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Formulation and evaluation of polarization-modulated triple-view information display with three TN-LCD layers

Keitaro Uchida[1,2] · Shiro Suyama[1] · Hirotsugu Yamamoto[1]

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Abstract
We have derived formulation and evaluation of polarization-modulated triple-view information display with three TN-LCD layers. The proposed display is composed of three liquid–crystal display (LCD) panels that are placed at a certain gap between polarizers. The viewed image is generated after polarization modulations by the three LCD panels. The polarization modulated results depend on the view directions. Thus, information of three binary images is shared between the displayed images on the three LCD panels.

Keywords LCD panels · Polarization modulation · Multiple-view display · Information sharing

1 Introduction

By layering LCD panels, multiple viewpoints can be displayed with no degradation in resolution. Our previous work which used two-layered LCD panels, a dual-views display was realized without reducing the resolution of the observed images by utilizing an information sharing algorithm between the LCD panels [1]. Recently, multi-layered LCD panels were utilized for light-field 3D displays [2, 3], where the displayed images were obtained by optimization methods. We proposed securing information display techniques [4]. Secure display to generate two limited viewing zones was realized by use of optical encryption codes [5, 6]. Secure display based on polarization-modulation by use of a patterned retarder array was reported [7, 8]. This display could also be used to limit the direction of observation for aerial imaging by retro-reflection (AIRR) [9, 10]. We further increased the number of layers of LCD panels and fabricated a three-layered LCD display. By using a three-layered liquid LCD panels, horizontal triple-view display and secure display with strictly restricted viewing angles were realized [11, 12]. In addition, triple-view display was realized, allowing observation from the left side, center, and top position [13, 14]. However, a general formulation of the triple-view display has not yet been done.

The purpose of this paper is to clarify general formulation for triple-view display using three-layered LCD panels. The purpose of this paper is to clarify the existence of general formulation for generating images for triple-view display using the three-layer LCD panels. The general formulation for triple-view display was reported for the case in which the center viewpoint is always included among the five directions (upper, bottom, left, right, and center) [15]. In this paper, we discuss the process of deriving the general formulation in more detail. Additionally, we examine the general formulation for the case where the center viewpoint is not included. This allows the viewpoint position to be selected more freely.

2 Structure and function of triple-view display

In an ordinary TN-LCD display, LCD panel is placed between two cross-nichol polarizers. When displaying white, the LCD panel rotates the polarization by 90 degrees, allowing it to pass through the cross-nichol polarizers. The display we produced to realize our triple-view display [11–14], three LCD panels

✉ Hirotsugu Yamamoto
hirotsugu@yamamotolab.science

[1] Utsunomiya University, Yoto 7-1-2, Utsunomiya City, Tochigi 321-0904, Japan

[2] NIPPON SIGNAL CO., LTD., 1836-1, Ezura, Kuki, Saitama 346-8524, Japan

are placed between cross-nichol polarizers with 1 cm gap. Figure 1 shows the structure of our proposed triple-view display by use of three-layered LCD panels. TN-LCD panels are used, with a screen size of 10.4″, resolution of 800×600, and pixel pitch of 0.264×0.264 mm. In the triple-view display, which limits the viewpoint to any three directions, black-and-white binary images are displayed on three LCD panels. In this case, the rotation angles of polarization are additive and rotate up to 270 degrees; white is observed when the rotation of 90 degrees is odd, and black is observed when it is even. This relationship can be expressed by XOR, where white is 1 and black is 0.

3 Formation for triple-view display

Figure 2 shows the structure of the display which enables any triple-view display. Pixel values of the displayed images in a position (k, l) on the rear, the middle, and the front LCD panel are expressed by $r_{k,l}$, $m_{k,l}$, and $f_{k,l}$, where

$$r_{k,l}, m_{k,l}, f_{k,l} \in \{0, 1\}. \tag{1}$$

Because of the certain gap between the three LCD panels, represented pixel values at each viewing positions, depend

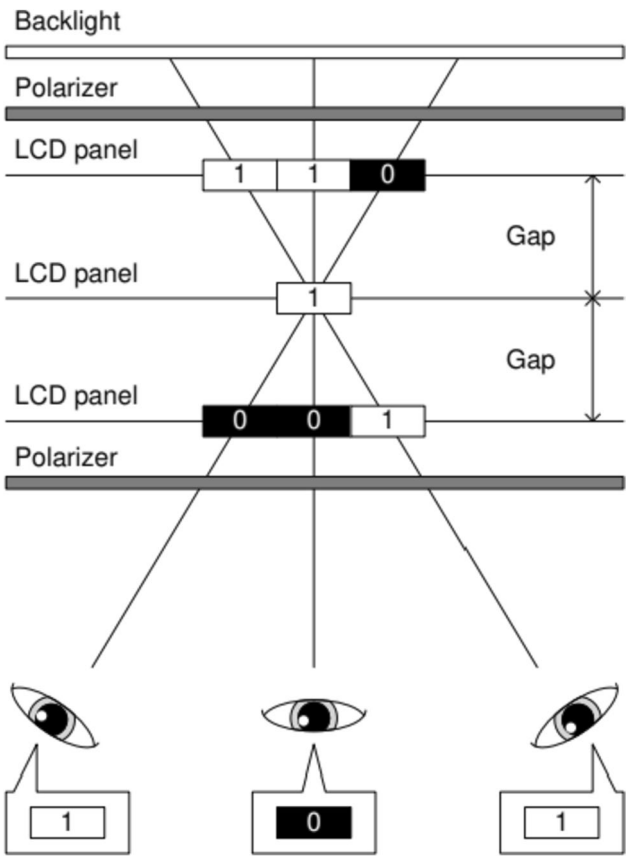


Fig.1 Relationship between observed results and pixel values for three-layered LCD panels

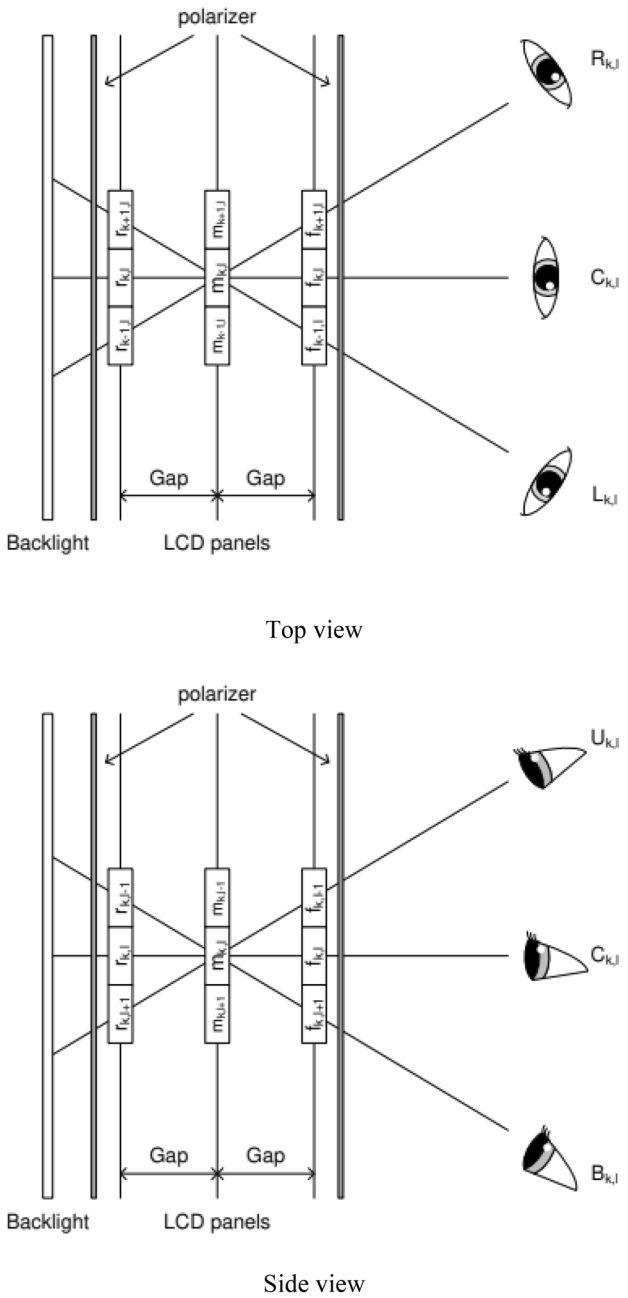


Fig. 2 Pixel arrangements and geometries to generate any triple views by use of three-layered LCD panels. The pixel size of LCD panel was 0.264×0.264 mm. The gap was 1 cm. The pixel size of the displayed image was set so that the viewing angle for each viewpoint was ±4 degrees

on the overlapped pixel values. Thus, these represented pixel values are obtained by

$$U_{k,l} = r_{k,l+1} \oplus m_{k,l} \oplus f_{k,l-1}, \tag{2}$$

$$L_{k,l} = r_{k+1,l} \oplus m_{k,l} \oplus f_{k-1,l}, \tag{3}$$

$$C_{k,l} = r_{k,l} \oplus m_{k,l} \oplus f_{k,l}, \tag{4}$$

$$R_{k,l} = r_{k-1,l} \oplus m_{k,l} \oplus f_{k+1,l}, \tag{5}$$

and

$$B_{k,l} = r_{k,l-1} \oplus m_{k,l} \oplus f_{k,l+1}. \tag{6}$$

Note that $\oplus$ expresses XOR operation. In order to reproduce the desired pixel values of the triple views, Three equations from Eqs. (2–6) must be maintained simultaneously. By alliance of three formulas, following equations are determined (See appendix for derivation process):

For triple-view display (left, center, right)

$$r_{k-1,l} \oplus r_{k,l} \oplus r_{k+1,l} \oplus r_{k+2,l} = L_{k+1,l} \oplus C_{k,l} \oplus C_{k+1,l} \oplus R_{k,l} \tag{7}$$

$$m_{k,l} \oplus m_{k+2,l} = L_{k+1,l} \oplus C_{k,l} \oplus C_{k+2,l} \oplus R_{k+1,l} \tag{8}$$

$$f_{k-1,l} \oplus f_{k,l} \oplus f_{k+1,l} \oplus f_{k+2,l} = L_{k,l} \oplus C_{k,l} \oplus C_{k+1,l} \oplus R_{k+1,l} \tag{9}$$

For triple-view display (upper, center, bottom)

$$r_{k,l-1} \oplus r_{k,l} \oplus r_{k,l+1} \oplus r_{k,l+2} = U_{k,l+1} \oplus C_{k,l} \oplus C_{k,l+1} \oplus B_{k,l} \tag{10}$$

$$m_{k,l} \oplus m_{k,l+2} = U_{k,l+1} \oplus C_{k,l} \oplus C_{k,l+2} \oplus B_{k,l+1} \tag{11}$$

$$f_{k,l-1} \oplus f_{k,l} \oplus f_{k,l+1} \oplus f_{k,l+2} = U_{k,l} \oplus C_{k,l} \oplus C_{k,l+1} \oplus B_{k,l+1} \tag{12}$$

For triple-view display (upper, center, left)

$$\begin{aligned} &r_{k,l} \oplus r_{k,l+1} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+2,l-1} \oplus r_{k+2,l} \\ &= U_{k,l} \oplus U_{k+1,l} \oplus C_{k,l} \oplus C_{k+1,l-1} \oplus L_{k+1,l-1} \oplus L_{k+1,l} \end{aligned} \tag{13}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+2,l-1} \oplus m_{k+2,l} \\ &= U_{k,l} \oplus U_{k+2,l} \oplus C_{k,l+1} \oplus C_{k+2,l-1} \oplus L_{k+1,l-1} \oplus L_{k+1,l+1} \end{aligned} \tag{14}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k,l+1} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+2,l-1} \oplus f_{k+2,l} \\ &= U_{k+1,l} \oplus U_{k+2,l} \oplus C_{k+1,l+1} \oplus C_{k+2,l} \oplus L_{k+1,l} \oplus L_{k+1,l+1} \end{aligned} \tag{15}$$

For triple-view display (upper, center, right)

$$\begin{aligned} &r_{k,l} \oplus r_{k,l+1} \oplus r_{k+1,l} \oplus r_{k+1,l+2} \oplus r_{k+2,l+1} \oplus r_{k+2,l+2} \\ &= U_{k+1,l+1} \oplus U_{k+2,l+1} \oplus C_{k+1,l} \oplus C_{k+2,l+1} \oplus R_{k+1,l} \oplus R_{k+1,l+1} \end{aligned} \tag{16}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+2,l+1} \oplus m_{k+2,l+2} \\ &= U_{k,l+1} \oplus U_{k+2,l+1} \oplus C_{k,l} \oplus C_{k+2,l+2} \oplus R_{k+1,l} \oplus R_{k+1,l+2} \end{aligned} \tag{17}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k,l+1} \oplus f_{k+1,l} \oplus f_{k+1,l+2} \oplus f_{k+2,l+1} \oplus f_{k+2,l+2} \\ &= U_{k,l+1} \oplus U_{k+1,l+1} \oplus C_{k,l+1} \oplus C_{k+1,l+2} \oplus R_{k+1,l+1} \oplus R_{k+1,l+2} \end{aligned} \tag{18}$$

For triple-view display (left, center, bottom)

$$\begin{aligned} &r_{k,l} \oplus r_{k,l+1} \oplus r_{k+1,l} \oplus r_{k+1,l+2} \oplus r_{k+2,l+1} \oplus r_{k+2,l+2} \\ &= L_{k+1,l+1} \oplus L_{k+1,l+2} \oplus C_{k,l+1} \oplus C_{k+1,l+1} \oplus B_{k,l+1} \oplus B_{k+1,l+1} \end{aligned} \tag{19}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+2,l+1} \oplus m_{k+2,l+2} \\ &= L_{k+1,l} \oplus L_{k+1,l+2} \oplus C_{k,l} \oplus C_{k+2,l+2} \oplus B_{k,l+1} \oplus B_{k+2,l+1} \end{aligned} \tag{20}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k,l+1} \oplus f_{k+1,l} \oplus f_{k+1,l+2} \oplus f_{k+2,l+1} \oplus f_{k+2,l+2} \\ &= L_{k+1,l} \oplus L_{k+1,l+1} \oplus C_{k+1,l} \oplus C_{k+2,l+1} \oplus B_{k+1,l+1} \oplus B_{k+2,l+1} \end{aligned} \tag{21}$$

For triple-view display (right, center, bottom)

$$\begin{aligned} &r_{k,l} \oplus r_{k,l+1} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+2,l-1} \oplus r_{k+2,l} \\ &= R_{k+1,l} \oplus R_{k+1,l+1} \oplus C_{k+1,l+1} \oplus C_{k+2,l} \oplus B_{k+1,l} \oplus B_{k+2,l} \end{aligned} \tag{22}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+2,l-1} \oplus m_{k+2,l} \\ &= R_{k+1,l-1} \oplus R_{k+1,l+2} \oplus C_{k,l+1} \oplus C_{k+2,l-1} \oplus B_{k,l} \oplus B_{k+2,l} \end{aligned} \tag{23}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k,l+1} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+2,l-1} \oplus f_{k+2,l} \\ &= R_{k+1,l-1} \oplus R_{k+1,l} \oplus C_{k,l} \oplus C_{k+1,l-1} \oplus B_{k,l} \oplus B_{k+1,l} \end{aligned} \tag{24}$$

For triple-view display (upper, left, right)

$$\begin{aligned} &r_{k,l} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+3,l-1} \oplus r_{k+3,l+1} \oplus r_{k+4,l} \\ &= U_{k+1,l} \oplus U_{k+3,l} \oplus L_{k+2,l-1} \oplus L_{k+3,l} \oplus R_{k+1,l} \oplus R_{k+2,l-1} \end{aligned} \tag{25}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+3,l-1} \oplus m_{k+3,l+1} \oplus m_{k+4,l} \\ &= U_{k,l} \oplus U_{k+4,l} \oplus L_{k+1,l-1} \oplus L_{k+3,l+1} \oplus R_{k+1,l+1} \oplus R_{k+3,l-1} \end{aligned} \tag{26}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+3,l-1} \oplus f_{k+3,l+1} \oplus f_{k+4,l} \\ &= U_{k+1,l} \oplus U_{k+3,l} \oplus L_{k+1,l} \oplus L_{k+2,l+1} \oplus R_{k+2,l+1} \oplus R_{k+3,l} \end{aligned} \tag{27}$$

For triple-view display (upper, left, bottom)

$$\begin{aligned} &r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+1,l+3} \oplus r_{k+2,l} \oplus r_{k+2,l+2} \\ &= U_{k,l+1} \oplus U_{k+1,l+2} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l} \end{aligned} \tag{28}$$

$$\begin{aligned} &m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l-1} \oplus m_{k+1,l+3} \oplus m_{k+2,l} \oplus m_{k+2,l+2} \\ &= U_{k,l} \oplus U_{k+2,l+2} \oplus L_{k+1,l-1} \oplus L_{k+1,l+3} \oplus B_{k,l+2} \oplus B_{k+2,l} \end{aligned} \tag{29}$$

$$\begin{aligned} &f_{k,l} \oplus f_{k,l+2} \oplus f_{k+1,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \oplus f_{k+2,l+2} \\ &= U_{k+1,l} \oplus U_{k+2,l+1} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k+1,l+2} \oplus B_{k+2,l+1} \end{aligned} \tag{30}$$

For triple-view display (left, right, bottom)

$$r_{k,l} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+3,l-1} \oplus r_{k+3,l+1} \oplus r_{k+4,l} = L_{k+2,l+1} \oplus L_{k+3,l} \oplus R_{k+1,l} \oplus R_{k+2,l+1} \oplus B_{k+1,l} \oplus B_{k+3,l} \quad (31)$$

$$m_{k,l} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+3,l-1} \oplus m_{k+3,l+1} \oplus m_{k+4,l} = L_{k+1,l+1} \oplus L_{k+3,l-1} \oplus R_{k+1,l-1} \oplus R_{k+3,l+1} \oplus B_{k,l} \oplus B_{k+4,l} \quad (32)$$

$$f_{k,l} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+3,l-1} \oplus f_{k+3,l+1} \oplus f_{k+4,l} = L_{k+1,l} \oplus L_{k+2,l-1} \oplus R_{k+2,l-1} \oplus R_{k+3,l} \oplus B_{k+1,l} \oplus B_{k+3,l} \quad (33)$$

For triple-view display (upper, right, bottom)

$$r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+1,l+3} \oplus r_{k+2,l} \oplus r_{k+2,l+2} = U_{k+2,l+1} \oplus U_{k+1,l+2} \oplus R_{k+1,l} \oplus R_{k+1,l+2} \oplus B_{k+1,l} \oplus B_{k+2,l+1} \quad (34)$$

$$m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l-1} \oplus m_{k+1,l+3} \oplus m_{k+2,l} \oplus m_{k+2,l+2} = U_{k,l+2} \oplus U_{k+2,l} \oplus R_{k+1,l-1} \oplus R_{k+1,l+3} \oplus B_{k,l} \oplus B_{k+2,l+2} \quad (35)$$

$$f_{k,l} \oplus f_{k,l+2} \oplus f_{k+1,\,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \oplus f_{k+2,l+2} = U_{k,l+1} \oplus U_{k+1,l} \oplus R_{k+1,l} \oplus R_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l+2} \quad (36)$$

4 Evaluation results (compared with formulation)

Create a display image based on Eqs. (7–36), display the image on the three-layered LCD panels, and confirm that the general formulation is correct by displaying the image on triple-view display. Equations (7–36) are used to determine the pixel values of the images shown on the three LCD panels. As can be seen from the general formulation, the pixel values of the display image are determined by the pixel values of the observation image, so the observation image is determined. In this time, the observation image is 50 × 30 pixel image representing a letter corresponding to the viewpoint as shown in Fig. 3. The displayed image created based on the pixel values of these observation images and the general formulation is shown in Fig. 4. The display image is created at 60 × 30 pixels and displayed on the rear LCD panel at a size of 200 × 100 mm. The pixel size of the binary image to be displayed is sufficiently larger than one pixel of the LCD panel. The ratio of the size of the image displayed on each LCD panel determines the appropriate viewing distance. This time, each displayed image is scaled down by 2%, and since the gap between the LCD panels is 1 cm, the appropriate viewing distance is 50 cm. Based on the pixel size of the binary displayed image, the top, bottom, left, and right viewpoints are offset by 18 degrees from the front.

Figure 5 shows the result of displaying the created display image on three-layered LCD panels and observing it from set viewpoint. The selected viewpoint produced the same observation results as in Fig. 3, while a scrambled image was observed at different viewpoints. The allowable viewing angle was approximately ± 4 degrees. Due to wavelength dispersion caused by multiple polarization rotations, portions rotated 180 degrees are reproduced in red. This can be improved by using a waveplate.

From this result, we established a method to display different images in any three directions by general formulation revealed in this paper. As a future work, we would like to work on a three-viewpoint display that follows three observers.

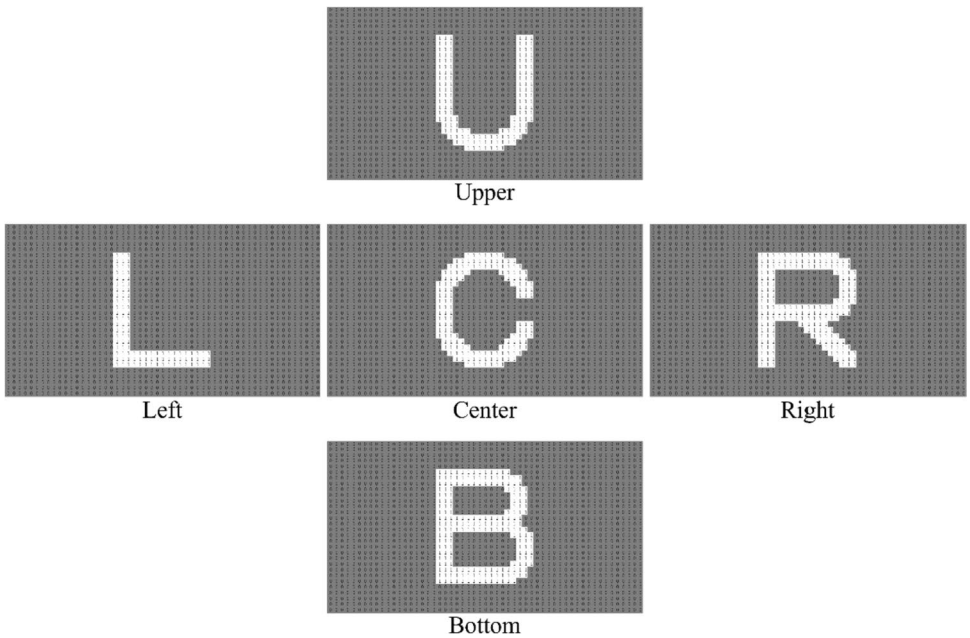


Fig. 3 Observation image corresponding to the viewpoint

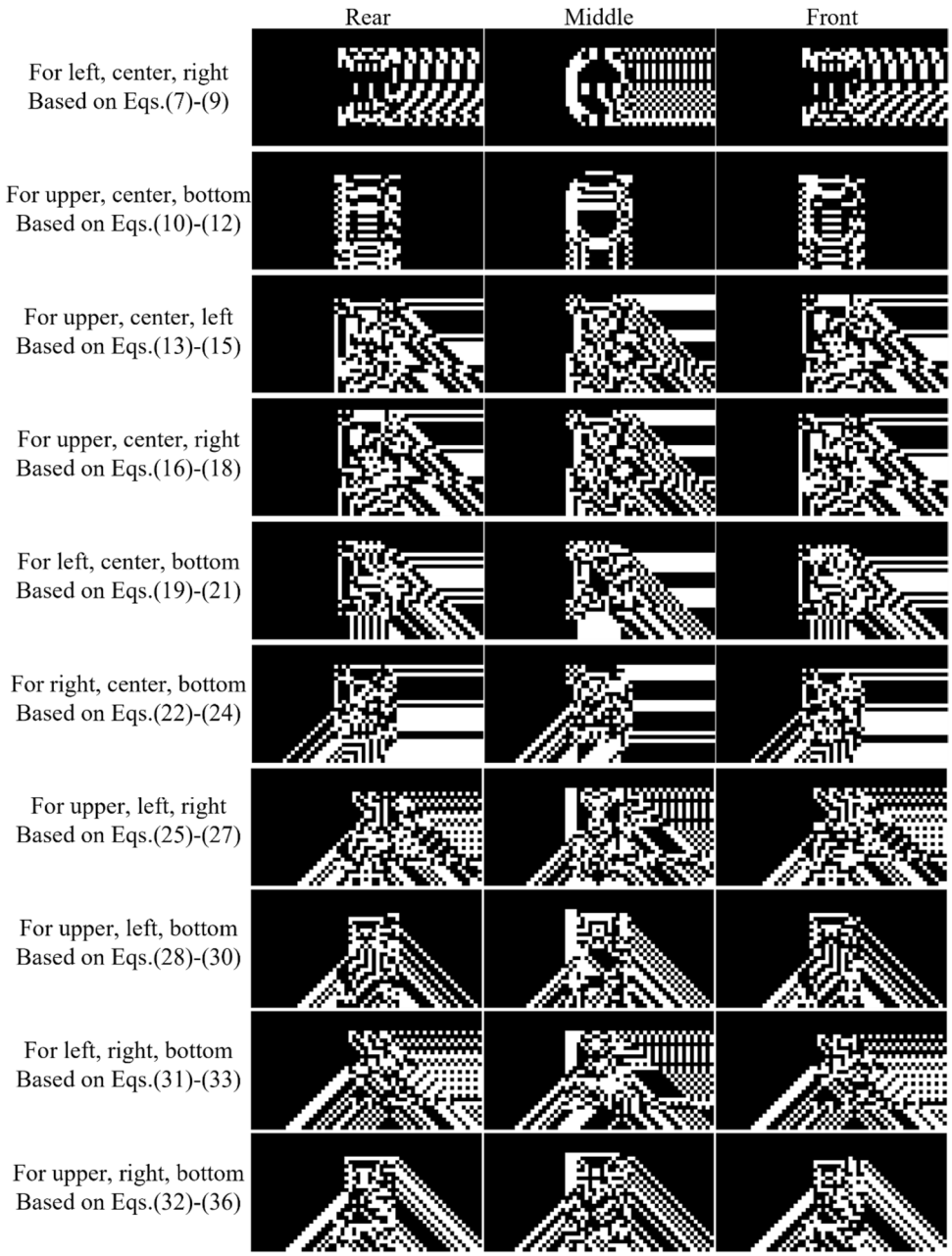


Fig. 4 Display image created based on the general formula

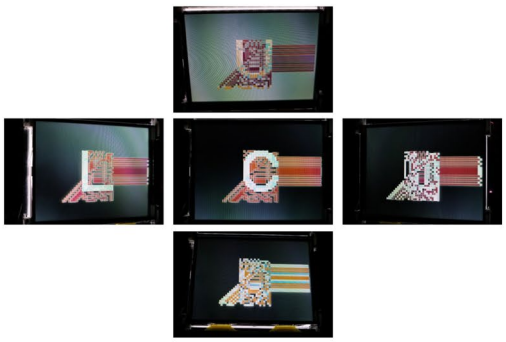
5 Conclusion

The general formulation for determining the pixel value of the displayed image in a polarization-modulated triple-view information display with three TN-LCD layers was clarified. Binary images to be displayed were created to achieve triple-view display of all combinations of top, bottom, left, right, and center. The size of the display image created for observation at 50 cm was adjusted and displayed on a three-layer LCD panel with polarization modulation to realize a triple-view display at the set viewpoint, thereby confirming that the general formula derived was correct.

Appendix

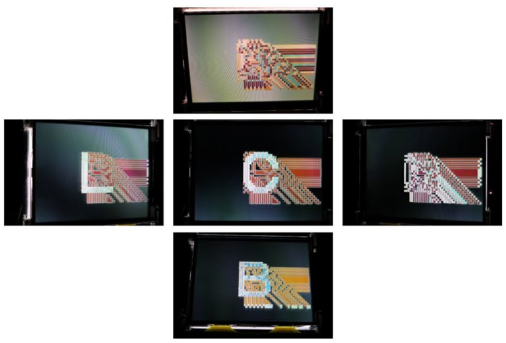
The procedure for deriving the general solution is described. When the center viewpoint is included, Eqs. (16–18) are used as examples; when the center

Upper, center and bottom triple-view display based on Eqs.(10)-(12)



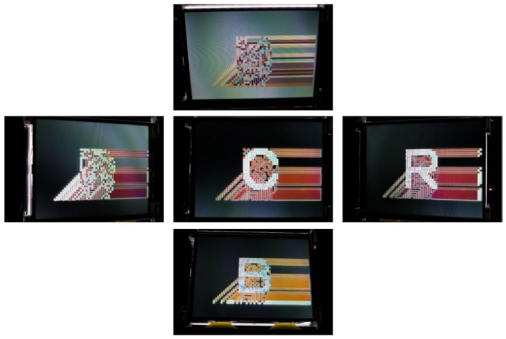
Upper, center and left triple-view display based on Eqs.(13)-(15)

Upper, center and right triple-view display based on Eqs.(16)-(18)



left, center and bottom triple-view display based on Eqs.(19)-(21)

Fig. 5 Images observed at each viewpoint position when the image in Fig. 4 is displayed on the three-layered LCD display



Right, center and bottom triple-view display based on Eqs.(22)-(24)

Upper, left and right triple-view display based on Eqs.(25)-(27)

Upper, left and bottom triple-view display based on Eqs.(28)-(30)

Left, right and bottom triple-view display based on Eqs.(31)-(33)

Upper, right and bottom triple-view display based on Eqs.(34)-(36)

Fig. 5 (continued)

viewpoint is not included, the results of the study for all combinations are noted.

$$U_{k,l} = r_{k,l+1} \oplus m_{k,l} \oplus f_{k,l-1} \tag{37}$$

$$C_{k,l} = r_{k,l} \oplus m_{k,l} \oplus f_{k,l} \tag{38}$$

$$R_{k,l} = r_{k-1,l} \oplus m_{k,l} \oplus f_{k+1,l} \tag{39}$$

To solve the simultaneous equation of XOR, we make a sum of the same terms.

$(37) \oplus (38)_{l+1}$

$$U_{k,l} \oplus C_{k,l+1} = r_{k,l+1} \oplus r_{k,l+1} \oplus m_{k,l} \oplus m_{k,l+1} \oplus f_{k,l-1} \oplus f_{k,l+1} \Rightarrow \\ U_{k,l} \oplus C_{k,l+1} = m_{k,l} \oplus m_{k,l+1} \oplus f_{k,l-1} \oplus f_{k,l+1} \quad (40)$$

(37) ⊕(38)

$$U_{k,l} \oplus C_{k,l} = r_{k,l} \oplus r_{k,l+1} \oplus f_{k,l-1} \oplus f_{k,l} \quad (41)$$

$(37)_{l+1}$ ⊕(38)

$$U_{k,l+1} \oplus C_{k,l} = r_{k,l} \oplus r_{k,l+2} \oplus m_{k,l} \oplus m_{k,l+1} \quad (42)$$

(39) $\oplus(39)_{l+1}$

$$R_{k,l} \oplus R_{k,l+1} = r_{k-1,l} \oplus r_{k-1,l+1} \oplus m_{k,l} \oplus m_{k,l+1} \oplus f_{k+1,l} \oplus f_{k+1,l+1} \quad (43)$$

(42) ⊕(43)

$$U_{k,l+1} \oplus C_{k,l} \oplus R_{k,l} \oplus R_{k,l+1} = r_{k-1,l} \oplus r_{k-1,l+1} \oplus r_{k,l} \oplus r_{k,l+2} \oplus f_{k+1,l} \oplus f_{k+1,l+1} \quad (44)$$

$(41)_{k+1,\, l+1}$ ⊕(44)

$$U_{k,l+1} \oplus U_{k+1,l+1} \oplus C_{k,l} \oplus C_{k+1,l+1} \oplus R_{k,l} \oplus R_{k,l+1} \\ = r_{k-1,l} \oplus r_{k-1,l+1} \oplus r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l+1} \oplus r_{k+1,l+2} \quad (45)$$

(39) $\oplus(39)_{l+2}$

$$R_{k,l} \oplus R_{k,l+2} = r_{k-1,l} \oplus r_{k-1,l+2} \oplus m_{k,l} \oplus m_{k,l+2} \oplus f_{k+1,l} \oplus f_{k+1,l+2} \quad (46)$$

$(40)_{k+1,\, l+1}$ ⊕(46)

$$U_{k+1,l+1} \oplus C_{k+1,l+2} \oplus R_{k,l} \oplus R_{k,l+2} \\ = r_{k-1,l} \oplus r_{k-1,l+2} \oplus m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l+1} \oplus m_{k+1,l+2} \quad (47)$$

(42) $\oplus(47)_{k+1}$

$$U_{k,l+1} \oplus U_{k+2,l+1} \oplus C_{k,l} \oplus C_{k+2,l+2} \oplus R_{k+1,l} \oplus R_{k+1,l+2} \\ = m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+2,l+1} \oplus m_{k+2,l+2} \quad (48)$$

(40) ⊕(43)

$$U_{k,l} \oplus C_{k,l+1} \oplus R_{k,l} \oplus R_{k,l+1} \\ = r_{k-1,l} \oplus r_{k-1,l+1} \oplus f_{k,l-1} \oplus f_{k,l+1} \oplus f_{k+1,l} \oplus f_{k+1,l+1} \quad (49)$$

(41) $\oplus(49)_{k+1}$

$$U_{k,l} \oplus U_{k+1,l} \oplus C_{k,l} \oplus C_{k+1,l+1} \oplus R_{k+1,l} \oplus R_{k+1,l+1} \\ = f_{k,l-1} \oplus f_{k,l} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+2,l} \oplus f_{k+2,l+1} \quad (50)$$

From these results, the general formula for triple-view display (upper, center, right) is as follows

$$r_{k,l} \oplus r_{k,l+1} \oplus r_{k+1,l} \oplus r_{k+1,l+2} \oplus r_{k+2,l+1} \oplus r_{k+2,l+2} \\ = U_{k+1,l+1} \oplus U_{k+2,l+1} \oplus C_{k+1,l} \oplus C_{k+2,l+1} \oplus R_{k+1,l} \oplus R_{k+1,l+1}$$

$$m_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+2,l+1} \oplus m_{k+2,l+2} \\ = U_{k,l+1} \oplus U_{k+2,l+1} \oplus C_{k,l} \oplus C_{k+2,l+2} \oplus R_{k+1,l} \oplus R_{k+1,l+2}$$

$$f_{k,l} \oplus f_{k,l+1} \oplus f_{k+1,l} \oplus f_{k+1,l+2} \oplus f_{k+2,l+1} \oplus f_{k+2,l+2} \\ = U_{k,l+1} \oplus U_{k+1,l+1} \oplus C_{k,l+1} \oplus C_{k+1,l+2} \oplus R_{k+1,l+1} \oplus R_{k+1,l+2}$$

Solving simultaneous equations with $U_{k,l}$, $L_{k,l}$, and $R_{k,l}$.

$$U_{k,l} = r_{k,l+1} \oplus m_{k,l} \oplus f_{k,l-1} \quad (51)$$

$$L_{k,l} = r_{k+1,l} \oplus m_{k,l} \oplus f_{k-1,l} \quad (52)$$

$$R_{k,l} = r_{k-1,l} \oplus m_{k,l} \oplus f_{k+1,l} \quad (53)$$

Solve the above XOR simultaneous equations.

$(51)_{k+1}$ $\oplus(52)_{l+1}$

$$U_{k+1,l} \oplus L_{k,l+1} = m_{k,l+1} \oplus m_{k+1,l} \oplus f_{k-1,l+1} \oplus f_{k+1,l-1} \quad (54)$$

(51) ⊕(52)

$$U_{k,l} \oplus L_{k,l} = r_{k,l+1} \oplus r_{k+1,l} \oplus f_{k-1,l} \oplus f_{k,l-1} \quad (55)$$

$(51)_{l+1}$ $\oplus(52)_{k+1}$

$$U_{k,l+1} \oplus L_{k+1,l} = r_{k,l+2} \oplus r_{k+2,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \quad (56)$$

$(53)_{l+1}$ $\oplus(53)_{k+1}$

$$R_{k,l+1} \oplus R_{k+1,l} = r_{k-1,l+1} \oplus r_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus f_{k+1,l+1} \oplus f_{k+2,l} \quad (57)$$

$(55)_{k+2,\, l+1}$ ⊕(57)

$$U_{k+2,l+1} \oplus L_{k+2,l+1} \oplus R_{k,l+1} \oplus R_{k+1,l} \\ = r_{k-1,l+1} \oplus r_{k,l} \oplus r_{k+2,l+2} \oplus r_{k+3,l+1} \oplus m_{k,l+1} \oplus m_{k+1,l} \quad (58)$$

(56) ⊕(58)

$$U_{k,l+1} \oplus U_{k+2,l+1} \oplus L_{k+1,l} \oplus L_{k+2,l+1} \oplus R_{k,l+1} \oplus R_{k+1,l} \\ = r_{k-1,l+1} \oplus r_{k,l} \oplus r_{k,l+2} \oplus r_{k+2,l} \oplus r_{k+2,l+2} \oplus r_{k+3,l+1} \quad (59)$$

$(53)_{k+1,\, l+2}$ $\oplus(53)_{k+3}$

$$R_{k+1,l+2} \oplus R_{k+3,l} \\ = r_{k,l+2} \oplus r_{k+2,l} \oplus m_{k+1,l+2} \oplus m_{k+3,l} \oplus f_{k+2,l+2} \oplus f_{k+4,l} \quad (60)$$

(56) ⊕(60)

$$U_{k,l+1} \oplus L_{k+1,l} \oplus R_{k+1,l+2} \oplus R_{k+3,l} \\ = m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+3,l} \oplus f_{k+2,l+2} \oplus f_{k+4,l} \quad (61)$$

$(54)_{k+3,\, l+1}$ ⊕(61)

$$U_{k,l+1} \oplus U_{k+4,l+1} \oplus L_{k+1,l} \oplus L_{k+3,l+2} \oplus R_{k+1,l+2} \oplus R_{k+3,l} = m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+3,l} \oplus m_{k+3,l+2} \oplus m_{k+4,l+1} \tag{62}$$

$(53)_{l+1} \oplus (53)_{k+1}$

$$R_{k,l+1} \oplus R_{k+1,l} = r_{k-1,l+1} \oplus r_{k,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus f_{k+1,l+1} \oplus f_{k+2,l} \tag{63}$$

$(54) \oplus (63)$

$$U_{k+1,l} \oplus L_{k,l+1} \oplus R_{k,l+1} \oplus R_{k+1,l} = r_{k-1,l+1} \oplus r_{k,l} \oplus f_{k-1,l+1} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+2,l} \tag{64}$$

$(55) \oplus (64)_{k+1}$

$$U_{k,l} \oplus U_{k+2,l} \oplus L_{k,l} \oplus L_{k+1,l+1} \oplus R_{k+1,l+1} \oplus R_{k+2,l} = f_{k-1,l} \oplus f_{k,l-1} \oplus f_{k,l+1} \oplus f_{k+2,l-1} \oplus f_{k+2,l+1} \oplus f_{k+3,l} \tag{65}$$

From these results, the general formula for triple-view display (upper, left, right) is as follows

$$r_{k,l} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+3,l-1} \oplus r_{k+3,l+1} \oplus r_{k+4,l} = U_{k+1,l} \oplus U_{k+3,l} \oplus L_{k+2,l-1} \oplus L_{k+3,l} \oplus R_{k+1,l} \oplus R_{k+2,l-1}$$
$$m_{k,l} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+3,l-1} \oplus m_{k+3,l+1} \oplus m_{k+4,l} = U_{k,l} \oplus U_{k+4,l} \oplus L_{k+1,l-1} \oplus L_{k+3,l+1} \oplus R_{k+1,l+1} \oplus R_{k+3,l-1}$$
$$f_{k,l} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+3,l-1} \oplus f_{k+3,l+1} \oplus f_{k+4,l} = U_{k+1,l} \oplus U_{k+3,l} \oplus L_{k+1,l} \oplus L_{k+2,l+1} \oplus R_{k+2,l+1} \oplus R_{k+3,l}$$

Solving simultaneous equations with $U_{k,l}$, $L_{k,l}$, and $B_{k,l}$.

$$U_{k,l} = r_{k,l+1} \oplus m_{k,l} \oplus f_{k,l-1} \tag{66}$$

$$L_{k,l} = r_{k+1,l} \oplus m_{k,l} \oplus f_{k-1,l} \tag{67}$$

$$B_{k,l} = r_{k,l-1} \oplus m_{k,l} \oplus f_{k,l+1} \tag{68}$$

Solve the above XOR simultaneous equations.

$(66)_{k+1} \oplus (67)_{l+1}$

$$U_{k+1,l} \oplus L_{k,l+1} = m_{k,l+1} \oplus m_{k+1,l} \oplus f_{k-1,l+1} \oplus f_{k+1,l-1} \tag{69}$$

$(66) \oplus (67)$

$$U_{k,l} \oplus L_{k,l} = r_{k,l+1} \oplus r_{k+1,l} \oplus f_{k-1,l} \oplus f_{k,l-1} \tag{70}$$

$(66)_{l+1} \oplus (67)_{k+1}$

$$U_{k,l+1} \oplus L_{k+1,l} = r_{k,l+2} \oplus r_{k+2,l} \oplus m_{k,l+1} \oplus m_{k+1,l} \tag{71}$$

$(68)_{l+1} \oplus (68)_{k+1}$

$$B_{k,l+1} \oplus B_{k+1,l} = r_{k,l} \oplus r_{k+1,l-1} \oplus m_{k,l+1} \oplus m_{k+1,l} \oplus f_{k,l+2} \oplus f_{k+1,l+1} \tag{72}$$

$(71) \oplus (72)$

$$U_{k,l+1} \oplus L_{k+1,l} \oplus B_{k,l+1} \oplus B_{k+1,l} = r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+2,l} \oplus f_{k,l+2} \oplus f_{k+1,l+1} \tag{73}$$

$(70)_{k+1,\,l+2} \oplus (73)$

$$U_{k,l+1} \oplus U_{k+1,l+2} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l} = r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+1,l+3} \oplus r_{k+2,l} \oplus r_{k+2,l+2} \tag{74}$$

$(68)_{l+2} \oplus (68)_{k+2}$

$$B_{k,l+2} \oplus B_{k+2,l} = r_{k,l+1} \oplus r_{k+2,l-1} \oplus m_{k,l+2} \oplus m_{k+2,l} \oplus f_{k,l+3} \oplus f_{k+2,l+1} \tag{75}$$

$(69)_{k+1,\,l+2} \oplus (75)$

$$U_{k+2,l+2} \oplus L_{k+1,l+3} \oplus B_{k,l+2} \oplus B_{k+2,l} = r_{k,l+1} \oplus r_{k+2,l-1} \oplus m_{k,l+2} \oplus m_{k+1,l+3} \oplus m_{k+2,l} \oplus m_{k+2,l+2} \tag{76}$$

$(71) \oplus (76)_{l+1}$

$$U_{k,l+1} \oplus U_{k+2,l+3} \oplus L_{k+1,l} \oplus L_{k+1,l+4} \oplus B_{k,l+3} \oplus B_{k+2,l+1} = m_{k,l+1} \oplus m_{k,l+3} \oplus m_{k+1,l} \oplus m_{k+1,l+4} \oplus m_{k+2,l+1} \oplus m_{k+2,l+3} \tag{77}$$

$(69) \oplus (72)$

$$U_{k+1,l} \oplus L_{k,l+1} \oplus B_{k,l+1} \oplus B_{k+1,l} = r_{k,l} \oplus r_{k+1,l-1} \oplus f_{k-1,l+1} \oplus f_{k,l+2} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \tag{78}$$

$(70) \oplus (78)_{l+1}$

$$U_{k,l} \oplus U_{k+1,l+1} \oplus L_{k,l} \oplus L_{k,l+2} \oplus B_{k,l+2} \oplus B_{k+1,l+1} = f_{k-1,l} \oplus f_{k-1,l+2} \oplus f_{k,l-1} \oplus f_{k,l+3} \oplus f_{k+1,l} \oplus f_{k+1,l+2} \tag{79}$$

From these results, the general formula for triple-view display (upper, left, bottom) is as follows

$$r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+1,l+3} \oplus r_{k+2,l} \oplus r_{k+2,l+2} = U_{k,l+1} \oplus U_{k+1,l+2} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l}$$
$$m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l-1} \oplus m_{k+1,l+3} \oplus m_{k+2,l} \oplus m_{k+2,l+2} = U_{k,l} \oplus U_{k+2,l+2} \oplus L_{k+1,l-1} \oplus L_{k+1,l+3} \oplus B_{k,l+2} \oplus B_{k+2,l}$$
$$f_{k,l} \oplus f_{k,l+2} \oplus f_{k+1,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \oplus f_{k+2,l+2} = U_{k+1,l} \oplus U_{k+2,l+1} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k+1,l+2} \oplus B_{k+2,l+1}$$

Solving simultaneous equations with $L_{k,l}$, $R_{k,l}$, and $B_{k,l}$.

$$L_{k,l} = r_{k+1,l} \oplus m_{k,l} \oplus f_{k-1,l} \tag{80}$$

$$R_{k,l} = r_{k-1,l} \oplus m_{k,l} \oplus f_{k+1,l} \tag{81}$$

$$B_{k,l} = r_{k,l-1} \oplus m_{k,l} \oplus f_{k,l+1} \tag{82}$$

Solve the above XOR simultaneous equations.

$(80) \oplus (81)_{k+2}$

$$L_{k,l} \oplus R_{k+2,l} = m_{k,l} \oplus m_{k+2,l} \oplus f_{k-1,l} \oplus f_{k+3,l} \tag{83}$$

(80) ⊕(81)

$$L_{k,l} \oplus R_{k,l} = r_{k-1,l} \oplus r_{k+1,l} \oplus f_{k-1,l} \oplus f_{k+1,l} \tag{84}$$

$(80)_{k+2}$ ⊕(81)

$$L_{k+2,l} \oplus R_{k,l} = r_{k-1,l} \oplus r_{k+3,l} \oplus m_{k,l} \oplus m_{k+2,l} \tag{85}$$

(82) $\oplus(82)_{k+2}$

$$B_{k,l} \oplus B_{k+2,l} = r_{k,l-1} \oplus r_{k+2,l-1} \oplus m_{k,l} \oplus m_{k+2,l} \oplus f_{k,l+1} \oplus f_{k+2,l+1} \tag{86}$$

$(84)_{k+1,\,l+1}$ ⊕(86)

$$\begin{aligned} &L_{k+1,l+1} \oplus R_{k+1,l+1} \oplus B_{k,l} \oplus B_{k+2,l} \\ &= r_{k,l-1} \oplus r_{k,l+1} \oplus r_{k+2,l-1} \oplus r_{k+2,l+1} \oplus m_{k,l} \oplus m_{k+2,l} \end{aligned} \tag{87}$$

(85) ⊕(87)

$$\begin{aligned} &L_{k+1,l+1} \oplus L_{k+2,l} \oplus R_{k,l} \oplus R_{k+1,l+1} \oplus B_{k,l} \oplus B_{k+2,l} \\ &= r_{k-1,l} \oplus r_{k,l-1} \oplus r_{k,l+1} \oplus r_{k+2,l-1} \oplus r_{k+2,l+1} \oplus r_{k+3,l} \end{aligned} \tag{88}$$

$(82)_{l+1}$ $\oplus(82)_{k+4,\,l+1}$

$$B_{k,l+1} \oplus B_{k+4,l+1} = r_{k,l} \oplus r_{k+4,l} \oplus m_{k,l+1} \oplus m_{k+4,l+1} \oplus f_{k,l+2} \oplus f_{k+4,l+2} \tag{89}$$

$(85)_{k+1}$ ⊕(89)

$$\begin{aligned} &L_{k+3,l} \oplus R_{k+1,l} \oplus B_{k,l+1} \oplus B_{k+4,l+1} \\ &= m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+3,l} \oplus m_{k+4,l+1} \oplus f_{k,l+2} \oplus f_{k+4,l+2} \end{aligned} \tag{90}$$

$(83)_{k+1,\,l+2}$ ⊕(90)

$$\begin{aligned} &L_{k+1,l+2} \oplus L_{k+3,l} \oplus R_{k+1,l} \oplus R_{k+3,l+2} \oplus B_{k,l+1} \oplus B_{k+4,l+1} \\ &= m_{k,l+1} \oplus m_{k+1,l} \oplus m_{k+1,l+2} \oplus m_{k+3,l} \oplus m_{k+3,l+2} \oplus m_{k+4,l+1} \end{aligned} \tag{91}$$

$(82)_{l+1}$ $\oplus(82)_{k+2,\,l+1}$

$$B_{k,l+1} \oplus B_{k+2,l+1} = r_{k,l} \oplus r_{k+2,l} \oplus m_{k,l+1} \oplus m_{k+2,l+1} \oplus f_{k,l+2} \oplus f_{k+2,l+2} \tag{92}$$

$(84)_{k+1}$ ⊕(92)

$$\begin{aligned} &L_{k+1,l} \oplus R_{k+1,l} \oplus B_{k,l+1} \oplus B_{k+2,l+1} \\ &= m_{k,l+1} \oplus m_{k+2,l+1} \oplus f_{k,l} \oplus f_{k,l+2} \oplus f_{k+2,l} \oplus f_{k+2,l+2} \end{aligned} \tag{93}$$

$(83)_{l+1}$ ⊕(93)

$$\begin{aligned} &L_{k,l+1} \oplus L_{k+1,l} \oplus R_{k+1,l} \oplus R_{k+2,l+1} \oplus B_{k,l+1} \oplus B_{k+2,l+1} \\ &= f_{k-1,l+1} \oplus f_{k,l} \oplus f_{k,l+2} \oplus f_{k+2,l} \oplus f_{k+2,l+2} \oplus f_{k+3,l+1} \end{aligned} \tag{94}$$

From these results, the general formula for triple-view display (left, right, bottom) is as follows

$$\begin{aligned} &r_{k,l} \oplus r_{k+1,l-1} \oplus r_{k+1,l+1} \oplus r_{k+3,l-1} \oplus r_{k+3,l+1} \oplus r_{k+4,l} \\ &= L_{k+2,l+1} \oplus L_{k+3,l} \oplus R_{k+1,l} \oplus R_{k+2,l+1} \oplus B_{k+1,l} \oplus B_{k+3,l} \\ &m_{k,l} \oplus m_{k+1,l-1} \oplus m_{k+1,l+1} \oplus m_{k+3,l-1} \oplus m_{k+3,l+1} \oplus m_{k+4,l} \\ &= L_{k+1,l+1} \oplus L_{k+3,l-1} \oplus R_{k+1,l-1} \oplus R_{k+3,l+1} \oplus B_{k,l} \oplus B_{k+4,l} \\ &f_{k,l} \oplus f_{k+1,l-1} \oplus f_{k+1,l+1} \oplus f_{k+3,l-1} \oplus f_{k+3,l+1} \oplus f_{k+4,l} \\ &= L_{k+1,l} \oplus L_{k+2,l-1} \oplus R_{k+2,l-1} \oplus R_{k+3,l} \oplus B_{k+1,l} \oplus B_{k+3,l} \end{aligned}$$

Solving simultaneous equations with $U_{k,l}$, $R_{k,l}$, and $B_{k,l}$.

$$U_{k,l} = r_{k,l+1} \oplus m_{k,l} \oplus f_{k,l-1} \tag{95}$$

$$R_{k,l} = r_{k-1,l} \oplus m_{k,l} \oplus f_{k+1,l} \tag{96}$$

$$B_{k,l} = r_{k,l-1} \oplus m_{k,l} \oplus f_{k,l+1} \tag{97}$$

Solve the above XOR simultaneous equations.

(95) $\oplus(96)_{k+1,\,l+1}$

$$U_{k,l} \oplus R_{k+1,l+1} = m_{k,l} \oplus m_{k+1,l+1} \oplus f_{k,l-1} \oplus f_{k+2,l+1} \tag{98}$$

(95) ⊕(96)

$$U_{k,l} \oplus R_{k,l} = r_{k-1,l} \oplus r_{k,l+1} \oplus f_{k,l-1} \oplus f_{k+1,l} \tag{99}$$

$(95)_{k+1,\,l+1}$ ⊕(96)

$$U_{k+1,l+1} \oplus R_{k,l} = r_{k-1,l} \oplus r_{k+1,l+2} \oplus m_{k,l} \oplus m_{k+1,l+1} \tag{100}$$

(97) $\oplus(97)_{k+1,\,l+1}$

$$B_{k,l} \oplus B_{k+1,l+1} = r_{k,l-1} \oplus r_{k+1,l} \oplus m_{k,l} \oplus m_{k+1,l+1} \oplus f_{k,l+1} \oplus f_{k+1,l+2} \tag{101}$$

(100) ⊕(101)

$$\begin{aligned} &U_{k+1,l+1} \oplus R_{k,l} \oplus B_{k,l} \oplus B_{k+1,l+1} \\ &= r_{k-1,l} \oplus r_{k,l-1} \oplus r_{k+1,l} \oplus r_{k+1,l+2} \oplus f_{k,l+1} \oplus f_{k+1,l+2} \end{aligned} \tag{102}$$

$(99)_{l+2}$ ⊕(102)

$$\begin{aligned} &U_{k+1,l+1} \oplus U_{k,l+2} \oplus R_{k,l} \oplus R_{k,l+2} \oplus B_{k,l} \oplus B_{k+1,l+1} \\ &= r_{k-1,l} \oplus r_{k-1,l+2} \oplus r_{k,l-1} \oplus r_{k,l+3} \oplus r_{k+1,l} \oplus r_{k+1,l+2} \end{aligned} \tag{103}$$

(97) $\oplus(97)_{k+2,\,l+2}$

$$B_{k,l} \oplus B_{k+2,l+2} = r_{k,l-1} \oplus r_{k+2,l+1} \oplus m_{k,l} \oplus m_{k+2,l+2} \oplus f_{k,l+1} \oplus f_{k+2,l+3} \tag{104}$$

$(98)_{l+2}$ ⊕(104)

$$\begin{aligned} &U_{k,l+2} \oplus R_{k+1,l+3} \oplus B_{k,l} \oplus B_{k+2,l+2} \\ &= r_{k,l-1} \oplus r_{k+2,l+1} \oplus m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l+3} \oplus m_{k+2,l+2} \end{aligned} \tag{105}$$

$(100)_{k+1}$ $\oplus(105)_{l+1}$

$$U_{k,l+3} \oplus U_{k+2,l+1} \oplus R_{k+1,l} \oplus R_{k+1,l+4} \oplus B_{k,l+1} \oplus B_{k+2,l+3} = m_{k,l+1} \oplus m_{k,l+3} \oplus m_{k+1,l} \oplus m_{k+1,l+4} \oplus m_{k+2,l+1} \oplus m_{k+2,l+3} \quad (106)$$

$(99)_{k+1} \oplus (101)_{l+1}$

$$U_{k+1,l} \oplus R_{k+1,l} \oplus B_{k,l+1} \oplus B_{k+1,l+2} = m_{k,l+1} \oplus m_{k+1,l+2} \oplus f_{k,l+2} \oplus f_{k+1,\,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \quad (107)$$

$(98)_{l+1} \oplus (107)$

$$U_{k,l+1} \oplus U_{k+1,l} \oplus R_{k+1,l} \oplus R_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l+2} = f_{k,l} \oplus f_{k,l+2} \oplus f_{k+1,\,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \oplus f_{k+2,l+2} \quad (108)$$

From these results, the general formula for triple-view display (upper, left, bottom) is as follows

$$r_{k,l} \oplus r_{k,l+2} \oplus r_{k+1,l-1} \oplus r_{k+1,l+3} \oplus r_{k+2,l} \oplus r_{k+2,l+2} = U_{k,l+1} \oplus U_{k+1,l+2} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k,l+1} \oplus B_{k+1,l}$$

$$m_{k,l} \oplus m_{k,l+2} \oplus m_{k+1,l-1} \oplus m_{k+1,l+3} \oplus m_{k+2,l} \oplus m_{k+2,l+2} = U_{k,l} \oplus U_{k+2,l+2} \oplus L_{k+1,l-1} \oplus L_{k+1,l+3} \oplus B_{k,l+2} \oplus B_{k+2,l}$$

$$f_{k,l} \oplus f_{k,l+2} \oplus f_{k+1,l-1} \oplus f_{k+1,l+3} \oplus f_{k+2,l} \oplus f_{k+2,l+2} = U_{k+1,l} \oplus U_{k+2,l+1} \oplus L_{k+1,l} \oplus L_{k+1,l+2} \oplus B_{k+1,l+2} \oplus B_{k+2,l+1}$$

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