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The lattice world, quantum foam and the universe as a metamaterial

The use of black holes as hyper-resolving microscopes to probe the fine structure of space

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Abstract This work describes the gravitational effects of an ordered array of Planck masses, arrayed in a cubic lattice with a lattice constant equal to the Planck length. This work adds the property of spatial order to the concept of quantum foam that was introduced by John Wheeler over 50 years ago. It is shown that this ordering results in new gravitational phenomena that affect elementary particles with masses close to the Planck mass. The only known elementary particles with a mass comparable to or greater than the Planck mass are black holes. Calculated in this work are the energies of particles within the crystal, their dispersion curves, group velocities and effective inertial masses. It is shown that, for particles having particular energies and momenta, the crystal can modify the inertial mass such that it is no longer equal to the gravitational mass—a violation of Einstein's equivalence principle in appearance only. Under certain conditions, particular particles can have a negative effective inertial mass such that it is pushed by the pull of an external gravitational force produced by sources other than the crystal. The connections between the effects of the gravity crystal and dark energy and dark matter are discussed. Also discussed is how to determine the properties of the universe-wide gravity crystal by studying the motion of black holes and galaxies—in effect using black holes as hyper-resolving microscopes to study the fine structure of the universe.

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1 Introduction

In this paper, the concepts of the fundamental length of space (i.e., the Planck length $l_p = \sqrt{\hbar G/c^3} = 1.62 \times 10^{-35}$ m), Werner Heisenberg's "Lattice World" and John Wheeler's quantum foam theory will be combined, resulting in a universe-wide gravity crystal (GC) that permeates all space (Fig. 1). The GC will manifest itself in the gravitational and inertial properties of point-like particles that have masses on the order of the Planck mass $m_p = \sqrt{\hbar c/G} = 2.18 \times 10^{-8}$ kg. By using well-developed concepts and techniques from the fields of quantum electrodynamics and solid-state physics, it will be shown that the inertial masses (m_i) of black holes become different than their gravitational masses (m_g) when they have particular momenta. For particular ranges of momenta, m_i can have values either much larger or smaller than m_g, be near zero, or even be negative. The possibility of non-equality of inertial and gravitational masses of particles—a violation of Einstein's equivalence principle in appearance only—is not a surprising result given the fact that the particles are traveling through a material, namely the GC. Such a phenomenon occurs for electrons in regular crystals (e.g., silicon, germanium) and is a well understood and experimentally verified phenomenon. As will be discussed in this paper, the momentum-dependent inertial masses of black holes may have a connection to dark matter, dark energy and other anomalies observed in the motion of black holes and galaxies. Also discussed in this paper is how one can use black holes as hyper-resolving microscopes to study the properties of space at length scales comparable to l_p.

This paper is organized as follows. In Sect. 2, a short summary is given of relevant episodes of the 2500-year-long history of the study of the fine structure of space. In Sect. 3, an introduction is given for the concept of

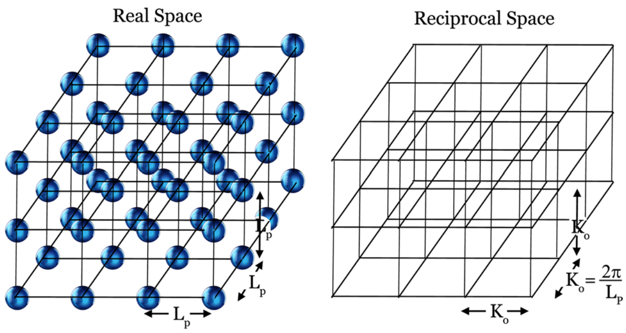

Fig. 1 *Left* A schematic of the Leopold crystal—the universe-wide GC. The basis (the *blue spheres*) is assumed to be particle of mass m_p that produces a spherically symmetric $1/r$ potential energy profile. The crystal is cubic in structure with a lattice constant of l_p. *Right* The dual or reciprocal lattice, which is also cubic with a lattice constant of $2\pi/l_p$

gravitational flux, gravitational permeability and the relation between gravitational flux and field (namely the constitutive equation). Also, the effective mass theory commonly used to describe electron motion in regular crystals will be adopted to describe the motion of point masses in the GC. In Sect. 4, standard methods to calculate energy band diagrams in crystals will be used, including the non-relativistic empirical pseudopotential method (EPM) and a *relativistic* Kronig–Penney model. Dispersion curves of a particle within the GC are calculated, as well as the particle's group velocity and inertial mass as a function of momentum and gravitational mass. In Sect. 5, the results of the calculations will be discussed along with the gravitational and inertial properties of black holes.

2 History of the inquiries on the fine structure of space

The discussion of whether space itself is discrete (i.e., atomized) or continuous (i.e., infinitely divisible) is as old as Democritus's atomic hypothesis of materials [1–3]. Starting in the fifth century BC, Parmenides and his student Zeno of Elea inquired about the nature of space, change, motion and plurality [4]. Their inquiries resulted in the famous Zeno's paradoxes that address motion, time and change. Many later philosophers and physicists have tried to resolve these and other paradoxes concerning time and space; these philosophers include Rene Descartes, Leonhard Euler, Gottfreid Leibniz, George Berkeley, David Hume, David Hilbert, and many others have contributed to this discussion, as described by Amit Hagar in [1]. However, despite over twenty-five centuries of philosophical musings on the matter by these and other philosophers, Hagar concludes that logic alone cannot disqualify either the discrete picture or the continuous picture of space [5].

If space–time is discretized, it is generally believed that the most probable atom of space (i.e., the smallest allowable length of space) is l_p, and the atom of time is $t_p = l_p/c = 5.39 \times 10^{-44}$ s [1]. Other Planck values include: the Planck mass m_p being the most mass a point particle can have, Planck charge $q_p = \sqrt{\hbar c}$ (in Gaussian units), Planck temperature $T_p = m_p c^2/k_B$ (where k_B is Boltzmann's constant) and others. It is often believed that the question of the discretization of space is not testable, either now or in the future, because of the extraordinarily small size of l_p. However, as will be shown in this work, the effects of the discreteness of space will manifest itself in measurable phenomena, namely the motion of black holes.

In this work, the role played by Heisenberg's lattice world is relatively minor but nonetheless important. In 1930, Heisenberg introduced the concept of space being discretized in unit cells of size of r_o^3 with $r_o = 10^{-15}$ m [6]. Along with this discretization, he proposed to replace the differential equations used in physics with difference equations. Niels Bohr and Wolfgang Pauli pointed out obvious problems with the concept, including the breaking of isotropy of space, and non-conservation of energy and momentum [7, 8]. Additionally, it appeared that the concept of a fundamental or smallest length breaks Lorentz invariance—this ostensibly smallest length would be Lorentz-contracted to a yet smaller value in a moving reference frame. In spite of these problems, the lattice world concept was further studied by Heisenberg and others, including Matvei Bronstein [9]. In this work, the concepts of a fundamental length and crystalline order will be used, but at a length scale twenty orders of magnitude smaller than what Heisenberg proposed.

The final and most important historical concept exploited in this work is John A. Wheeler's concept of quantum foam that he introduced in 1957 in an attempt to show that all of classical physics, particle physics included, is "purely geometrical and based throughout on the most firmly established principles of electromagnetism and general relativity" [10]. Wheeler made use of well-developed concepts in quantum electrodynamics, in which the probability amplitude of a transition of a system from an initial configuration to a final configuration is obtained by summing the Feynman–Huygens exponent over all possible histories:

$$\langle C_2\sigma_2|C_1\sigma_1\rangle = \sum_H e^{iI_H/\hbar} \tag{1}$$

In order for a particular history to contribute to the transition probability, the phase of the exponent in Eq. (1), which is dependent on the metric g and the electric vector potential **A**, should be small (~ 1 radian or less) to avoid destructive interference. This limits fluctuations in g (*i.e.,* Δg) that can occur over a volume of space–time L^4 to be on the order of $\Delta g \sim l_p/L$, where l_p is the Planck length, and fluctuations in **A** to be on the order of $\sqrt{\hbar c}/L$. The fluctuations in the metric Δg remains small relative to g until L approaches l_p, at which point, $\Delta g \sim g$ and "the character of the space undergoes an essential change... and multiple connectedness develops" [10] that results in a disordered collection of "wormholes" in the topology of space with a spacing of approximately l_p; he calls this wormhole collection "quantum foam".

Each wormhole has an associated pair of positive and negative electrical charges of charge $\pm q_{\text{Planck}} = \pm\sqrt{\hbar c}$ that produces an intense electromagnetic field energy E that has associated with it a mass of $m = c^{-2}E = m_p$. After further discussion, he summarizes his findings as follows [10]:

1. Quantization of the physics of Maxwell and Einstein forces upon space a foam-like structure... on the scale of l_p.
2. In the vacuum, virtual pairs of charges are being continually created and annihilated.
3. With these pairs are associated charges and especially masses "far larger than anything familiar from the elementary particle problem" [10].
4. The gravitational energy density is negative and approximately equal in magnitude to the positive electromagnetic energy density, and "circumstances are favorable for the local *compensation* of electromagnetic energy by gravitational energy," such that "to the extent this compensation holds locally, nearby wormholes exert no gravitational attraction on remote concentrations of mass-energy."

Item 4 states that the formation of the foam is an energy conserving process. This property makes these fluctuations highly probable to occur and contribute to the transition probability. However, in this work, it will *not* be assumed that "nearby wormholes exert no gravitational attraction on remote concentrations of mass-energy." Also, in this work it will be assumed, contrary to Item 2, that these wormholes (i.e., Planck masses) are stable in time. And most importantly in this work, rather than a random distribution of these wormholes with an average spacing of l_p as Wheeler assumed, a regular cubic lattice of lattice constant l_p will be assumed (Fig. 1). Note that the GC does not need to have a cubic structure to produce the gravitational anomalies described later in this work—the same anomalies will occur for other lattice structures, such as dodecahedron [11].

Thus, we arrive at the GC described in this work—a lattice with a spatial period (i.e., lattice constant) that is the smallest that nature allows, namely the Planck length l_p, and a basis composed of an elementary point particle with a mass that is the most that nature allows, namely the Planck mass m_p. This crystal, which I call the Leopold crystal, is assumed to fill up all space with no defects of any kind, including edges. Any dislocations or interstitial sites would necessarily involve distances less than l_p—something thought not to be possible. It is well known that an elementary particle with mass m_p has a reduced Compton wavelength $\lambda_c = \hbar/m_p c$ that is equal to l_p. With the λ_c being the fundamental limit on measurements of the position of a particle, the question as to where the Planck particle is within unit cell of volume l_p^3 is unanswerable. Also, because the lattice constant l_p is one-half of Schwarzschild radius ($R_c = 2Gm_p/c^2$), the unit cells (composed of a Planck particle) form an array of quantum black holes, each with the smallest allowable mass for a black hole.

To get a sense of what particles would experience the GC to any significant degree, one can compare the relative magnitudes of a particle's kinetic energy $KE = \hbar^2|\mathbf{k}|^2/2m$ and gravitational potential energy $|V| = Gmm_p/l_p$, with m being the gravitational *and* inertial mass of the particle. In the physics of crystals, $|\mathbf{k}|$ can be assumed to be within the first Brillouin zone (BZ), i.e., $-\pi/l_p <= |\mathbf{k}| <= \pi/l_p$, and that the effects of the crystal most often manifest themselves at the BZ boundary of $|\mathbf{k}| = \pi/l_p$. Thus, when an elementary particle's mass is large enough, namely when $m = (\pi/\sqrt{2})m_p = 2.22m_p, KE$ will equal $|V|$. Particles with less (more) mass experience the effects of the crystal to a lesser (greater) degree. Since we have already stated that masses greater than m_p within a volume l_p^3 are not possible, let us consider particles within this crystal having a mass m_p. Such a mass, namely m_p, is very large—almost no elementary particle has a mass close to this value. The exceptions to this are black holes, and to a much lesser extent the Higgs Boson ($m = 2.25 \times 10^{-25}$ kg), top-quark ($m = 3.09 \times 10^{-25}$ kg), and exceptionally high-energy cosmic rays. All heavier entities (e.g., atoms, molecules, neutron stars) have their mass distributed over a volume such that the mass density is much less than m_p/l_p^3. Black holes are predicted to compress the mass of many stars to a single, infinitesimally small point. Even if this mass is not compressed to a singularity but instead limited to a density of $\rho_{\max} = m_p/l_p^3$ (as assumed later in this work), the inertial properties of the black hole will be significantly influenced by the GC.

3 Gravitational flux and effective mass model of gravity

Non-relativistic gravitational fields produced by point masses have a spatial dependence the same as electric fields produced by point charges, and thus, the tools used in effective mass theory of crystals can be used for gravity. One can start with the equation:

$$\nabla \cdot \mathbf{D} = 4\pi G \rho_{\text{free}} \tag{2}$$

Because the right side of Eq. (2) only contains the free mass density ρ_{free} produced by non-lattice entities (e.g., stars, planets, electrons) and not the masses comprising the crystal, $\mathbf{D}$ is not the gravitational field but is instead the gravitational displacement (or flux density). The total gravitational field $\mathcal{G}_{\text{total}}$ is given by sum of the gravitational field $\mathcal{G}_{\text{free}}$ produced by non-lattice entities (again stars, planets, electrons) and the gravitation field $\mathcal{G}_{\text{lattice}}$ produced by the GC.

Similar to electromagnetics, we can relate the $\mathbf{D}$ and $\mathcal{G}_{\text{total}}$ via a constitutive equation with a constant of proportionality we can call the gravitational permeability μ_g. Also, if *all* the gravitational forces on a particle (that is traveling within the crystal) are taken into account and summed, then this sum (under non-relativistic conditions) is equal to the product of its gravitational mass m_g and acceleration ($m_g\mathbf{a} = m_g\mathcal{G}_{\text{total}}$) and is in agreement with Einstein's equivalence principle. These two relations lead to:

$$\mathbf{D} = \mu_{\mathbf{g}}\mathcal{G}_{\text{total}} \quad \mathbf{a} = \mathcal{G}_{\text{total}} \tag{3}$$

In effective mass theory, the forces due to external (*non-lattice*) sources, such as an electric field produced by an applied voltage, are equal to the product of the effective mass and acceleration. Similarly to gravitational forces, we can write:

$$\mathbf{F} = m_i\mathbf{a} = F_{\text{external}} = m_g\mathbf{D} \tag{4}$$

Substituting Eq. (3) in Eq. (4) leads to the following relation:

$$\mu_g = m_i/m_g \tag{5}$$

The value of m_g is a constant, but the next section will describe methods to calculate m_i, and it will be shown that m_i is dependent on the momentum of the particle and can take on values much different than unity (what μ_g would be absent the lattice), including positive values much greater than m_g, values near zero, and can also be negative.

4 The band diagram of the universal gravity crystal

One can draw upon the tools (long developed and experimentally verified) from the fields of solid-state physics and crystallography. For particles with $m < m_p$, the particle only weakly experiences the effects of the crystal and one can use the nearly free particle model or empirical pseudopotential method (EPM) to calculate the energy band diagram [12, 13]. For particles with $m > 2.22m_p$ one has to use the tight-binding (TB) model or another model that assumes that the particles are predominately localized around one lattice site and only weakly experiences the Planck particles at neighboring lattice sites. The TB model has been used to model particles with $m > 2.22m_p$, but due to space constraints, this work cannot be included in this paper but will be made public elsewhere. The same qualitative behavior of m_i is seen using the TB model as compared to the EPM model. Namely, the TB model predicts non-equality of m_i and m_g, and a momentum dependence for m_i such that it can be much larger than m_g, be negative or near zero. With either the EPM or TB models, one starts with the time independent form of Schrodinger's equation where $V(\mathbf{r})$ includes the gravitational potential energy profile produced by *all* the particles comprising the crystal:

$$\frac{\hbar^2}{2m_g}\nabla^2\psi + V(\mathbf{r})\psi = E\psi \tag{6}$$

Identical to what is done with the periodic Coulombic potential energy profile created by the positively charged nuclei in crystal, $V(\mathbf{r})$ in Eq. (6) for an electrically neutral particle of mass m_g is:

$$V(\mathbf{r}) = -Gm_g m_p \sum_{\mathbf{R}} \frac{1}{|\mathbf{r} - \mathbf{R}|} \tag{7}$$

where m_g is *both* the inertial mass and gravitational mass of the particle in all the terms in Eqs. (6) and (7), m_p is the mass of each particle comprising the crystal, and $\mathbf{R}$ are the spatial translation vectors $\mathbf{R} = n_1\mathbf{a_x} + n_2\mathbf{a_y} + n_3\mathbf{a_z}$ with n_1, n_2 and n_3 being integers, and $\mathbf{a_x} = l_p\hat{x}, \mathbf{a_y} = l_p\hat{y}, \mathbf{a_z} = l_p\hat{z}$ are the primitive spatial translation vectors for the cubic lattice.

The Fourier transform of $V(\mathbf{r})$ is:

$$V(|\mathbf{K}|) = \frac{1}{l_p^3}\int_{\text{unitcell}} e^{-i\mathbf{K}\cdot\mathbf{r}}V_o(\mathbf{r})\,\mathrm{d}\mathbf{r} = -\frac{2\pi m m_p G}{l_p}\text{sinc}^2(|\mathbf{K}|) \tag{8}$$

with $\mathbf{K}$ being the reciprocal lattice vectors. Because the periodicity of the structure, the wave functions of the particles are Bloch functions:

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{\mathbf{K}'} U(\mathbf{K}') e^{i\mathbf{K}'\cdot\mathbf{r}} \tag{9}$$

Inserting Eqs. (8)–(9) into Eq. (6) yields the matrix equation:

$$\left[\frac{\hbar^2(\mathbf{k}+\mathbf{K})^2}{2m}\delta_{\mathbf{K},\mathbf{K}'} + \sum_{\mathbf{K}'} V(|\mathbf{K}-\mathbf{K}'|)\right] U(\mathbf{K}') = EU(\mathbf{K}) \tag{10}$$

One then simply solves Eq. (10) for the eigenvectors $U(\mathbf{K})$ and eigenvalues E along particular directions in $\mathbf{k}$-space (see Fig. 2) and plots the energy of the modes as a function of $|\mathbf{k}|$. Once the dispersion curves ($\omega - \mathbf{k}$ curves) are calculated, the effective inertial mass can be calculated by the well-known formula [12, 13] given below, and plotted in Fig. 3 as a function of $|\mathbf{k}|$.

$$\frac{1}{m_i} = \frac{1}{\hbar^2}\frac{\partial^2 \mathcal{E}_n(\mathbf{k})}{\partial k_i \partial k_j} \tag{11}$$

It is seen from Fig. 2 that the bands are parabolic near $k = 0$ (*i.e.,* the Γ point in momentum space), but near the BZ boundaries at X, M and $R, \partial^2 E/\partial k^2$ changes sign, leading to sharp changes and negative values for m_i. The salient point is that the mass given by Eq. (11) is the *inertial mass* m_i affecting properties such as acceleration, momentum and kinetic energy and *not* the gravitational mass m_g in Eqs. (7) and (8) involving the gravitational interaction between particles. Thus, for particles that experience the effects of the crystal, there will seemingly be a violation of Einstein's equivalence principle, in that m_i can be different than m_g. However, this violation is in appearance only and results from the fact that one typically calculates a particle's acceleration in space using only non-lattice contributions to the gravitation field. Also, with the system being comprised of the particle *and* the crystal, overall conservation of energy and momentum is maintained; any energy or momentum given up by the particle is taken up by the crystal. Thus, the motion of a particle with negative effective inertial mass will be accompanied by gravity waves within the crystal that propagate at some speed away from the particle, presumably c.

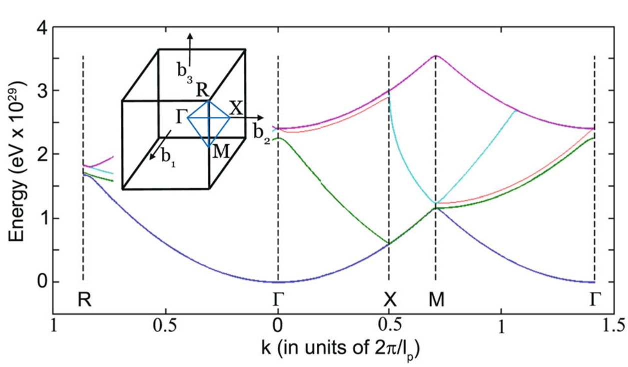

Fig. 2 Energy–momentum band diagram calculated using the EPM model. The structure of the GC is given in Fig. 1, and the particle is electrically neutral and has a gravitational mass of m_p. *Inset* One unit cell of reciprocal space showing the crystal directions

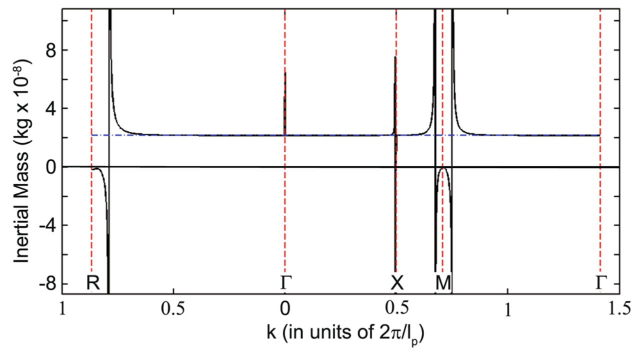

Fig. 3 Inertial mass m_i (*solid black line*) as a function of momentum for an electrically neutral particle with gravitational mass $m_g = m_p$ (*horizontal dashed blue line*). It is seen that near the BZ boundaries, m_i goes from being nearly equal to m_g (which is a constant $m_p = 2.18 \times 10^{-8}$ kg) to much greater than m_g and then changing sign to become negative

Once m_i has been calculated, all the effects of the crystal are contained within it—the potential energy profile created by the array of Planck masses can be ignored and only non-crystal sources of gravitational potential energy will contribute to $V_{\text{external}}(\mathbf{r})$ in Schrodinger's equation, and m_i is used in the denominator of the kinetic energy term in Schrodinger's equation:

$$-\frac{\hbar^2}{2m_i}\nabla^2\psi + V_{\text{external}}(\mathbf{r})\psi = E\psi \tag{12}$$

Both the EPM model and the TB model presented above are non-relativistic models. Even though the goal of this work is not to provide precise values for m_i for particular particles but only to introduce the gravitational and inertial anomalies that the GC produces, doubt can be cast on validity of these non-relativistic models. This is because, for particles with momenta close to the BZ boundary of $k = \pi/l_p$ and with a mass m_p, their velocities approach c. To demonstrate that the same phenomena are predicted using a relativistic method, a relativistic Kronig–Penney (KP) model described by Strange [14] will be used. The starting point is the one-dimensional Dirac equation:

$$\left(\frac{c\hbar}{i}\tilde{\alpha}_x\frac{\partial}{\partial x} + \tilde{\beta}mc^2 + V(x)\right)\psi(x) = W\psi(x) \tag{13}$$

where α and β are given below,

$$\tilde{\alpha}_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \tilde{\beta}_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{14}$$

ψ is a Bloch wave similar in form to Eq. (9) with the additional complexity of being a 4-vector. ψ also has to obey the Bloch boundary conditions:

$$e^{ika}\psi(x) = \psi(x+a) \tag{15}$$

Defining a value E as $E = W - mc^2$ to eliminate the rest energy, and applying the all necessary boundary conditions throughout one period of the structure, leads to the following equation:

$$\cos(ka) = \cos(k_1 b)\cos(k_2(a-b)) - \frac{\Gamma^{-1}+\Gamma}{2}\sin(k_1 b)\sin(k_2(a-b)) \tag{16}$$

where k_1, k_2 and Γ are given by:

$$k_1^2 = \frac{(E+V_o+2mc^2)(E+V_o)}{c^2\hbar^2} \qquad k_2^2 = \frac{E(E+2mc^2)}{c^2\hbar^2} \tag{17}$$

$$\Gamma = \frac{E+V_o}{E}\frac{k_2}{k_1} \tag{18}$$

Shown in Fig. 4 is the energy–momentum band diagram for a one-dimensional period potential well structure with a period a equal to l_p, a well width b of $0.952 l_p$ and the well depth of $V_o = Gm_p^2/l_p$. The same qualitative behavior observed with the non-relativistic EPM model is observed with the relativistic KP model, namely the occurrence of large positive, near zero and negative inertial mass as the momentum of the particle approaches the BZ boundary at $k = \pi/l_p$, as shown in Fig. 5.

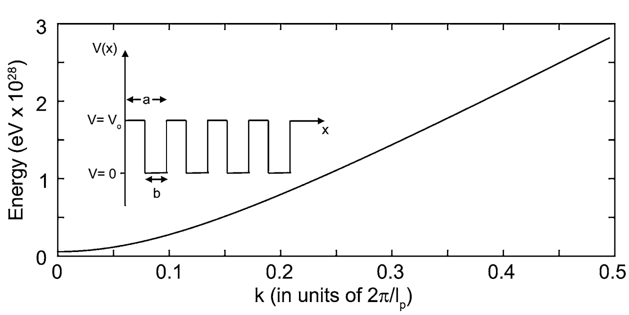

Fig. 4 Energy–momentum band diagram calculated using the one-dimensional relativistic KP model. The energies of the states are relative to the bottom of the potential wells. The structure is described in the text and a few unit cells are shown in the *inset*

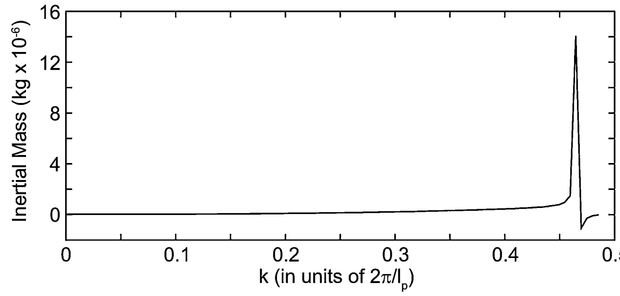

Fig. 5 Effective inertial mass m_i for a particle with gravitational mass $m_g = m_p$ width varying momentum. Similar to what is predicted by the non-relativistic EPM model, the relativistic KP model predicts large positive, near zero and negative inertial mass as the momentum of the particle approaches the BZ boundary at $k = pi/l_p$

5 Discussion

For the purposes of presenting this work at the Meta'15 conference, similarities were described between the properties of GC and electromagnetic metamaterials. The accepted definition of metamaterials given by Rodger Walser is that they are "...macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation." Additionally, the materials should: be periodic composite materials, have constituent elements that are substantially smaller than the wavelength and have properties that can be described by materials parameters (e.g., ϵ, μ). The GC is periodic and is a composite of Planck particles and the void, its constituent elements are smaller than the wavelengths of any excitation, and its properties can be described by the parameter μ_g, and μ_g can take on values not commonly thought to occur in nature (i.e., negative and near zero values). Of course, the GC is not optimized by humankind for any application and not even "manmade." In fact, this metamaterial is not "meta" at all—it is not beyond what nature provides but is the very fabric of nature, the very fabric of space–time. Nevertheless, the GC is an extraordinary material that is omnipresent. If it can be harnessed or exploited in some way by future technologies, it would allow extraordinary advancements in many scientific and technological fields, including propulsion systems.

At present, the GC may be affecting the motion of black holes and the evolution of the universe. It may be that regular black holes (i.e., not quantum black holes of $m \sim m_p$) do not concentrate their mass to an infinitely small singularity but instead to a mass density of m_p/l_p^3, such a thing would be consistent with the view that the maximum mass an elementary particle can have is m_p and any particle contained within a volume of l_p^3 is necessarily elementary.

In this case, the models described in this work for particles with $m_g = m_p$ traveling within the GC are applicable and black holes' inertial properties will be significantly affected only when they have a very large velocity relative to the GC. Fast moving black holes may then have very large positive inertial masses and would therefore be relatively unresponsive to the gravitational field produce by all the stars and planets of the universe. At other momenta, a black hole's m_i may be negative. If, for example, a supermassive black hole at the center of a galaxy is moving fast away from the center of the observable universe, it may have a negative value for m_i that is approximately equal in magnitude to its gravitational mass m_g. If this is the case, then the only observable effect may be that fast moving galaxies (perhaps at the edges of the observable universe) may be increasing their acceleration away from the center of the universe. Another example is for a supermassive black hole has a momentum such that its inertial mass is near zero; in this case the black hole may be ejected from the parent galaxy.

If black holes can compress significantly more mass than m_p into a volume of l_p^3, then the models described in this work, and the accompanying work using the TB model, predict that the black hole will become trapped around one or several lattice points of the GC. If too massive, the black holes may generate defects in the GC or locally obliterate the GC. Either way, by observing the motion of fast moving black holes, one should be able to assess the structure and properties of any universe-wide GC, in effect using black holes as a probe to study the fine structure of space.

Additionally, one can look at large-scale structural features of the universe for experimental evidence of the discretization of space. Similar to regular crystals, the GC may possess crystal "facets" at its surface. The existence of facets of the universe will leave tell-tale patterns on the cosmic microwave background (CMB) as recently recorded by the Wilkinson Microwave Anisotropy Probe (WMAP) [11]. The orientations of these enormous facets can provide information about the minuscule lattice and unit cell as to whether the lattice type is cubic, dodecahedron or something else.

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