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Analysis of floating distance of arc 3D display with respect to inclination angle of substrate

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Abstract

We derive an equation that enables to get the floating distance of floating images of an arc 3D display by the radius of the arc, the angle of the arc 3D substrate, the light source illumination angle, and the observer's angle. Conventional theoretical expression for the positions of the light source and observer relative to the center of the arc have been used to calculate the floating distance. However, when the arc3D substrate is inclined, it becomes more difficult to determine the floating distance from the actual positions of the light source and observer. In this paper, we derive an equation to approximate the floating distance from the positions of the light source and the observer while considering the tilt of the arc3D substrate and check the accuracy of the derived equation through experiments.

Keywords Aerial display · Arc 3D display · Directional sign · Arc

1 Introduction

Currently, arrow signs called direction signs are used to control one lane of traffic during nighttime construction on highways. Figure 1 shows an outline of the installation of directional signs. Direction signs are installed at 20 m intervals starting 1 km before the construction site, and 50 signs are required for each construction site, causing installation to be costly and time-consuming. Figure 2 shows the current specific installation environment of direction signs. The direction signs are freestanding and installed directly on the road. The direction signs are installed directly on the road one by one by workers. Therefore, we are aiming to reduce the number of direction signs by using aerial display technology. Displaying an aerial image of an arrow floating 20 m in front of the actual direction signs may replace some of the direction signs with floating images and reduce the number of direction signs installed Aerial display technologies are AIRR (Aerial Imaging by Retro-Reflection) [1], which uses a display and retro-reflector to form a floating image in the air. AIRR is being considered for application as a real-time aerial display and a touchless display and also it has also been proposed to use this technology for aerial display of road signs [2]. This system uses AIRR technology to create multiple floating images from a single light source, enabling the same floating image to be presented to drivers in different lanes. Although this system is suitable for permanently installed signs, direction signs must be installed and taken down repeatedly at each construction site, so a simple, large floating aerial display must be realized. For such aerial display, we propose to use arc 3D display, one of the aerial display technologies, to directional signs.

The arc 3D display is a floating display technology that displays by illuminating a single light source through an arc-shaped scratches on a transparent substrate. In 1992, Plummer reported a phenomenon in which "an imaginary image appears in the space above and below the plate" when a nickel plate is rotary polished [3]. This technology attracted attention when William J. Beaty made a presentation entitled "Drawing Holograms by Hand" at SPIE in 2003 [4]. Since it has been shown that an arc 3D display has a large viewing area in the horizontal and floating distance directions and that it is possible to perceive aerial images floating more than one meter [5], it is considered to be a suitable aerial display technology as a suitable substitute for direction signs.

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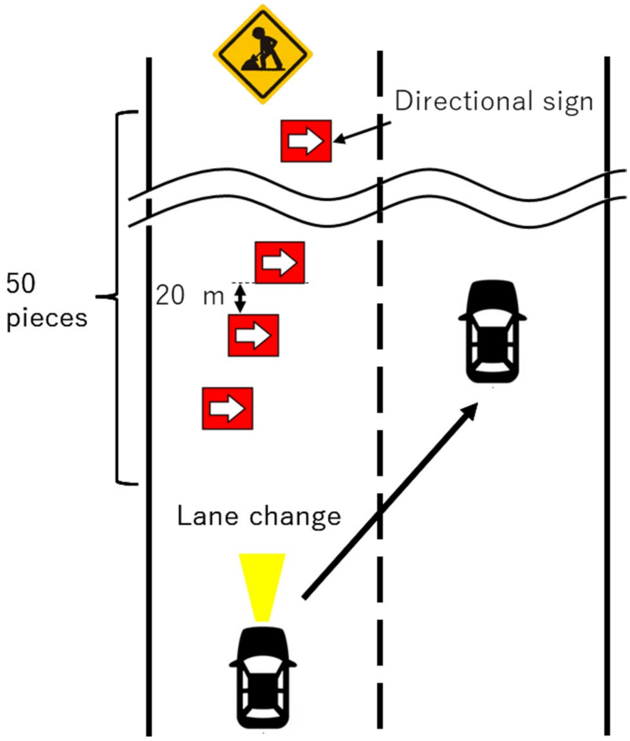


Fig. 1 Outline of current direction sign installation

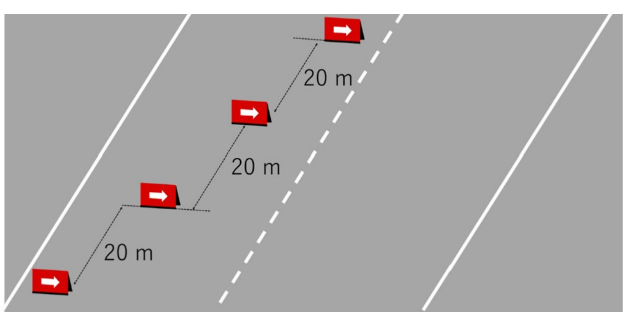


Fig. 2 Current detailed installation environment of directional signs

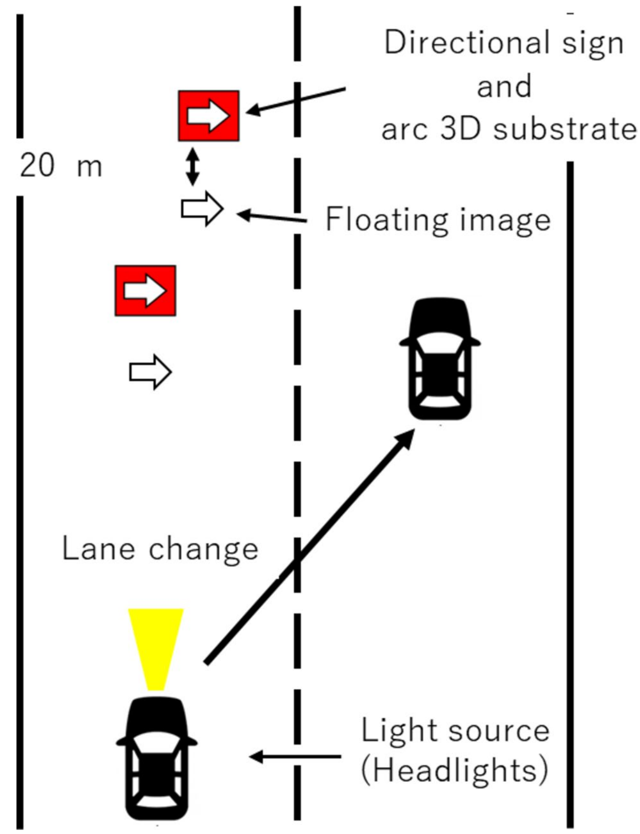


Fig. 3 Application of Arc 3D display to directional signs

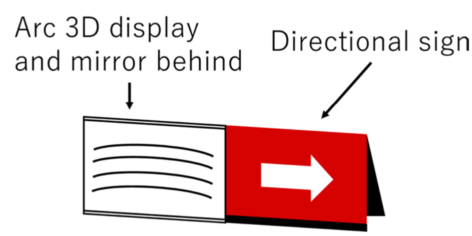


Fig. 4 Proposed installation of arc 3D display on directional signs

In addition, this arc 3D display has a method to solve the blurring phenomenon of floating images [5] by applying a special process to a glass plate, which enables the perception of a clear aerial image [6]. Figure 3 shows an outline of the reduction in the number of direction signs using the arc 3D display. Using an arc 3D substrate in front of the direction indicators and presenting a floating image floating 20 m in front of them, the number of installations can be reduced by half by replacing the direction indicators in front with floating images by arc 3D displays. We are considering using car headlights as the light source. Since the installation height, color, and illumination requirements of car headlights are defined, it is considered that there are no significant differences in headlight characteristics among different types of cars. Therefore, headlights are used as light sources because the aerial image does not change depending on the type of car. Some headlights may have multiple light sources built into the unit. In the case of such light sources, there is a possibility of blurring of the aerial image, but the effect of blurring is considered to be negligible because the illumination distance is sufficiently far away from the light sources. The arc 3D display is considered to be installed directly on the road, and a proposed installation of a specific arc 3D display is shown in Fig. 4. The arc 3D display is installed next to the existing direction sign, and a mirror is placed behind the arc 3D display. Figure 5 shows an image of the specific installation of the arc 3D substrate and direction signs on the expressway. The arc 3D substrate adjacent to

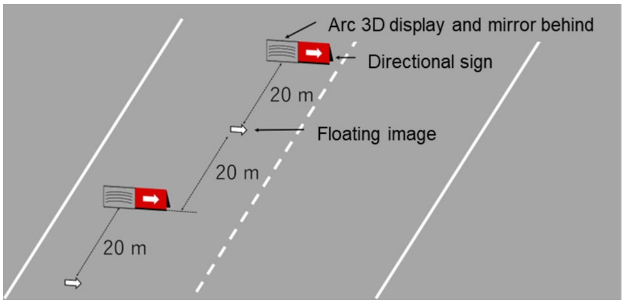


Fig. 5 Specific installation environment and floating image presentation when combining arc 3D display with direction signs

the direction sign presents a floating image that floats 20 m in front. The floating distance of the arc 3D floating image can be calculated from the position of the light source from the center of the arc and the position of the observer from the center of the arc [7, 8]. However, when the arc 3D substrate is inclined, it becomes difficult to obtain the floating distance from the actual positions of the light source and observer.

The purpose of this paper is to derive a first-order approximate equation that can determine the floating distance of a floating image from the angle of arc 3D substrate, light source, observation from the horizontal plane and arc radius while considering the tilt of the arc 3D substrate, and to evaluate the accuracy of the equation by perceptual experiments.

By using this approximate equation, we analyzed the floating distance as the angles of the arc 3D substrate, light source, and observation change, and analyzed the change in floating distance when the light source and observer simultaneously approach the substrate in LDC 2023[9]. To use the arc 3D substrate as a direction sign, it is necessary to achieve a long floating distance. Therefore, it is important to derive a first-order approximate equation to obtain the floating distance from the parameters expressed in the actual observation environment, which are the angle of the arc 3D substrate, the angle of the light source, the observation angle, and the arc radius, for future research on the floating distance.

2 Principle

Arc 3D display is composed of many arc-shaped scratches and provides 3D images in air through binocular disparity and motion parallax. Figure 6 shows the principle of arc 3D display. When light is incident from a single light source on a transparent substrate with very narrow engravings in the shape of an arc (called arc-shaped scratch), the light is irradiated in a directional manner and two specific locations in the arc-shaped scratches appears to be bright for one eye. P_{1R} and P_{2R} are perceived as bright locations by the right eye, and P_{1L} and P_{2L} by the left eye. Because the position of the bright spots differs for each eye, G_1 and G_2 are perceived as floating images at the back and front of the substrate, by binocular parallax. The position of one bright spot for one eye changes in response to the movement of the eye in the horizontal direction and in the observation distance direction. This enables smooth motion parallax.

3 Derivation of first-order approximation of the theoretical equation

We derive an approximate equation for large observation distance that determines the floating distance of the floating image generated by the arc 3D display by the angle of arc 3D substrate to the horizontal plane, the angle of the light source to the horizontal plane and the angle of the observation to the horizontal plane. The floating image in the arc 3D display is determined by the position of the arc-shaped bright spots perceived by both eyes and the positions of the left and right eyes, and the floating image Z_G is expressed by the following Eq. (1).

$$Z_G = \frac{(X_{pr}-X_{pl})Ze}{(X_{el}-X_{er})+(X_{pr}-X_{pl})} \tag{1}$$

Each parameter in Eq. (1) is shown in Fig. 7 (a). X_{el} is the position of the left eye, X_{er} is the position of the right eye, X_{pl} is the position of the bright spot perceived by the left eye, X_{pr} is the position of the bright spot perceived by the right eye and Z_e is the distance from the centre of the arc in the viewing direction. The coordinates of the arc-shaped bright spots are expressed by the following Eqs. (2, 3).

$$X_{pl} = r\left\{\left(\frac{Z_e\sin\theta - Y_e\cos\theta\cos\varphi}{X_{el}\cos\theta\cos\varphi}\right)^2 + 1\right\}^{-\frac{1}{2}} \tag{2}$$

$$X_{pr} = r\left\{\left(\frac{Z_e\sin\theta - Y_e\cos\theta\cos\varphi}{X_{er}\cos\theta\cos\varphi - Z_e\cos\theta\sin\varphi}\right)^2 + 1\right\}^{-\frac{1}{2}} \tag{3}$$

The parameters of Eqs. (2, 3) are shown in Fig. 7 (b). r is the radius of the arc, Y_e is the distance from the centre of the arc in the height direction. The angles θ and φ are shown in Fig. 8 and are polar coordinates transformed from the position of the point light source. θ is the horizontal angle of the light source and φ is the vertical angle.

Considering that the light source is placed in front of the substrate, $\varphi = 0$ in Eq. (2), the position of the bright spot X_{pl} is obtained from Eq. (4) to Eq. (5) below.

$$X_{pl} = r\left\{\left(\frac{Z_e\sin\theta - Y_e\cos\theta}{X_{el}\cos\theta}\right)^2 + 1\right\}^{-\frac{1}{2}} \tag{4}$$

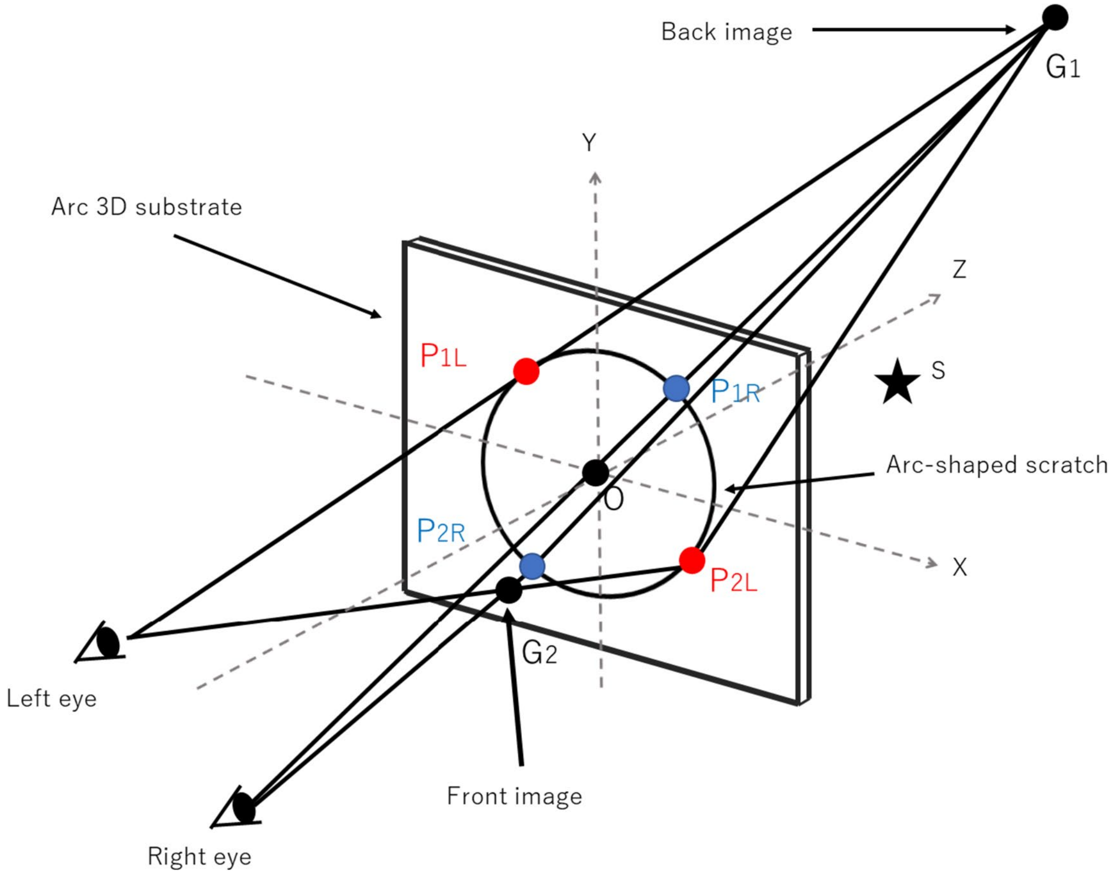
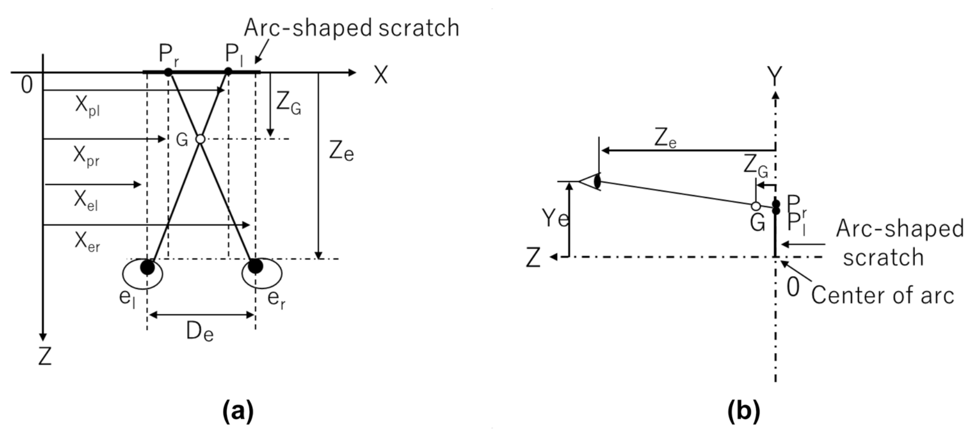


Fig. 6 Principle of Arc 3D Display

Fig. 7 Composition of floating image perception with arc 3D display and notation of eye, bright spot, observation position, and arc radius. **a** Ground view. **b** Side view



$$X_{pl} = r\frac{X_{el}\cos\theta}{Z_e\sin\theta - Y_e\cos\theta}\left\{1 + \left(\frac{1}{\frac{Z_e\sin\theta - Y_e\cos\theta}{X_{el}\cos\theta}}\right)^2\right\}^{-\frac{1}{2}} \quad (5)$$

Considering that the observation distance Z_e is varied from tens to hundreds of meters, an approximation that eliminates the second term within the – 1/2 square yields the following Eq. (6).

$$X_{pl} \fallingdotseq r\frac{X_{el}}{Z_e}\left(\frac{1}{\tan\theta - \frac{Y_e}{Z_e}}\right) \quad (6)$$

Equation (3) is approximated in the same way.

$$X_{pr} \fallingdotseq r\frac{X_{er}}{Z_e}\left(\frac{1}{\tan\theta - \frac{Y_e}{Z_e}}\right) \quad (7)$$

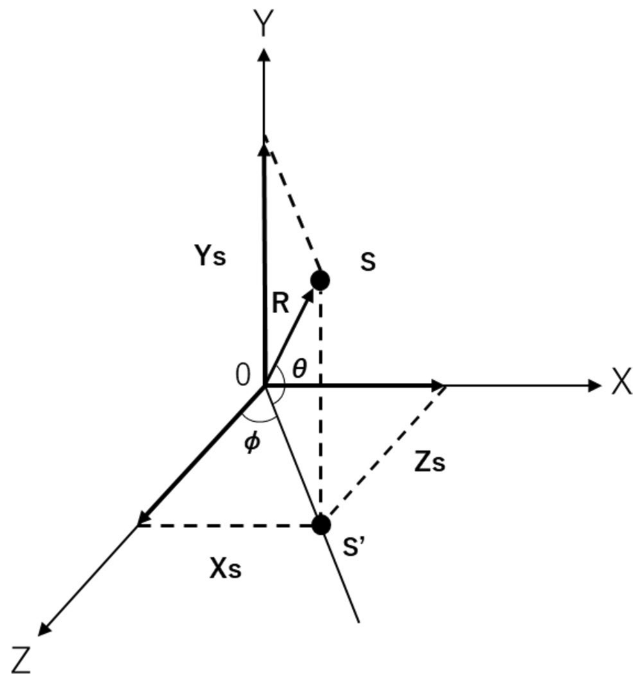


Fig. 8 Polar coordinate transformation of light source position

From Eqs. (6, 7), $X_{pr} - X_{pl}$ becomes the following Eq. (8).

$$X_{pl} - X_{pr} = r\frac{1}{Z_e}\left(X_{el} - X_{er}\right)\left(\frac{1}{\tan\theta - \frac{Y_e}{Z_e}}\right) \tag{8}$$

where $X_{er} - X_{el}$ is the distance between the eyes, so it is assumed to be the parameter De. Z_G is expressed from Eq. (9) to Eq. (10).

$$Z_G = \frac{-De*Ze\left(\frac{1}{tan\theta - \frac{Ye}{Ze}}\right)}{De - \frac{r}{Ze}*De\left(\frac{1}{tan\theta - \frac{Ye}{Ze}}\right)} \tag{9}$$

$$Z_G = -r\left\{\frac{1}{tan\theta - \frac{1}{Ze}(Ye + r)}\right\} \tag{10}$$

When the substrate is inclined to α with respect to the ground, the coordinates including the positions of the observer and the light source are shown in Fig. 9. If the angle between the light source and the ground is δ and the angle between the observer and the substrate is β, $\frac{1}{Ze}(Ye + r)$ in Eq. (10) is shown in Eq. (11) below. In addition, since a mirror is placed behind the arc 3D substrate in this paper, θ is shown by the following Eq. (12).

$$\frac{1}{Ze}(Ye + r) = tan(90 - \alpha - \beta) \tag{11}$$

$$\theta = \alpha + \delta - 90 \tag{12}$$

From the above, the floating image floating distance Z_G can be expressed by Eq. (13). Figure 10 show the parameters of the Eq. (13). The floating distance can be determined from the arc radius, the angle of the arc 3D substrate, the angle of the light source, and the angle of observation.

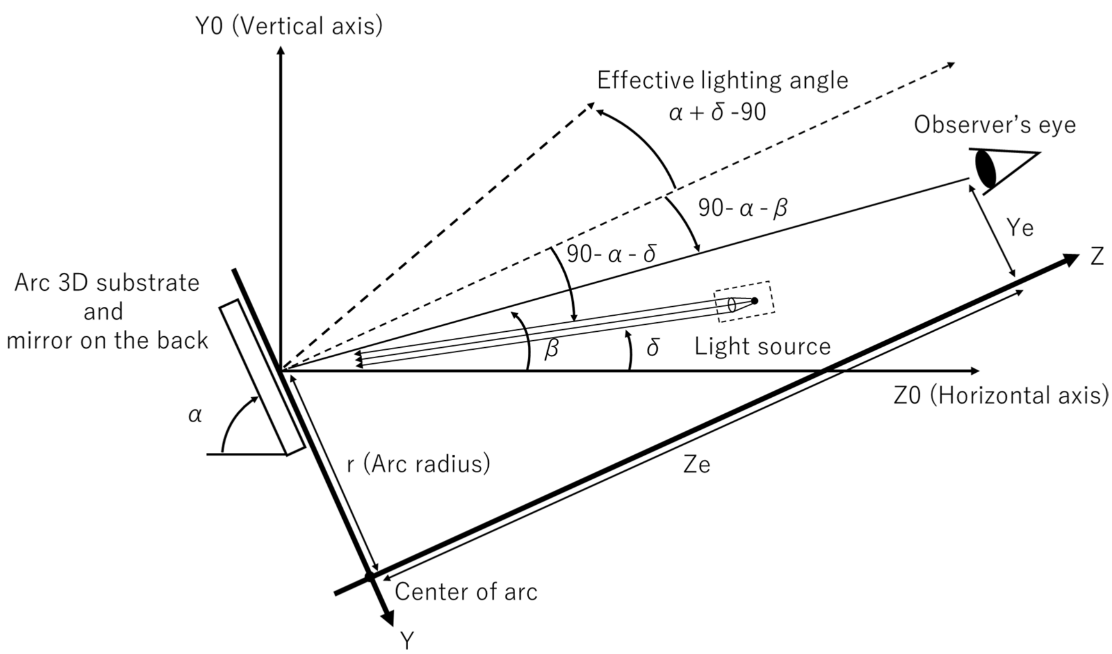


Fig. 9 Coordinates including the positions of the observer and the light source when the substrate is inclined at α with respect to the ground

$$Z_G = -r\left\{\frac{1}{\tan(\alpha+\delta-90)-tan(90-\alpha-\beta)}\right\} \quad (13)$$

4 Experiments in measurement of floating image in arc 3D display by changing parameters

To evaluate the accuracy of the first-order approximation, we measured the distance of the floating images at each angle change. The experimental arrangement for measurement the floating distance of floating image formed by arc 3D display is shown in Fig. 11. In this experiment, the light source was placed on a height-adjustable table and the arc 3D substrate was placed on a height and angle adjustable table. A chinrest was used to fix the observer's head height and viewpoint. The distance of the aerial image from the substrate was measured with a mobile marker to the position perceived by the observer to be floating by using a remote-control operation.

4.1 Arc 3D substrate angle changes

The environment of the experiment for the angular change of the arc 3D substrate is shown in Fig. 12. The observation distance was 10 m from the substrate and the eye height was 1.2 m from the ground, while the light source illumination

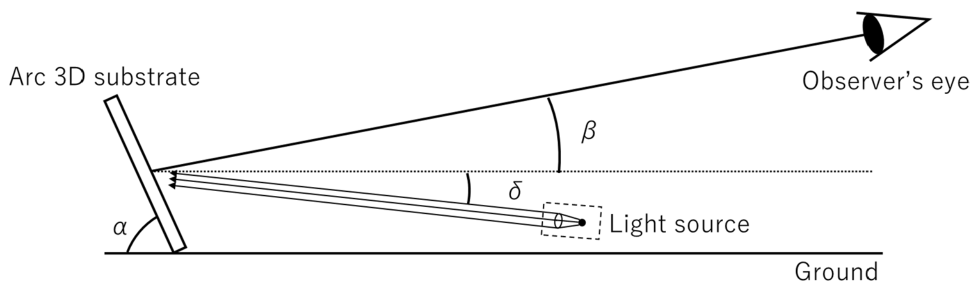


Fig. 10 Angle of substrate (α), angle of substrate (β), and angle of light source (δ) in Eq. (13)

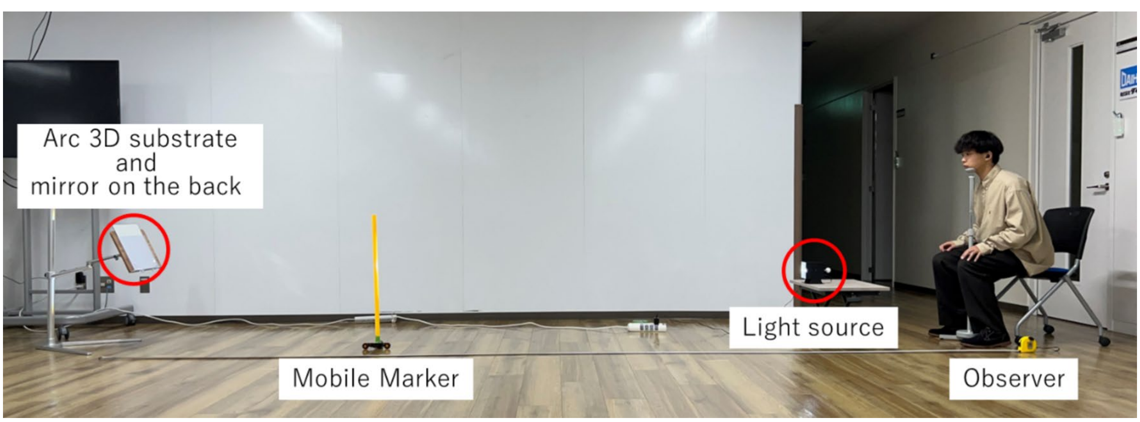


Fig. 11 Experimental arrangement of measurement the floating distance of arc 3D display

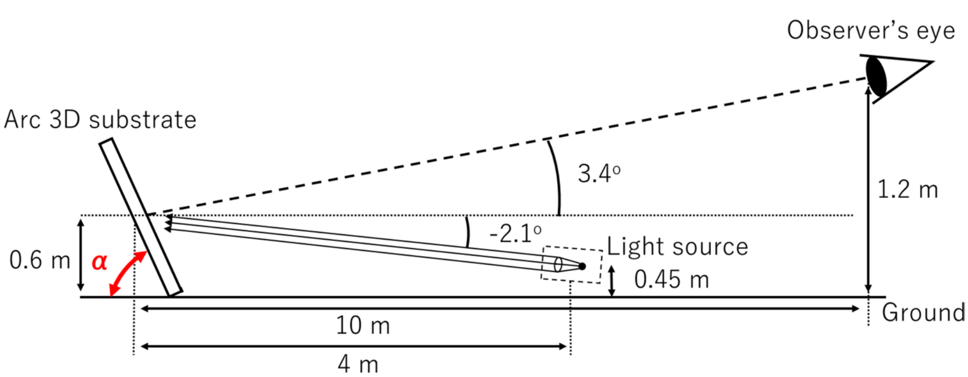


Fig. 12 Diagram of the experimental setup for the angular change of the arc 3D substrate

distance was 4 m from the substrate and the height was 0.45 m from the ground. The height of the substrate was set at 0.6 m from the ground. In this case, the observation angle was 3.4 degrees and the light source angle was − 2.1 degrees. Under these conditions, we measured the floating distance at substrate angles of 60 degrees, 65 degrees and 75 degrees.

4.2 Light source angle changes

The environment of the experiment for changing the angle of the light source is shown in Fig. 13. The observation distance was 10 m from the substrate and the eye height was 1.2 m from the ground. The illumination distance of the light source was 4 m from the substrate, the substrate height was 0.6 m from the ground, and the angle was set at 70 degrees. In this case, the observation angle was 3.4 degrees. Under these conditions, we measure the change in the floating distance when the angle of the light source was changed. The angle of the light source was adjusted by changing the height of the light source.

The angle of the light source was − 2 degrees when the light source height was 0.46 m to illuminate with a height of 0.6 m. When the light source height was 0.39 m to illuminate with a height of 0.6 m, the angle was -3 degrees; when the light source height was 0.32 m to illuminate with a height of 0.6 m, the angle was − 4 degrees; when the light source height was 0.25 m to illuminate with a height of 0.6 m, the angle was set to − 5 degrees. The floating distance was measured at these four angles.

4.3 Observation angle changes

The environment of the experiment for changing the observation angle is shown in Fig. 14. The observation distance was 10 m from the substrate, the light source illumination distance was 4 m from the substrate, and the height was 0.45 m from the ground. The height of the substrate was 0.6 m from the ground and the angle was set at 70 degrees. In this case, the light source angle was − 2.1 degrees. Under these conditions, we measure the change in the floating distance when the angle of the observation is changed. The observation angle was adjusted by changing the height of the head of observer. We measured the floating distance at three different observation angles of 3.4, 4.6, and 5.7 degrees at 1.2, 1.4, and 1.6 m height from the ground.

5 Results

Figure 15 shows the analysis results of floating distance when the angle of the arc 3D substrate is changed. Both the theoretical value of the floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment confirm that the closer the arc

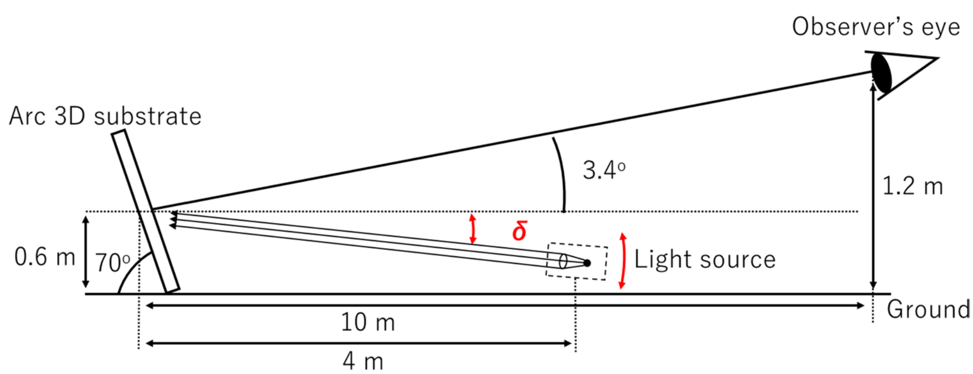


Fig. 13 Diagram of the experimental setup for the angular change of the light source

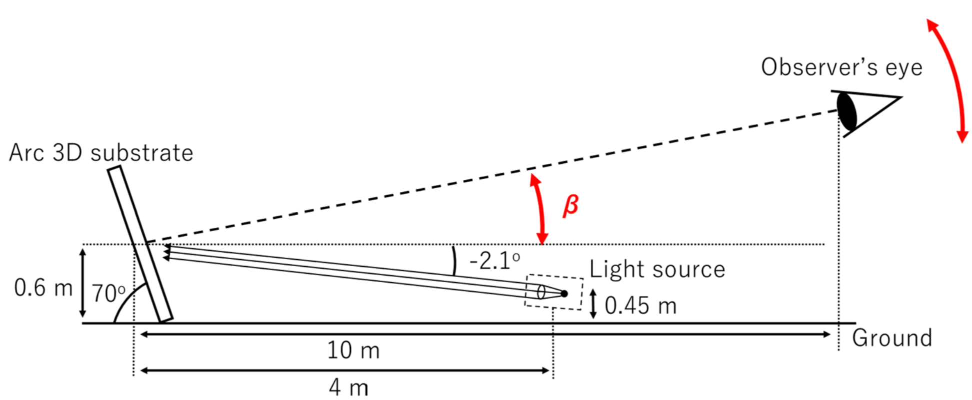


Fig. 14 Diagram of the experimental setup for the angular change of the observation

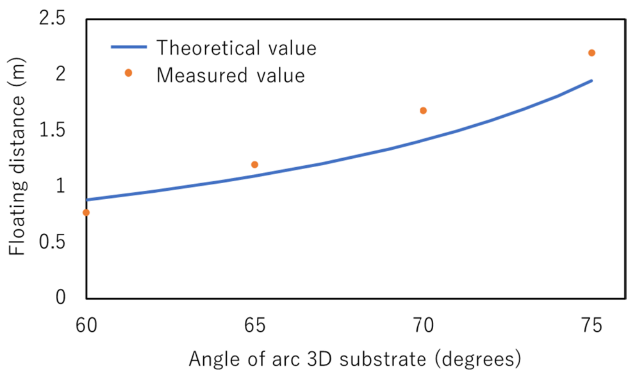


Fig. 15 Measurement results of the floating distance for the angular change of the arc 3D substrate

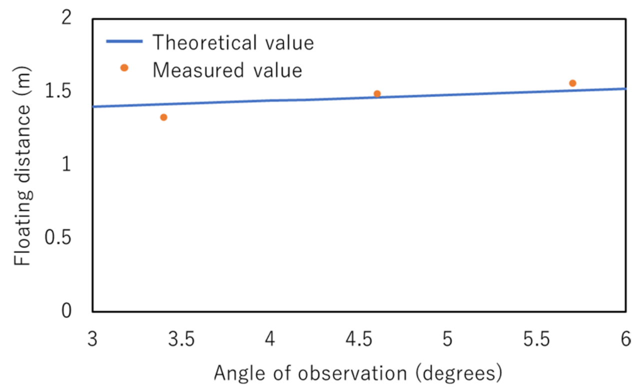


Fig. 17 Measurement results of the floating distance for the angular change of the observation

3D substrate is perpendicular to the ground, the larger the floating distance of the floating image becomes. The difference between the theoretical floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment is at most 0.27 m at an arc 3D substrate angle of 70 degrees.

Figure 16 shows the analysis results of floating distance when the light source illumination angle is changed. Both the theoretical value of the floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment confirmed that the floating distance of the floating image gradually increases as the light source illumination angle is increased. The difference between the theoretical value of the floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment was at most 0.16 m at the light source angle of − 5 degrees.

Figure 17 shows the analysis results of floating distance when the observation angle is changed. Both the theoretical value of the floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment confirmed that the floating distance of the floating image gradually increases as the observation angle is increased. The difference between the theoretical value of the floating distance obtained by the first-order approximate equation and the measured value obtained by the experiment was at most 0.09 m at the observation angle of 3.4 degrees.

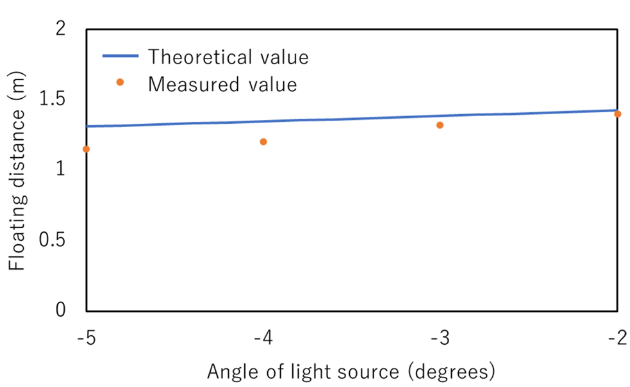


Fig. 16 Measurement results of the floating distance for the angular change of the light source

In the measurement of each parameter, the distance between the observer and the perceived aerial image is about 7 to 8 m. This is considered to have caused a small discrepancy between the theoretical and measured values. In this experiment, the maximum difference between the theoretical and measured values is about 0.27 m for this large distance, confirming that the discrepancy between the measured and theoretical values is very small as compared to the large observation distance. This shows that a good approximation can be obtained with a first-order approximation equation.

6 Discussion

An approximation equation was obtained to simply calculate the floating distance on arc 3D display from parameters under actual observation conditions. In order to reduce the number of direction signs to be installed in the future by using arc 3D display as a direction sign, the realization of floating images with a floating distance as long as 20 m is a significant issue. The first-order approximate equation derived in this paper is considered important because it is necessary to conduct future research on the floating distance using arc 3D display. In this experiment, we evaluated that the first-order approximate equation is a good approximation because the difference between the theoretical value and the measured value is small compared to the observation distance. The change in floating distance as the arc 3D substrate changes is larger than the change in floating distance as the light source illumination angle and the observation angle change. This is because the change in the angle of the arc 3D substrate simultaneously changes α, which is expressed as the angle of the arc 3D substrate and φ, which is expressed as the illumination angle of the light source in Eq. (13). Therefore, the installation angle of the arc 3D substrate is considered to be important for increasing the floating distance.

The problem of floating image perception is a problem in substituting an arc 3D display as direction signs. In the arc 3D display, the observation distance must be at least twice the floating distance to perceive the floating image, so the observation distance must be at least 40 m to realize a floating image that floats 20 m. However, there is the problem of whether or not large floating images can actually be perceived at such long observation distances. Arc 3D displays use binocular disparity and motion disparity to perceive floating images, but it has been confirmed that binocular stereopsis is effective in-depth perception at observation distances of approximately 10 m or less [10] so it is thought that there is an upper limit to stereopsis based on binocular disparity. It has been confirmed that motion parallax is important for stereopsis at farther viewing distances if the motion speed is appropriate [10], but it is unclear whether stereopsis is effective for floating images of several tens of meters. It is also possible that the fusion of motion parallax and binocular disparity enables the perception of larger floating distances, but it has been shown that crossed disparity (which is usually perceived in the front) has a limited amount of disparity compared to non-crossed disparity (which is usually perceived in the back) [11]. Thus, when the observation distance and floating distance are long, there is considered to be an issue with floating image perception.

Therefore, it is necessary to conduct experiments to elucidate the cognitive characteristics of humans with respect to arc 3D displays in order to clarify whether large floating images can be perceived by binocular disparity, motion disparity, or a fusion of both.

7 Conclusion

We have derived an equation that enables to determine the floating distance of the floating image from the actual positions of the light source and observer and the radius of the arc when the substrate is installed at an inclined angle in the arc 3D display. We conducted experiments to measure the floating distance of the floating image at each change of angle to evaluate the accuracy of the proposed equation. The measured and theoretical values are close for all angular changes, showing that a good approximation can be obtained with the first-order approximation equation. This equation makes it possible to simply calculate the floating distance on the arc 3D display from parameters under actual observation conditions. Therefore, this equation can be used in future studies of application as directional sign when it is necessary to analyze the floating distance on the arc 3D display.

Authors contributions HO contributed for this paper as 1st author. He derived the equations and conducted the experiments and wrote the original draft. KF, MY and SS checked the equations and edited the manuscript. HY designed the experiments and edited the manuscript.

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Data availability The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare no conflicts of interest associated with this manuscript.

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