right)$, $Q\left(\frac{\mathrm{PL}_q-T_{i,q}-b_q^{(i)}}{\sigma_q^{(i)}}\right)$ should be smaller than 1, otherwise $Q\left(\frac{\mathrm{PL}_q-T_{i,q}-b_q^{(i)}}{\sigma_q^{(i)}}\right)P(H_i)$ will be larger than $\mathrm{PHMI}_q\left(1-\frac{P_{\mathrm{unmonitored}}}{\mathrm{PHMI}_{\mathrm{overall}}}\right)$ and the equation cannot be established. Thus, the PL_q should be larger than $T_{i,q}-b_q^{(i)}$ at that time. In other words, the risk of this fault is not ignorable in the PHMI budgets and thus the estimated PLs should be larger than the threshold of SST with a bias for this fault. Therefore, due to the large threshold of SST for the subset excluding the wrong ambiguity-fixing, it can be easily noticed that the effect of considering wrong fixing is not significant if the probability of wrong ambiguity fixing is smaller than the PHMI budgets excluding the probability of unmonitored faults allocated to each direction. Otherwise, the effect of considering wrong fixing is significant.

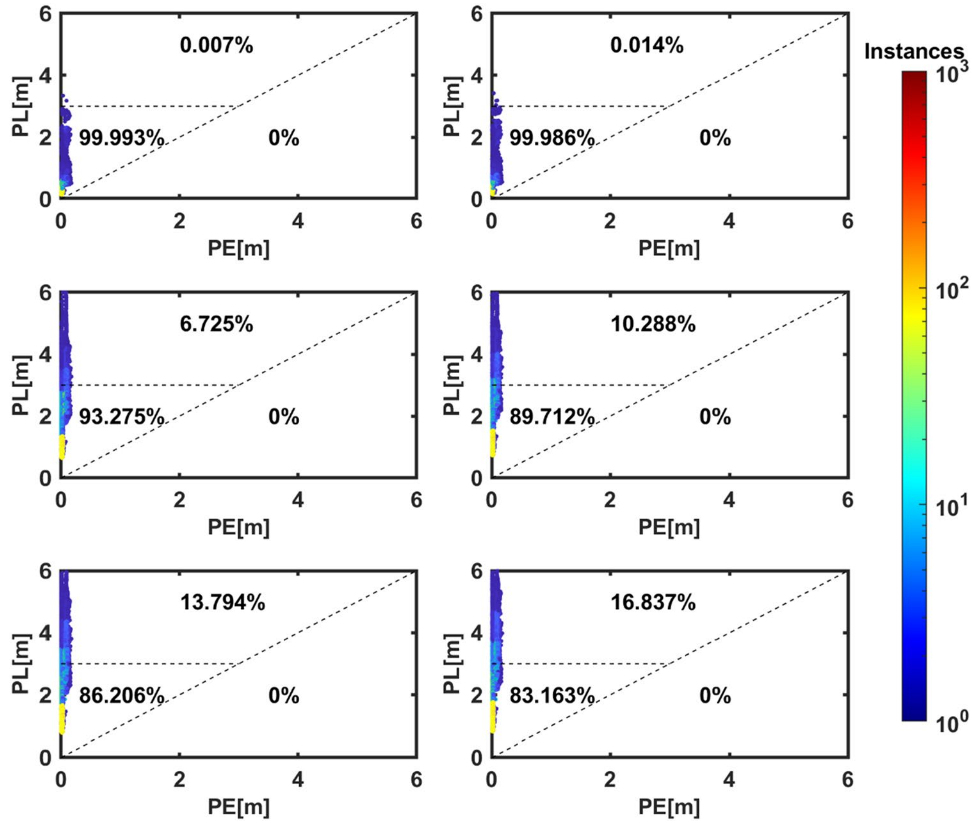

Fig. 13 Stanford integrity diagram of the PPP-RTK ambiguity-fixed results without considering risks of wrong ambiguity fixing (top left) and considering risks of wrong ambiguity fixing with different probabilities (10^{-9}: top right; 10^{-8}: middle left; 10^{-7}: middle right; 10^{-6}: bottom left; 10^{-5}: bottom right)

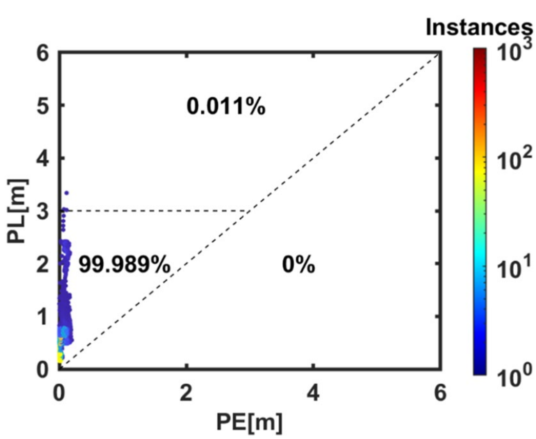

Fig. 14 Stanford integrity diagram of the PPP-RTK ambiguity-fixed results for monitoring incorrect fixing from satellites and constellations

Furthermore, an alternative approach is to monitor and identify satellite and constellation integer ambiguity faults rather than all-satellite integer ambiguity faults only. As depicted in Fig. 14, this alternative approach provides the outcomes where the probability of incorrect integer ambiguity fixing from satellites and constellations is assumed to be equivalent to the occurrence of faults within satellites and constellations. It is evident that the performance of this alternative approach closely resembles the one that disregards the potential risks associated with incorrect ambiguity fixing in Fig. 13. However, it is important to note that the ambiguities arising from multiple satellites and constellations might concurrently be incorrectly resolved due to their inherent correlation. Further investigation is required in a separate study.

Conclusion and discussion

This study proposes and analyzes a method for considering external risks arising from wrong integer ambiguity fixing. In the existing ARAIM with the framework of MHSS, the classic least squares models are adopted, which do not consider the constraints for the parameters. In the case of integer ambiguity fixing, however, we need to add the integer ambiguity parameter constraints. Therefore, the proposed new method innovatively monitors carrier phase integer ambiguity fixing as pseudo-measurements, which may contain an individual type of fault named as wrong integer ambiguity fixing. The solution separation statistics to detect the wrong integer ambiguity fixing can be calculated by excluding all pseudo-measurements of integer ambiguity fixing and returning to the ambiguity-float solution in the subset.

The numerical results have demonstrated that when the probability of wrong ambiguity resolution is assumed to be small, the positioning protection levels (PLs) are very close to those obtained when ignoring the risks of wrong integer ambiguity fixing. However, when the probability of wrong integer ambiguity fixing increases, the PLs will also increase. In general, as expected, the PLs with the new method considering the wrong integer ambiguity fixing will fall between the PLs of the ambiguity-float solutions and the PLs of ambiguity-fixed solutions ignoring the risks of wrong integer ambiguity fixing.

It can be noted that, with the proposed new integrity monitoring method considering wrong ambiguity fixing, the positioning protection levels will undergo a converging process similar to the positioning accuracy of the ambiguity-float solutions. Such a converging process is closely linked to the geometric strength within the precise point positioning solution and the uncertainty of carrier phase ambiguity parameters. In addition, considering the ambiguity fixing process in PPP-RTK and RTK are essentially identical, this method could be also implemented in RTK.

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Author contributions W. Z. and J. W. designed the research, W. Z. conducted the experiments, W. Z. and J. W. contributed to the writing of the paper.

Data availability The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Competing interests The authors declare no competing interests.

References

Blanch J, Walker T, Enge P, Lee Y, Pervan B, Rippl M, Spletter A, Kropp V (2015) Baseline advanced RAIM user algorithm and possible improvements. IEEE Trans Aerosp Electron Syst 51(1):713–732

Blanch J, Walter T, Enge P, Lee Y, Pervan B, Rippl M, Spletter A (2012) Advanced RAIM user algorithm description: integrity support message processing, fault detection, exclusion, and protection level calculation. ION GNSS 2012, US Institute of Navigation, Nashville, pp 2828–2849

Blanch J, Walter T, Norman L, Gunning K, De Groot L (2020) Solution separation-based FD to mitigate the effects of local threats on PPP integrity. In: 2020 IEEE/ION Position, location and navigation symposium (PLANS), Portland, pp 1085–1092

Blanch J (2011) MHSS ARAIM algorithm and constrained constellation faults. Briefing presented at the EU-US cooperative WG-C ARAIM subgroup meeting, Portland

Brown RG (1996) Receiver autonomous integrity monitoring theory and applications, vol II. Published by American Institute of Aeronautics and Astronautics, Washington, pp 143–165

Feng S, Ochieng W, Moore T, Hill C, Hide C (2009) Carrier phase-based integrity monitoring for high-accuracy positioning. GPS Solut 13(1):13–22

Feng S, Ochieng W, Samson J, Tossaint M, Hernandez-Pajares M, Juan JM, Sanz J, Aragón-Àngel À, Ramos-Bosch P, Jofre M (2012) Integrity monitoring for carrier phase ambiguities. J Navig 65(1):41–58

Gao Y, Yang J, Gao Y, Huang G (2021) A linear Kalman filter-based integrity monitoring considering colored measurement noise. GPS Solut 25(2):1–13

Green GN, Humphreys TE (2018) Data-driven generalized integer aperture bootstrapping for high-integrity positioning. IEEE Trans Aerosp Electron Syst 55(2):757–768

Gunning K, Blanch J, Walter T, De Groot L, Norman L (2018) Design and evaluation of integrity algorithms for PPP in kinematic applications. ION GNSS 2018, US Institute of Navigation, Miami, pp 1910–1939

Gunning, K, Blanch J, Walter T, De Groot L, Norman L (2019) Integrity for tightly coupled PPP and IMU. ION GNSS+ 2019, US Institute of Navigation, Miami, pp 3066–3078

Jokinen A, Feng S, Schuster W, Ochieng W, Hide C, Moore T, Hill C (2013a) Integrity monitoring of fixed ambiguity precise point positioning (PPP) solutions. Geospat Inf Sci 16:141–148

Jokinen A, Feng S, Schuster W, Ochieng W, Yang L, Moore T, Hill C (2013b) Improving ambiguity validation and integrity monitoring of precise point positioning (PPP). In: Proceedings ION GNSS+ 2013, institute of navigation, Nashville, pp 1224–1233

Khanafseh S, Joerger M, Pervan B (2012) Integrity risk of cycle resolution in the presence of bounded faults. In: Proceedings of IEEE/ION PLANS 2012, Myrtle Beach, South Carolina, pp 664–672

Khanafseh S, Langel S (2011) Implementation and experimental validation of cycle ambiguity resolution with position domain integrity risk constraints. Navigation 58(1):45–58

Khanafseh S, Pervan B (2010) A new approach for calculating position domain integrity risk for cycle resolution in carrier phase navigation systems. IEEE Trans Aero Electron Syst 46(1):296–307

Li T, Wang J (2014) Analysis of the upper bounds for the integer ambiguity validation statistics. GPS Solut 18(1):85–94

Liu T, Jiang W, Laurichesse D, Chen H, Liu X, Wang J (2020) Assessing GPS/Galileo real-time precise point positioning with ambiguity resolution based on phase biases from CNES. Adv Space Res 66:810–825

Pervan BS, Pullen SP, Christie JR (1998) A multiple hypothesis approach to satellite navigation integrity. Navigation 45(1):61–71

Pervan BS, Chan FC (2001) System concepts for cycle ambiguity resolution and verification for aircraft carrier landings. In Proceedings of the 14th international technical meeting of the satellite division of the institute of navigation (ION GPS 2001) Salt Lake City, pp 1228–1237

Simon D, Chia TL (2002) Kalman filtering with state equality constraints. IEEE Trans Aerosp Electron Syst 38(1):128–136

Teunissen PJG, Khodabandeh A (2015) Review and principles of PPPRTK methods. J Geodesy 89(3):217–240

Teunissen PJG, Odijk D, Zhang B (2010) PPP-RTK: Results of CORS network-based PPP with integer ambiguity resolution. J Aeronaut Astronaut Aviat Ser A 42(4):223–230

Teunissen PJG, De Jonge PJ, Tiberius CCJM (1995) The LAMBDA method for fast GPS surveying. In: Proceedings of the international symposium GPS technology applications, Bucharest, Romania, pp 203–210

Teunissen PJG, Joosten P, Tiberius CCJM (2002) A comparison of TCAR, CIR and LAMBDA GNSS ambiguity resolution. In:

Proceedings of ION GPS, institute of navigation, Portland, pp 2799–2808
Verhagen S, Li B, Teunissen PJG (2013) Ps-LAMBDA: ambiguity success rate evaluation software for interferometric applications. Comput Geosci 54:361–376
Wang K, El-Mowafy A (2021) Effect of biases in integrity monitoring for RTK positioning. Adv Space Res 67(12):4025–4042
Wang J, Stewart M, Tsakiri M (1998) A discrimination test procedure for ambiguity resolution on-the-fly. J Geodesy 72(11):644–653
Wang K, El-Mowafy A, Rizos C, Wang J (2020) Integrity monitoring for horizontal RTK positioning: new weighting model and overbounding CDF in open-sky and suburban scenarios. Rem Sens 12(7):1173
Wang K, EI-Mowafy A, Qin W, Yang X (2022) Integrity monitoring of PPP-RTK positioning; part I: GNSS-based IM procedure. Rem Sens 14(1):44
Wang S, Zhan X, Zhai Y, Gao Z (2023) solution separation-based integrity monitoring for integer ambiguity resolution-enabled GNSS positioning. In: Proceedings of the 2023 international technical meeting of the institute of navigation, Long Beach, California, pp 492–513
Working Group C (2016) ARAIM technical subgroup milestone 3 report of the EU-US cooperation on satellite navigation, Working Group C. https://www.gps.gov/policy/cooperation/europe/2016/working-group-c/.
Zhang W, Wang J (2023) Integrity monitoring scheme for single-epoch GNSS PPP-RTK positioning. Satell Navig 4:10. https://doi.org/10.1186/s43020-023-00099-1
Zhang W, Wang J, El-Mowafy A, Rizos C (2023) Integrity monitoring scheme for undifferenced and uncombined multi-frequency multi-constellation PPP-RTK. GPS Solut 27(2):68. https://doi.org/10.1007/s10291-022-01391-4

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